

# **Measurement of SME credit risk using different default criterions**

**Michel DIETSCH\***  
**Université Robert Schuman of Strasbourg**  
47, avenue de la Fôret Noire,  
67000 Strasbourg - France

## **Abstract**

In the Basel II capital reform, the Basel Committee defined default as any credit loss event associated with any obligation of the borrower, including a very large varieties of defaults, going from a lender's simple doubt to legal bankruptcy. The objective of this paper is to compare measures of credit risk computed by using three different definitions of default : bank loan default, default in payment of legal liabilities to institutional creditors, and legal bankruptcy. We take the distance to default as the measure of credit risk. Data cover a large population of 100.000 French firms regularly monitored by Coface, a large French credit insurance company, what allows to know day by day situation of firms regarding the different types of default. Our results show that, at least in the French SME population, ratings systems calibrated using legal bankruptcy or bank loan default criterions give quite close measures of the borrower's credit risk. In fact, the degree of proximity of distances to default depends largely on firm's size and on institutional factors that determine the reaction of creditors.

\*Correspondance to : [michel.dietsch@urs.u-strasbg.fr](mailto:michel.dietsch@urs.u-strasbg.fr)

We thank Coface group, which provided the data, Gilles Baugey, Evelyne Guilly, Bill Lang, Nicolas Lemette, François Meunier, Vichett Oung, Arnaud Tisseyre and the participants at the "Basel II and Small Business Credit Risk" special session of the XII International Tor Vergata Conference on Banking and Finance, for their helpful comments. Remaining errors are our own.

## 1. Introduction

In Basel II bank capital ratio reform, the Basel Committee propose banks to compute their capital charges either by using a “Standard” approach or an “Internal Ratings-Based” (IRB) approach. Under the IRB approach, banks should provide estimates of probability of default for each loan of their portfolios. So, the IRB approach requires banks to develop their own models to assess credit risk and allocate economic capital to different segments of their portfolios. The Basel Committee defined default as any credit loss event associated with any obligation of the borrower, including distressed restructuring, involving the forgiveness or postponement of principal and interest, and delay in payment of the obligor of more than 90 days (BIS, 2001). According to the new Basel II accord, banks will have to use such definition of default for estimating internal rating-based models. However, current credit risk models in retail banking are characterized by a wide divergence in approaches (Lang and Santomero, 2002). In particular, large banks and credit bureaus have often calibrated their own credit scoring models by using legal bankruptcy criterion. The widespread reliance of this criterion raises the issue of the sensitivity of the outputs of credit score models to the definition of default. Because the likelihood of default change from one form of default to another one, banks could possibly benefit a competitive advantage if their internal models are calibrated using a bank default criterion that differs from the default criterions used by the other banks.

Theoretically, the issue is to know if different default criterions give different distances to default. According to Merton's structural model of default (1974), a borrower defaults when the value of its assets falls below the value of its liabilities. Under this structural approach, the distance to default is a measure of a borrower's leverage relative to the volatility of its asset values. As the value a borrower's assets changes over time, its distance to default changes as well. If assets fall below the value of fixed liabilities, the distance to default drops, and the borrower becomes insolvent. Given assumptions about the asset return process, a borrower's distance to default is all what is needed to determine its default probability at a given future date (Gordy and Heitfield, 2002, Nickell and ali, 2000). A direct measure of distance to default is difficult to implement, in that it supposes to know with sufficient precision economic characteristics of the assets and liabilities of each borrower. However, ratings systems provide indirect measures of

this distance. Generally, a borrower's current rating is taken to be a sufficient statistic for this structural measure of its credit quality. In other words, borrowers with the same rating grade are assumed to share the same distance to default.

Following this structural approach, the distance to default of a given borrower should be the same, whatever the nature of the liability. However, in practice, different creditors could be more or less prompt to declare borrower's insolvency, due to differences judgements about the borrower's capacity to pay, to differences in the protection given by the laws to creditors, and also to differences in the behaviour of courts. So, distance to default could differ from one type of default to another one, because of the existence of various institutional factors independent of the debtor's insolvency.

The aim of this paper is to try to give responses to these issues and to provide new empirical evidence on differences and similarities of distances to default, when different default criterions are used. To deal with this issue, we built three different ratings systems we calibrated by using different criterions of default : legal bankruptcy, bank loan default - 90 days past due on commercial paper remitted to discount – and default in payment of liabilities (mainly, taxes) due to institutional creditors : mainly tax department and social security system (which benefit from “*Privilèges*” in the collection of these liabilities). Ratings system were estimated on a large database containing about 100.000 French firms, among which a large proportion of small and medium-sized enterprises (SME). Data cover the 1998-2002 period.

An important lesson of this paper is that, at least for the French SME population, a ratings system calibrated by using legal bankruptcy default as criterion of default provides distances to default measures which are quite close to those computed by using bank loan default as default criterion. Nevertheless, distances are quite different when we build a ratings system by using the third form of default. We show that the degree of proximity of distances to default depends on differences in firm size, what reflects the existence of differences of legal and institutional conditions in which creditors take their decisions to declare the insolvency of their debtors. If creditors who assign grades to the same borrowers have in mind, on average, the same distances to default for the same borrowers, institutional factors could affect their perception determining their promptness

to declare debtors insolvency or bankruptcy. The paper measures also the potential effect of the choice of default criterion on the level of bank capital charges induced by the new Basel II risk weight formulas. We show that different criterions of default could nevertheless induce very similar level of charges, due to the structure of the distribution of the borrowers in the different grades of different ratings systems. To our knowledge, only one paper explores the same issue in the recent literature (Hayden, 2002).

The paper is organized as follows. Section 2 presents the data and the three credit score models we used to build ratings systems and assign grades to borrowers. Section 3 illustrates the relative proximity of distances to default. Section 4 tries to provide some explanation of the distance between the different distances to default measures associated to different default criterions. Section 5 measures the impact of different definitions of default on the banks capital charges induced by the new Basel II risk weight formulas. Section 6 concludes.

## **2. The data and the three ratings systems**

The database comes from Coface, a large French credit insurance company. It contains information about the solvency of a panel of around 100.000 French firms, mostly composed of SME<sup>1</sup>, which were regularly monitored by Coface, what allows to know day by day borrower's situation regarding three different types of default : (1) legal bankruptcy, (2) bank loan default (that is 90 days past due on commercial paper remitted to discount) and (3) default in payment of State creditors, so-called "*Privilèges*" default, which corresponds to default in payment of legal liabilities (taxes and other liabilities) due to Public Treasury and Social Security System. Hereafter, this third type of default is called "privilèges" default. Database contains also balance sheet and other accounting information about the firms of the panel. The period of observation is the five years period 1998-2002. Default histories are available for a large number of firms and for each type of default.

---

<sup>1</sup> Following a quite conventional definition, SME are defined as incorporated firms with turnover under € 50 million (and over € 0,15 million).

A very large proportion (88 %) of firms belonging to the panel never defaulted, whatever the kind of default, over the period under study. Among the 12 % of firms that defaulted, the most frequent type of default is bank loan default. Moreover, the same firm can have encountered several types of default successively over the period. For instance, on average, 84 % of firms which encountered a banking default during quarter Q encountered another banking default during the following quarter Q+1. In this study, we choose to consider the quarter as the elementary period of observation.

We used available default histories to estimate simple point-in-time models of borrower default probability. More precisely, we estimated three different logit models of default calibrated on the three criteria of default. Following Carey and Hrycay (2001), we consider that the properties of such simulated ratings systems are likely to be representative of properties of internal ratings from systems which use a scoring model similar to ours to make rating assignments. In particular, dynamic properties are likely to be similar in that our scoring models have a one-year horizon and a “point-in-time” orientation.

We used the same limited number of independent variables – the same financial risk factors - as predictors of default in the three credit score models. Our goal in building default models is not to maximize model accuracy or to fine tune the variables set to improve models performance, but rather to provide relatively simple and easy to interpret models that would be applicable to a large fraction of typical borrowers and that could be relevant for the three types of default. It is always possible to improve model performances by introducing information that is specific to a lender. However, in our comparison exercise, it is first of all important to work with models that are broadly applicable. Here, in addition, broad applicability allows better comparisons between the three scoring models and the three ratings assignment systems, in that the underlying default model is the same.

Since Altman (1968), in the credit scoring literature, generally four types of variables explain a large proportion of the borrower default probability. First, higher leverage increases the borrower vulnerability to default. Second, borrowers with low profitability are also more vulnerable because generally low earnings today announce low earnings in the future. Third, higher level of financial charge in the operating income of the firm is also a signal of vulnerability and a good

predictor of financial difficulties in the present or in the future. Finally, less liquid borrowers have also a greater chance to default because they are more sensitive to liquidity crunches. Following these previous results in the literature, we introduced four set of variables in the three credit score models. The first two ratios measure the borrower leverage : (1) gearing ratio : equity / total balance sheet, and (2) coverage ratio : fixed capital and quasi-fixed working capital / long term stable financing. One ratio measures the profitability of the firm : (3) cash flows / turnover. One ratio measures the weight of financial charges (interest paid) in operating income. And three ratios measure the main components of borrower liquidity : (4) quick ratio, (4) delay of payment of customers in days and (5) delay of payment of furnishers in days. In addition, we introduce a ratio measuring the level of assets sales as compared to firm turnover. The reason is observation shows that sales of assets often precede default or bankruptcy.

We included two other variables as control variables in the logit models. The first is the sector to which each firm belongs, coming from the intuition that the borrower vulnerability depends also of the sensitivity of the industry as a whole to changes in current economic conditions. The second variable is the firm size, coming from the observation that, generally, large firms are less likely to default because they have a better access to various financing sources and a better diversification of their clients and products.

Parameter estimates of the three models are presented in Appendix A. All variables were transformed in several discrete variables by using the quartiles of each variable distribution. The logit models were ran with the stepwise option on a training sample and their performances were tested on a validation sample. All variables are significant and overall performances of the three models are good in terms of concordance ratios and of models capacity to discriminate defaulted from non-defaulted firms (Type I and II errors). Consequently, each credit score model produce quite satisfactory quantifications of grades.

Then, the three credit score models results were used to simulate rating grades assignment. We built three 10 grades ratings system, by dividing the probability interval into ten ranges of probability simply by taking deciles of scores. Again, our goal here is not to built the better ratings system in terms of credit decision and credit allocation to the best uses, but simply to

build the most homogenous ratings systems in order to compare the three types of distances to default. To compute default probability, we built two types of transition matrices taking each models scores : the first one was built with an horizon of a quarter and the second one with one year horizon. Then, we built one year horizon stationary transition matrixes over the 1998-2002 period by averaging annual matrices. These matrices are presented in appendix B.

### **3. The proximity of distances to default**

Each ratings system measures distances to default relative to its proper criterion of default and it assigns grades taking into account this distance. Because the default rate in the SME population differs from one criterion of default to another one, it is not relevant to compare directly the probabilities of default associated to the grades of the three ratings systems. However, ratings grades provide indirect measures of distances to default. Consequently, the objective of this section, is to show if the three “point-in-time” ratings systems give relatively close distances to default or, on the contrary, if the distances to default are very far from one type of default to another. This section begins by an illustration of the differences between probabilities of default. Then, we present two kinds of test we performed to assess the degree of proximity of distances to default given by the three default criterions.

#### **3.1 Rates of default and probabilities of default differences**

Probabilities of default should differ if default rates differ among criterions of default. To compute the probabilities of default, we built annual transition matrixes and we pooled these annual matrices over the period to get stationary probabilities of default. This process was repeated for each model of default (table 1).

Results in table 1 verify that the sets of legal probabilities of default diverge between the models. First, we can notice that the average bank loan rate of default is clearly higher than the average other default rates. Then, observation shows that the riskier borrowers - belonging to the 10<sup>th</sup> grade of a rating system - have an higher likelihood to encounter a bank loan default than a legal

bankruptcy default or a privilèges default. We neutralise these differences when we ran the following tests of distances to default proximity.

**Table 1 : Stationary probabilities of defaults in the three ratings systems**

<b>Ratings grades</b>	<i>legal bankruptcy model</i>	<i>bank default model</i>	<i>privilèges default model</i>
<b>1</b>	<b>0.19</b>	<b>1.29</b>	<b>0.33</b>
<b>2</b>	<b>0.60</b>	<b>1.80</b>	<b>0.52</b>
<b>3</b>	<b>0.31</b>	<b>2.08</b>	<b>0.89</b>
<b>4</b>	<b>0.71</b>	<b>2.80</b>	<b>0.90</b>
<b>5</b>	<b>0.77</b>	<b>3.14</b>	<b>1.38</b>
<b>6</b>	<b>0.69</b>	<b>3.93</b>	<b>1.20</b>
<b>7</b>	<b>0.79</b>	<b>5.34</b>	<b>2.05</b>
<b>8</b>	<b>2.00</b>	<b>7.41</b>	<b>2.46</b>
<b>9</b>	<b>3.90</b>	<b>7.60</b>	<b>4.12</b>
<b>10</b>	<b>10.61</b>	<b>17.99</b>	<b>6.93</b>
<b>Average default rate</b>	<b>2.44</b>	<b>5.50</b>	<b>2.07</b>

Source : Coface and our calculus

### 3.2 Do ratings systems provide close probabilities of the different types of defaults ?

Following the hypothesis that if default probabilities are close, the objective of the first test of the proximity of distances to default consists to assess the ability of a ratings system to give close distances to the different types of default. Consequently, we have measured the degree of proximity of distances to default. The issue is to know if the distance of a given default is the same in equivalent grades of the three ratings systems. Do borrowers belonging to a given risk class have close probabilities to encounter the three forms of default ? If that is the case, a given credit score model will provide good forecasts of the other types of default. Thus, the test allows to assess how equally risky borrowers are distant from the different forms of default.

To answer, we built annual transition matrixes that compute the probability that borrowers who are assigned a given grade in a ratings system will encounter the three defaults at one year horizon. These annual matrices were averaged over the period under study. So, three sets of average default probabilities are computed for each ratings system.



In table 3, the comparison of columns (1) to (3) shows that the set of legal bankruptcy probabilities given by the legal bankruptcy ratings system and the bank default ratings systems are very similar. That is true, in particular, if we consider the lower risk and higher risk grades. It is less verified if we consider the three intermediate grades, where average default probabilities differ. Nevertheless, legal bankruptcy probabilities assigned by the “privileges” system are very far from those given by the two previous models. Because the main risk factors introduced in the default models are the same, this difference does not come from differences in the fundamental risk factors. It should come from differences in the “loading” of these factors. Very similar results appear if we consider the bank loan default (column (4) to (6)). The probabilities of this type of default given by the legal bankruptcy model and the banking default model are again relatively quite close, while the default probabilities provided by the “privileges” model is quite far from the other ones. Finally, when we compare columns (7) to (8), we observe that the privileges default probabilities are very close in the three ratings systems. Thus, borrowers in a given risk class of this ratings system should have similar distances to the three types of default.

**Table 3 : Stationary probabilities of defaults in the three ratings systems**

ratings grades	<i>Probability of legal bankruptcy</i>			<i>Probability of bank loan default</i>			<i>Probability of "privileges" default</i>		
	<i>In the legal bankruptcy model grades (1)</i>	<i>In the bank loan default model grades (2)</i>	<i>In the privileges default model grades (3)</i>	<i>In the legal bankruptcy model grades (4)</i>	<i>In the bank loan default model grades (5)</i>	<i>In the privileges default model grades (6)</i>	<i>In the legal bankruptcy model grades (7)</i>	<i>In the bank loan default model grades (8)</i>	<i>In the privileges default model grades (9)</i>
1	0.19	0.18	0.23	2.07	1.29	2.22	0.62	1.07	0.33
2	0.60	0.59	0.25	2.61	1.80	3.33	0.97	1.26	0.52
3	0.31	0.36	0.40	3.20	2.08	3.41	0.84	1.21	0.89
4	0.71	0.67	0.46	3.90	2.80	3.86	1.04	1.43	0.90
5	0.77	1.07	0.61	4.44	3.14	4.45	1.26	1.36	1.38
6	0.69	1.07	0.87	5.15	3.93	3.29	1.39	1.60	1.20
7	0.79	1.22	1.73	4.11	5.34	6.46	2.11	1.79	2.05
8	2.00	2.00	2.06	6.31	7.41	6.78	2.60	2.29	2.46
9	3.90	4.19	4.26	8.59	7.60	8.53	3.29	3.15	4.12
10	10.61	11.99	4.85	14.79	17.99	7.35	6.30	5.39	6.93

Source : Coface and our calculus

These results tend to demonstrate that, all in all, different creditors have in mind quite close distances of default when they assign ratings grades to the same debtors. However, the bankruptcy model and the bank loan default model do not forecast very well the distance to

privileges default. Now, these differences should reflect, as we will see later, differences in the institutional factors that determine the speed of reaction of legal creditors to changes in the borrower's financial situation and to declare insolvency of their debtors.

### 3.3 Comparison of rating assignment by the three default models

The second test of the degree of proximity of distances to default consists to compare the rating assignments of the three ratings systems by building cross matrices which take the different ratings systems in pairs. In other words, the objective is to know if a borrower getting a given grade in a ratings system gets also the same grade in another ratings system. The intuition is that if the distances to default are close, borrowers should be assigned the same grades in the different systems. In that case, they will concentrate on the diagonal of the matrixes crossing grades of two different ratings system.

Tables 1a to 1c present cross rating assignments by the three rating systems. They show the cross distribution of borrowers among the grades of the three ratings systems taken two by two. In other words, they show joint probabilities of the borrowers to be assigned in the ten grades. To compute these probabilities, we proceeded in two steps. In a first step, we built stationary transition matrices by averaging one year transition matrixes for each ratings system. Thus, probabilities reported in tables 1 are average probabilities over the 1998-2002 period. Then, in a second step, we proceeded to joint ratings assignment. For instance, in table 1a, 43,59 % of borrowers assigned grade 1 in the legal bankruptcy model are also assigned grade 1 in the bank loan default model, 22,83 % of borrowers with grade 1 in the first model were assigned grade 2 by the second model, and so on.

We test the hypothesis that if the distance to default is the same in two models, firms will concentrate on the diagonal of the matrix. However, we do not expect a very high concentration on the first diagonal. The reason is that such "point-in-time" ratings systems are characterised by a high volatility of one year ratings, specially in the intermediate grades. In other words, generally, transition matrixes of these ratings systems are not very stable. Present ratings systems have this characteristic, as shown by transition matrixes in Appendix 2. Results in table 1 show

that few observations are located on the first diagonal, what simply reflects this characteristic of all “point-in-time” ratings systems. However, tables 1 show that borrowers are mainly located on a “large” diagonal, what militates in favour of the distance to default proximity hypothesis.

Results show also greater proximity between legal bankruptcy model and the other two default models than between bank loan default model and “privilèges” default model. Thus, on average, the same borrowers are assigned more or less in the same grades by two ratings systems among the three ones we built.

**Table 1 : Cross rating assignment**

*Table 1a : bank loan default model and bankruptcy model*

	bank loan default model									
bankruptcy model	1	2	3	4	5	6	7	8	9	10
1	<b>43.59</b>	<b>22.90</b>	16.24	10.82	4.86	1.21	0.31	0.06	0.00	0.00
2	<b>25.61</b>	<b>18.64</b>	<b>12.51</b>	14.68	14.51	9.82	3.35	0.85	0.03	0.00
3	15.91	<b>20.42</b>	<b>13.16</b>	<b>10.79</b>	10.70	13.83	10.59	4.05	0.56	0.00
4	7.78	17.28	<b>18.66</b>	<b>13.91</b>	<b>9.10</b>	9.19	12.05	8.63	3.38	0.02
5	3.32	9.83	17.71	<b>16.31</b>	<b>13.64</b>	<b>10.78</b>	9.31	10.82	7.89	0.40
6	1.95	4.64	9.80	14.76	<b>17.41</b>	<b>16.85</b>	<b>12.03</b>	9.62	10.12	2.81
7	1.23	3.62	5.85	8.51	13.62	<b>15.58</b>	<b>19.55</b>	<b>17.43</b>	9.07	5.55
8	0.53	2.28	4.87	7.25	9.75	10.63	<b>14.61</b>	<b>21.08</b>	<b>22.22</b>	6.77
9	0.04	0.31	1.20	2.77	6.24	11.52	15.56	<b>19.44</b>	<b>22.27</b>	<b>20.64</b>
10	0.00	0.00	0.02	0.04	0.15	0.62	2.64	8.05	<b>24.38</b>	<b>64.11</b>

*Table 1b : “privileges” default model and bankruptcy model*

	"privilèges" default model									
bankruptcy model	1	2	3	4	5	6	7	8	9	10
1	<b>53.26</b>	<b>24.26</b>	11.85	5.70	2.59	1.23	0.56	0.43	0.12	0.00
2	<b>21.50</b>	<b>24.61</b>	<b>21.03</b>	14.41	9.61	5.28	2.25	0.81	0.40	0.11
3	12.26	<b>16.50</b>	<b>19.38</b>	<b>17.68</b>	13.77	9.98	6.67	2.85	0.73	0.17
4	6.00	13.49	<b>14.71</b>	<b>15.83</b>	<b>15.79</b>	13.76	10.78	6.88	2.50	0.25
5	3.86	8.78	11.95	<b>13.82</b>	<b>15.72</b>	<b>14.73</b>	13.64	10.54	5.75	1.20
6	1.85	6.31	9.51	12.53	<b>13.44</b>	<b>14.57</b>	<b>15.62</b>	13.40	10.10	2.66
7	0.92	3.61	6.15	10.17	12.84	<b>15.10</b>	<b>15.21</b>	<b>15.03</b>	14.67	6.30
8	0.31	2.15	4.03	6.51	10.11	14.04	<b>16.89</b>	<b>18.67</b>	<b>17.22</b>	10.06
9	0.00	0.20	1.26	3.31	5.67	9.42	13.85	<b>20.98</b>	<b>25.99</b>	<b>19.33</b>
10	0.00	0.00	0.03	0.07	0.39	1.86	4.52	10.40	<b>22.40</b>	<b>60.32</b>

*Table 1c : bank loan default model and “privileges” default model*

	"privilèges" default model									
bank default model	1	2	3	4	5	6	7	8	9	10
1	<b>26.39</b>	<b>14.67</b>	13.08	12.20	11.12	9.76	6.82	4.52	1.33	0.11
2	<b>20.56</b>	<b>16.28</b>	<b>12.28</b>	11.02	9.26	8.94	9.27	7.75	4.03	0.60
3	14.55	<b>15.13</b>	<b>13.87</b>	<b>10.81</b>	10.68	9.22	9.15	8.89	6.36	1.35
4	13.45	14.17	<b>13.65</b>	<b>11.76</b>	<b>10.44</b>	9.46	9.09	8.16	7.47	2.35
5	9.67	12.35	13.63	<b>12.87</b>	<b>11.48</b>	<b>9.49</b>	9.34	8.65	8.56	3.97
6	9.77	10.98	11.46	12.02	<b>11.68</b>	<b>10.61</b>	<b>9.33</b>	8.99	9.25	5.91
7	4.79	10.00	9.81	11.74	12.05	<b>12.11</b>	<b>10.65</b>	<b>10.36</b>	10.48	8.02
8	0.78	5.36	8.16	9.81	11.64	13.32	<b>12.99</b>	<b>12.26</b>	<b>12.99</b>	12.71
9	0.06	1.02	3.87	7.04	8.80	11.32	14.27	<b>16.45</b>	<b>16.12</b>	<b>21.06</b>
10	0.00	0.01	0.13	0.78	2.78	5.78	9.17	14.03	<b>23.39</b>	<b>43.93</b>

Source : Coface and our calculus

To resume, previous results tend to verify that, at least in the French businesses population, the distances to default are generally quite close, especially if we consider legal bankruptcy default and bank loan default. The distance to these defaults are relatively similar among borrowers of any of the risk grades. A ratings system which is calibrated by using legal bankruptcy default as criterion of default provides distances to default measures which are quite close to those computed using bank loan default criterion. In other terms, they assign the same borrowers in

close grades. Nevertheless, the distance to the privilèges default is far from the distances to the other two defaults. Ratings systems calibrated on the two previous criterions of default do not very well measure the distance to the “privileges” default. As we will see now, that is largely due to the specificity of this type of default. In other words, privilèges creditors do not react to changes in the debtor’s financial situation as do the bankers or the courts.

### 3. Why distances to default are close ? Why are they different ?

Two main reasons help to explain the degree of proximity between the different distances to default. First, each type of default have its own characteristics, which are linked to institutional factors. Second, the different types of defaults are broadly linked together.

#### 3.1 Institutional factors and the unequal treatment of firms

It is largely acknowledged that the different probabilities of default vary with size, what can reflect the role of creditors behaviour and of legal and institutional factors. Firms of different sizes do not receive the same treatment from their creditors. Thus, if the distance to default varies inside the businesses population, whatever the type of default, that might mainly reflect the role of institutional factors or differences in creditors behavior. To illustrate this point, table 4 presents average rates of default for the three criterions of default by firms size. One can verify that all default rates decrease regularly with firm size, what can induce significant differences in the probabilities of default.

**Table 4 : Average one year rates of default by firm size**

	<b>Size (turnover in millions of euros)</b>				<b>All sample</b>
	<b>Lower than € 1 million</b>	<b>€ 1 to 5 millions</b>	<b>€ 5 to 50 millions</b>	<b>Higher than € 50 million</b>	
<b>Legal default</b>	5.77	3.56	2.07	1.01	2.55
<b>Bank loan default</b>	8.24	5.28	3.48	2.13	5.50
<b>Privilèges default</b>	2.15	2.32	2.04	1.70	2.07

Source : Coface

The relationship between the rate of default and the firm's size is particularly true for legal bankruptcy default and also for bank loan default. In fact, the social and economic consequences of firms failures can make courts reluctant to declare firm's bankruptcy when the firm is large. The consequences of default on the creditors revenues are also very dependant of the size of the firms. Banks will generally benefit more from restructuring debts of large firms than of small ones. Nevertheless, firms seems more equal in front of creditors which benefit from privilèges. The rate of privilèges default varies less with size than the other two default rates. That explains why the distances to default are not so widely widespread between the risk classes in the privilèges ratings system than in the two other ratings systems (tables 1 or 2 above). Firms are distributed on a narrower range of distances to default in the privilèges default model.

### 3.2 The links between different defaults

To give a first illustration of the links between the different forms of default, we computed the distribution by credit quality in year T-1 of firms which encountered one type of default in year T. To measure firm's credit quality, here we use a 9 grades ratings system provided by Coface SCRL, the largest French credit bureau. Table 5 presents the distribution (in percent) of defaulted firms by grade one year before the occurrence of different defaults. Average proportions are computed over 1998-2002 period.

**Table 5 : Distribution of SME which defaulted in year N by credit quality in year N-1 and by type of default**

<b>Year N-1 Rating (associated PD in parentheses)</b>	<b>Legal bankruptcy in year N</b>	<b>Privilèges default in year N</b>	<b>Bank loan default in year N</b>
1 (PD=0.12)	0.02 %	0.03 %	0.10 %
2 (PD=0.28)	0.31 %	0.54 %	2.32 %
3 (PD=0.33)	3.37 %	6.86 %	6.02 %
4 (PD=0.77)	3.52 %	4.57 %	5.95 %
5 (PD=1.39)	8.06 %	9.37 %	9.47 %
6 (PD=2.51)	17.08 %	14.95 %	16.11 %
7 (PD=4.36)	24.32 %	21.23 %	23.69 %
8 (PD=8.52)	19.21 %	17.27 %	14.80 %
9 (PD=14.1)	24.11 %	25.18 %	21.54 %
All	100 %	100 %	100 %

Source : Coface and our calculus

Results show that a large proportion of small and medium-sized firms which failed or defaulted during a given year were already located in the riskier (7 to 9) grades the year before, whatever the type of default. Thus, financially fragile firms are equally exposed to the different types of default in the short run.

A second more direct illustration of the proximity of distances to default consists in computing the frequency of other defaults in the population of failed firms during the period (here, the last quarter) preceding the bankruptcy. Table 6 shows the results by firm size. That allows to verify that a very large proportion of failed firms encountered a bank or a privilèges default before going bankrupt. Results also show that this result depends of the firm size. Small firms which went bankrupt have very frequently encountered a problem with their bank, while this situation is less frequent in large firms. Often, privilèges default or bank default serve as predictors of failure in the credit score models.

**Table 6 : Distribution of firms which failed in Quarter Q depending of the existence or the absence of other defaults in quarter Q-1 by firm size**

	<b>Lower than € 1million</b>	<b>€ 1- 5 millions</b>	<b>€ 5 - 50 millions</b>	<b>Higher than € 50 millions</b>	<b>All sizes</b>
<b>Absence of defaults in Q-1</b>	31.37 %	31.15 %	35.82 %	53.47 %	34.92 %
<b>Privilèges default in Q-1</b>	4.41 %	14.15 %	21.25 %	19.44 %	16.78 %
<b>Bank loan default in Q-1</b>	64.22 %	54.69 %	42.94 %	27.08 %	48.30 %
<b>All</b>	100 %	100 %	100 %	100 %	100 %

Source : Coface and our calculus

Previous results show that failed firms frequently encountered other forms of defaults before failure, especially when their size is small. Then, it is interesting to measure the probability to go bankrupt (here, at the one quarter horizon) in the population of firms encountering bank loan default or privilèges default. Table 7 presents average probabilities over the 1998-2002 period. One can show that the two probabilities of default are very high and decrease sharply with the firm's size. Again, observed differences between firms of different sizes likely reflect differences in the bank promptness to declare bankruptcy of borrowers.

**Table 7 : One Quarter Horizon Probability of Legal Bankruptcy in the populations of firms which encountered another default by size**

	<b>Firms which encountered privilèges default</b>	<b>Firms which encountered bank loan default</b>
<b>Size (turnover in millions of euros)</b>		
<b>Lower than € 1million</b>	<b>7.14</b>	<b>4.10</b>
<b>€ 1- 5 millions</b>	<b>11.69</b>	<b>4.38</b>
<b>€ 5 - 50 millions</b>	<b>11.39</b>	<b>6.91</b>
<b>Higher than € 50 millions</b>	<b>7.09</b>	<b>6.29</b>
<b>All sizes</b>	<b>10.76</b>	<b>5.17</b>

Source : Coface

#### **4. Are capital charges similar ? The results of a simulation exercise<sup>2</sup>**

Default probabilities (PD) and asset correlations (R) are key parameters in the calibration of any credit risk model. They also hold a central position in the new regulatory framework of Basel II (BIS, 2002, 2003). So, it is interesting to simulate the differences in capital charges when we use the probabilities of default given by the three default models. To this aim, we computed capital charges using the Basel II risk weight formulas. Following these formulas, we proceeded in two steps. The first step is devoted to the calculus of the assets correlation ( $R$ ). In the new risk weight formulas,  $R$  is given by the following equations :

1°) for corporate exposures (on firms with turnover higher than € 5 millions) :

$$R = 0.12 \times (1 - \exp(-50 \times PD)) / (1 - \exp(-50)) + 0.24 \times [1 - (1 - \exp(-50 \times PD)) / (1 - \exp(-50))]$$

2°) for retail exposures (on firms with turnover lower than € 5 millions):

$$R = 0.02 \times (1 - \exp(-35 \times PD)) / (1 - \exp(-35)) + 0.17 \times [1 - (1 - \exp(-35 \times PD)) / (1 - \exp(-35))]$$

Moreover, in Basel II formulas, the following adjustment is introduced for corporate exposures on firms whose turnover is lower than € 50 million (and higher than € 5 millions) :

$$-0.04 \times \left(1 - \frac{S-5}{45}\right)$$

<sup>2</sup> Thanks to Bill Lang, who suggested such simulation.



where  $S$  is the turnover ( in € millions) of the firm.

The new risk-weight formulas that apply both to corporate exposures and to retail exposures assume a negative relationship between PD and R. That means that firms with lower default risk are also more exposed to changes in economic conditions. Conversely, firms with higher default probabilities are less prone to default simultaneously. Their activities may be less dependent on the business cycle<sup>3</sup>.

Then, in a second step, values of R are integrated in the following risk weight formulas :

1°) for corporate exposures on firms with turnover over € 5 millions :

$$K = LGD \times \Phi \left[ (1 - R)^{-0.5} \times \Phi^{-1}(PD) + (R/(1 - R))^{0.5} \times \Phi^{-1}(0.999) \right] \times \frac{1 + (M - 2.5) \times b(PD)}{1 - 1.5 \times b(PD)}$$

2°) for retail exposures on firms with turnover lower than € 5 millions :

$$K = LGD \times \Phi \left[ (1 - R)^{-0.5} \times \Phi^{-1}(PD) + (R/(1 - R))^{0.5} \times \Phi^{-1}(0.999) \right]$$

where LGD measures the loss given default, P is the borrower's probability of default,  $\Phi$  is the standard normal CDF, M is the maturity of the asset, and  $b(PD)$  is an adjustment for maturity.

We applied these formulas to a portfolio of around 50 000 firms, which were randomly drawn in the SME population of our sample. In the computation of the capital charges, we assumed a recovery rate equal to zero, so that LGD is equal to each borrower's exposure. The exposure corresponds to the total of borrower's bank debt listed in its balance sheet. In addition, due to lack of information, we did not take into account the maturity adjustment. Results are presented in table 8.

---

<sup>3</sup> However, various empirical results do not completely verify this pattern (see Dietsch and Petey, 2004).

**Table 8 : Capital charges simulations using Basel II risk weight formulas**

	<b>Capital charges in € millions</b>	<b>Median capital charge in €</b>	<b>Total debt in € millions</b>
<b>Legal bankruptcy</b>	<b>3.365</b>	<b>2.443</b>	<b>65.383</b>
<b>Bank default</b>	<b>6.287</b>	<b>5.142</b>	<b>65.383</b>
<b>Privilèges default</b>	<b>6.370</b>	<b>3.415</b>	<b>65.383</b>

Results show that capital charges vary significantly from one model to another. In particular, they are lower when the probabilities of legal bankruptcy are used than when the two other sets of probabilities of default are used. That could not simply reflect the fact that average default rates are different. Indeed, the rates of legal bankruptcy and privilèges default are quite close together and both differ sharply from bank loan default rate. Recall that the former are equal to 2,44 % and 2,07 %, respectively, while the later is equal to 5,5 % (table 1). Nevertheless, it is quite surprising to observe that total capital charges computed by using bank default ratings system and privilèges default ratings system are very close, even if the average defaults rates are very different from one model to the other.

In fact, taking account for the steepness of the risk weight Basel II formulas and the existence of a negative relationship between the probabilities of default  $PD$  and the assets correlations  $R$ , the total amount of capital charges depends : (1) on the distribution of default probabilities by grade in each ratings system and, (2) on the distribution of debt amounts by grade in each system. Indeed, the higher level of capital charges induced by the privilèges default ratings system, when we compare this level to the level given by the legal bankruptcy system, is the joint result of the relatively higher values of the probabilities of default in the lower and intermediate grades in the former ratings system (table 1), on the one hand, and of a more uniform distribution of debt amounts between grades in the privilèges default ratings system, on the other hand. Larger SME, with correspondingly higher amounts of debt, are more present in the high risk grades of the privilèges ratings system than in the equivalent grades of the two other systems, what explain the strong difference of capital charges given by the bankruptcy ratings system and the privilèges default system and the quasi-equality of these charges in the privilèges ratings system and the bank default ratings systems. Finally, these simulations tend to demonstrate that, given Basel II formulas, the building of ratings grades scales and not only the structure of probabilities of default in the borrowers population largely determine the level of regulatory capital charges.

## 5. Conclusion

In the capital charges reform proposed by the Basel committee, banks are encouraged to build sophisticated risk measurement systems and to use their own internal systems to determine risk parameters that enter into the new regulatory capital calculation. However, few banks have already developed sophisticated credit risk models and they rarely have maintained historical databases with consistent data to estimate precisely risk characteristics of their exposures (Land and Santomero, 2002). In particular, banks have rarely maintained historical databases of defaulted customers. Instead, to calibrate their credit models, more advanced banks relied upon external databases of bankruptcy firms. This paper compares the risk parameters computed by using ratings systems which are calibrated on this currently used criterion of default with the outputs obtained when other criteria of default served to calibrate ratings systems. The main result is that ratings systems calibrated by using bank default and legal bankruptcy default criteria give on average relatively close distances to default. The same borrowers are assigned more or less in the same grades by two ratings systems. On the contrary, the distances to default given by a ratings system calibrated using the privileges criterion of default are quite far from those provided by the former two.

Thus, on average, creditors tend to have in mind quite close distances of default when they assign ratings grades to the same debtors. Nevertheless, their perception of the distance to default depends also on legal and institutional factors, which determine the quickness of creditors decisions to declare insolvency of their debtors. Consequently, distances to default could be quite different from one type of creditor to another one. The differences of distance to default by firm size reflect the existence of these differences of legal and institutional conditions.

Simulations of regulatory capital induced by the new Basel II risk weight formulas showed that these charges are also sensitive to the choice of the default criterion. In fact, this choice determines the architecture of the ratings systems and, in particular, it affects the distribution of borrowers in the different grades. So, ratings systems calibrated using different criteria of defaults could induce very similar level of charges, even if different criteria give very different average default rates.

## References

- Altman, E., 1968, Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy, *Journal of Finance* pp. 589–609.
- BIS Basle Committee on Banking Supervision, 2001 : The New Basel Capital Accord, and associated documents, January 2001.
- BIS Basle Committee on Banking Supervision, 2002 : « Quantitative Impact Study 3 : technical guidance », October.
- BIS Basle Committee on Banking Supervision, 2003 : « Nouvel Accord de Bâle sur les fonds propres », consultative document, April.
- Carey M. M. Hrycay, 2001, Parameterizing credit risk models with rating data, *Journal of Banking and Finance*, vol. 25, 197-270.
- Hayden, E. (2003) Are credit scoring models sensitive with respect to default definitions? Evidence from the Austrian market, working paper, University of Wien, department of business administration.
- Gordy, M., E. Heitfield, 2001, Of Moody's and Merton : A structural model of bond rating transitions, working paper, Federal Reserve Board.
- Lang W., Santomero A., 2002, Risk quantification of retail credit: Current practices and future challenges, working paper, Federal Reserve of Philadelphia.
- Merton, R. (1974) On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance*, vol 29, 449-470.
- Nickell, P., Perraudin, W. et S. Varotto, 2000, Stability of rating transitions, *Journal of Banking and Finance*, 24, 203-227.

## Appendix A : The three credits score models

	classes	Legal bankruptcy		Bank loan default		Privilèges default	
		Parameter estimates	Pr > ChiSq	Parameter estimates	Pr > ChiSq	Parameter estimates	Pr > ChiSq
Intercept		-3,8264	<.0001	-5.2421	<.0001	-4.5393	<.0001
class_R1	1	0,9849	<.0001	2.0704	<.0001	0.9393	<.0001
class_R1	2	0,4426	<.0001	0.0450	0.7240	0.3621	<.0001
class_R1	3	-0,0569	0.0824	-0.3054	0.0351	-0.1559	0.0576
class_R1	4	-0,4406	<.0001	-1.0067	<.0001	-0.5757	<.0001
class_R4	1	-0,198	<.0001	-0.2888	0.0088	0.0635	0.4001
class_R4	2	0,0404	0.1090	0.0584	0.5425	0.1321	0.0506
class_R5	1	0,2652	<.0001	0.3026	0.0018	0.2596	<.0001
class_R5	2	-0,0411	0.1022	-0.1729	0.0978	-0.2167	0.0016
class_R3	1	0,2177	<.0001	0.1427	0.2041	0.3646	<.0001
class_R3	2	0,0404	0.2228	-0.2400	0.0716	-0.1577	0.0668
class_R3	3	0,0352	0.2946	-0.0463	0.7030	-0.1833	0.0248
class_R7	1	0,2028	<.0001	0.4822	0.0016	0.4100	<.0001
class_R7	2	-0,153	<.0001	-0.0594	0.7613	0.1984	0.0355
class_R7	3	-0,2244	<.0001	-0.4823	0.0570	-0.2481	0.0252
class_R2	1	0,1715	0.0062	0.7529	0.0001	0.1819	0.3078
class_R2	2	0,5119	<.0001	0.4716	<.0001	0.2519	0.0026
class_R2	3	-0,0711	0.0549	-0.2424	0.0878	0.0307	0.7254
class_R2	4	-0,3171	<.0001	-0.4204	0.0093	-0.2083	0.0321
class_R6	1	0,1522	0.0001	1.1693	<.0001	0.0858	0.3328
class_R6	2	-0,2701	0.0095	-1.0013	0.2171	0.2912	0.1511
class_R6	3	0,0376	0.3388	-0.1244	0.6310	-0.1495	0.1162
class_R6	4	0,0423	0.3131	-0.2556	0.3486	-0.0839	0.4025
class_R8	1	-0,0406	0.1892	-0.1464	0.2443	-0.1874	0.0229
class_R8	2	-0,0608	0.0494	-0.0315	0.7949	0.00837	0.9118
class_R8	3	0,1	0.0006	0.2700	0.0079	0.3400	<.0001
class_sect	Housing,	0,1193	0.0095	0.2201	0.2102	-0.3972	0.0050
class_sect	Retail	0,2685	<.0001	-0.4347	0.0178	-0.4313	0.0023
class_sect	Wholesale	0,1195	0.0074	-0.5978	0.0034	-0.5016	<.0001
class_sect	HCR*	-0,0559	0.5097	0.3518	0.1998	0.8822	<.0001
class_sect	Manufact. ind	-0,0533	0.1940	0.3998	0.0033	-0.0157	0.8690
class_sect	Services	-0,1456	0.0043	0.2950	0.0765	0.5840	<.0001
class_taille	Large	-0,3526	0.0095	-0.9544	0.1098	0.3841	0.0111
class_taille	Medium +	-0,6267	<.0001	-0.1830	0.5174	-0.1856	0.0890
class_taille	Medium -	-0,1918	0.0035	0.1582	0.4927	0.1692	0.0726
class_taille	Small	0,2385	<.0001	0.4445	0.0167	-0.2915	0.0001

\* HCR : hotels, pubs, cafés, restaurants,

### Model Performances

Model	concordance rate	Global reclassification	reclassification of defaulted firms	reclassification of good firms
Legal bankruptcy	87,3 %	82,6 %	78,2 %	82,7 %
Bank loan default	77,0 %	70,3 %	71,9 %	70,2 %
Privilèges default	74,3 %	69,7 %	68,2 %	69,7 %

## Appendix B : Transition matrixes in the three ratings systems

**Table B1 : Transition matrix built from legal default model**

	1	2	3	4	5	6	7	8	9	10
1	46.28	20.23	10.67	6.50	4.53	3.63	3.29	2.73	1.67	0.48
2	19.15	25.26	18.23	10.47	8.00	5.88	4.81	4.38	2.89	0.92
3	11.14	16.59	19.78	15.72	11.49	8.06	6.36	5.45	4.09	1.33
4	6.79	11.05	14.50	18.03	15.18	11.51	8.21	7.30	5.42	2.00
5	4.75	7.87	11.24	14.53	17.88	15.34	10.67	8.36	6.49	2.85
6	3.96	6.21	7.94	11.57	14.29	17.82	15.36	10.12	8.57	4.17
7	3.35	5.15	6.47	7.94	10.46	13.92	19.71	16.14	10.99	5.87
8	3.14	4.51	5.39	6.95	7.87	9.90	14.19	21.91	16.77	9.37
9	1.29	2.58	4.06	5.84	6.80	8.85	10.64	14.62	26.74	18.58
10	0.48	0.91	1.91	2.36	3.35	4.98	6.74	9.20	16.47	53.61

**Table B2 : Transition matrix built from bank loan default model**

	1	2	3	4	5	6	7	8	9	10
1	48.91	20.67	11.33	7.18	4.73	3.05	1.95	1.31	0.64	0.23
2	19.59	24.93	17.27	13.57	9.88	6.09	4.24	2.59	1.30	0.53
3	11.48	16.54	20.84	15.34	12.82	9.41	6.81	3.99	2.07	0.71
4	7.14	13.00	14.28	18.85	14.05	13.15	9.04	6.15	3.21	1.12
5	4.74	9.06	12.92	13.00	18.72	14.51	12.24	8.58	4.58	1.65
6	3.19	6.67	9.19	12.42	12.95	18.70	15.49	11.97	6.82	2.61
7	2.34	4.57	6.55	8.85	11.91	13.42	19.77	16.30	11.51	4.78
8	1.35	2.69	4.39	6.09	7.92	11.92	15.31	21.68	19.28	9.38
9	0.74	1.45	2.33	3.31	4.87	7.26	10.61	18.33	29.31	21.79
10	0.33	0.65	0.84	1.35	1.94	3.02	4.66	9.22	21.31	56.69

**Table B3: Transition matrix built from "privileges" default model**

	1	2	3	4	5	6	7	8	9	10
1	46.12	22.47	13.35	7.67	4.82	2.76	1.51	0.83	0.35	0.12
2	22.80	25.16	18.40	13.08	8.82	5.60	3.40	1.70	0.77	0.27
3	13.15	18.58	19.96	16.22	12.31	8.28	5.98	3.35	1.56	0.62
4	7.92	12.88	16.14	18.37	15.17	12.06	8.45	5.45	2.66	0.91
5	4.71	8.80	12.10	15.06	17.04	15.64	11.65	8.41	4.80	1.78
6	2.85	5.67	8.74	12.16	14.60	17.23	16.04	12.32	7.44	2.93
7	1.59	3.24	5.76	8.34	12.13	15.19	18.87	16.79	12.78	5.31
8	0.83	1.87	3.32	5.34	8.52	12.31	16.85	21.89	19.39	9.68
9	0.45	1.07	1.68	2.79	4.78	8.25	12.46	19.47	27.77	21.29
10	0.12	0.25	0.70	0.92	1.76	2.96	5.01	9.78	22.29	56.19

Source : Coface and our calculus