

Resilience and contagion in a connected economy*

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Abstract

This paper argues that financial links between agents may lead to the resilience or to the contagion of financial distress. Our model details the real effects of agents' beliefs on the resilience of the economy. When the economy is connected enough, it is subject to an unstable equilibrium. Our model therefore delivers various implications for crisis workouts.

JEL classification: G01, C02.

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1 Introduction

We analyze the contagion from a small number of defaulting agents to other agents via the potential occurrence of different types of shocks affecting the economy. Answering how problems within one economic sector may provoke a dislocation of the financial system is crucial to determining how policy should respond.

In this paper, we argue that financial connections between economic agents may lead to a crisis. We base our analysis on two cornerstones: (i) the connection of financial activities and (ii) the resilience of the network created by these connections and (iii) the beliefs of market participants about the resilience of the network. Indeed, two main features characterize contemporary finance.

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- **Interconnection:** One of the most pervasive aspects of the contemporary financial environment is the rich network of interconnections among firms and financial entities, especially via securitization. Securitization first aimed at better risk management, at improving market completeness and therefore at preventing contagion effects. However, securitization has also created interconnections between financial actors which generated contagion. We raise the question of whether this interconnectedness strengthens or weakens the financial system as a whole.
- **Uncertainty:** The existence and the positions of special entities concentrating credit risks is generally little known outside expert circles; they contribute to the opaqueness of overall risk vulnerability.

To model a financial system sharing these two characteristics, we consider an economy populated by n agents represented by dots. Each of these dots is to be thought of as a distinct economic entity, or financial dot, participating in the financial system. Each such entity may have liabilities to other entities in the system. We assume that agents know the origin of financial problems: the number of defaulting (or junk) sources within the economy, and the number of agents contaminated by these sources. However agents are uncertain regarding the resilience and the connectivity of the economy. More precisely, agents build expectations about the resilience of the network economy.

We study the impact of different shocks in the number of junk agents on the economy. A positive shock on the number of defaulting agents might have different origins. On the one hand a shock on the number of contaminated agents is caused by a shock on connectivity and resilience. On the other hand a positive shock on the number of defaulting sources provokes a large shock on the number of defaulting contaminated agents, keeping the connectivity and the resilience constant. The behavior of agents depends on the influence of shocks on the market sentiment. For example the reaction of economic agents to a small increase in the number of junk agents with an optimistic market sentiment consists in increasing the connectivity.¹ A large positive shock on the number of defaulting agents induces agents to decrease their connectivity.² The intermediary cases where the market sentiment can easily change creates different equilibria.

¹Following a negative shock on the number of junk agents, agents decide to increase the number of their financial links to benefit from the high network resilience they expect because of optimism.

²Because of the large positive shock they are pessimistic and expect resilience to be lower.

Rational bubbles and unstable equilibrium We define the resilience of the economic network by the global availability of capital. We make the hypothesis - without loss of generality - that agents are homogenous in size, and that financial flows have the same financial weight. We consider that a "healthy" agent becomes "junk" if he has not enough funds. Indeed we calculate the sum of the incoming flows of his "junk" neighbors and compare to the sum of the incoming financial flows from his "healthy" neighbors multiplied by the availability of capital. Since the global availability of capital is the sum of individual sharings of capital, the resilience of the network is an endogenous variable. Bubbles appear when this resilience is kept high with an increasing number of contaminating agents.

We state our main results:

First, when the economy is slightly connected, a small increase in the number of junk agents raises the number of junk agents and therefore increases the number of pessimistic agents. The result is a disconnection, that reduces the number of junk agents. Therefore this equilibrium is stable. Second, when the economy is highly connected, a small increase in the number of junk agents induces an overall disconnection that increases the number of junk agents. This creates more pessimistic agents. The disconnection is somewhat "accelerated". This generates a crisis. On the other way, a small decrease in the number of junk agents moves the economy towards a better state. This second equilibrium is unstable. So when the economy is connected and subject to shocks in the number of junk agents or sources, it might be prone to unstable situations. Moreover, the unstable equilibrium might converge with a crisis to an inefficient stable equilibrium because of agents' rational behavior. This stable equilibrium is inefficient because agents reduce connections too much. It may otherwise diverge to some other equilibrium, stable in the degree of connectivity p , but unstable in the number of junk agents J or sources s . This other equilibrium is however subject to crisis.

These features may rationalize two recent episodes as detailed in the last part.

Related literature In common with Eisenberg and Noe (2001), Cifuentes et al. (2005)³, Nier et al. (2007), Rotemberg (2008)⁴, the current paper uses graph-theoretic

³Cifuentes et al. (2005) combine liquidity risk with externally imposed regulatory solvency requirements, when mark-to-market accounting rules of firms' assets are effective. Their model incorporates two channels of contagion – direct balance sheet interconnections among financial institutions and contagion via changes in asset prices.

⁴Rotemberg (2008) uses numerical techniques only sparingly, with most results established analytically. His analysis is limited to simple environments. He focuses on liquidity issues (limits on the amount of liquid assets that are available to the financial system for the payment of debts can lead to more defaults when the web of debts is more densely interconnected).

techniques. In these papers, interconnectedness has two mutually opposing effects. Interconnections appear to have the potential both of spreading the failure of a single institution but also of cushioning its impact (by ensuring that a single institution only has a moderate effect on any other given institution). Focardi and Fabozzi (2004) and Egloff et al. (2007) present contagion models using percolation and random graphs, taking into account the impact of the credit portfolio's interdependence structure. However, none of these models account for a resilience effect.

Watts (2002) presents a simple model of global cascades in terms of a random network of interacting agents. He determines the probability of global cascade in the economy, that is contagion as a function of network resilience and connectivity. Our resilience effect is similar to that of Watts. Compared to Watts, we introduce a new feature, i.e. agents' beliefs and we endogenize the network resilience ϕ .

The basic structure of our financial system is close to Eisenberg and Noe (2001) who aim at showing the existence of a clearing vector. Their contagion effect differs from ours as they have a payoff matrix. They study the transfers of cash and debts among the network. By contrast, we have a matrix showing the connections between the different agents. The financial weights of a connection between two agents is represented by the number of connections between both agents. Eisenberg and Noe introduce black holes while we have junk agents in our model.

Allen and Gale (1998) and Allen and Gale (2000) explain how small shocks in a given sector may transform a banking crisis into a more widespread financial crisis. They model financial crises as part of the business cycle and study the transmission mechanisms of financial risks between interconnected regions.

The paper proceeds as follows. Section 2 presents in a simple manner the financial system of interconnected lending. Section 3 solves the model owing to simulations. Section 4 determines agents' behavior in response to shocks on the number of "junk" agents. Section 5 applies our model to two recent episodes of crisis and section 6 concludes.

2 Description of the financial system

An example of the financial system is given in Figure 1.

Agents are heterogenous in size and structure There are two issues to be considered here. First, the way we describe financial links between the different agents. And second, the way we describe the heterogeneous agents among their financial structures and their financial weights. We use basic graph theory to deal with these issues.

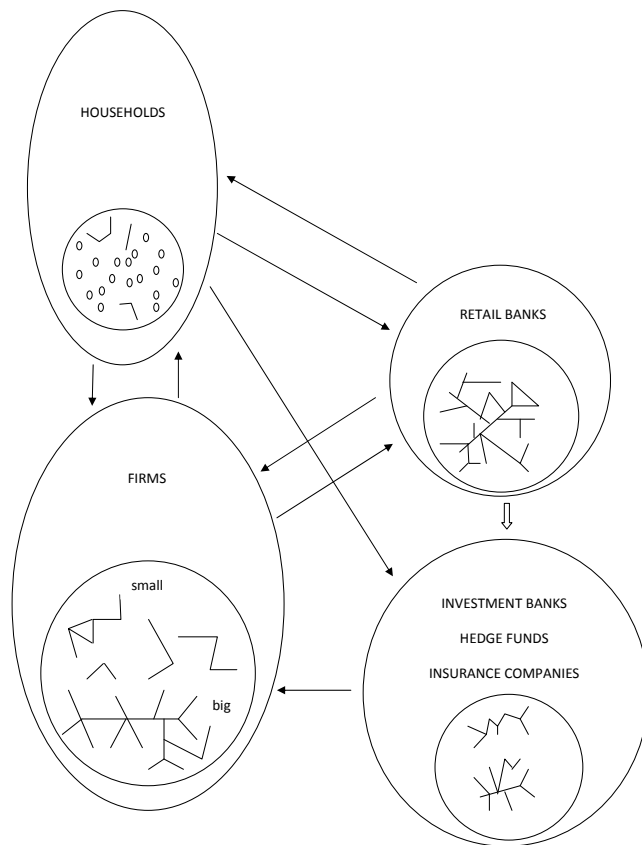


Figure 1: Example of a representation of the financial system

Financial links are represented by oriented edges between the different agents. Arrows represent the direction of investment. Agents are represented by dots. About financial weight and financial structure, we assume that there is a financial entity. Such an entity is represented by a dot on the graph. We represent an agent by a dot or a couple of dots proportionally to its financial weight. Within an agent, the edges are non oriented because they represent the repartition of financial weight inside the household. The connections within an agent depend on the structure of the agent.

To relate our analysis to graph theory, a complete undergraph represents a perfect share and mobility of financial wealth. The activities are linked by simple edges, because they cannot easily transfer their financial value.

Finally, since a dot represents a financial weight, it means that the financial flows entering and going out of that dot are approximatively of the same size as the dot. This allows us not to consider the weightmatrix of the graph but only the existence of a link between two dots. Without loss of generality, we assume that the oriented links between different agents can be considered as bi-directional. Indeed to add an oriented edge on the graph, we would add the number 1 on line i and column j if the link is an investment from agent i to agent j in the weightmatrix.⁵ In case of a bi-directional link we would also add in the graph-link matrix the number 1 on line j and column i . The weightmatrix representing only the agents is symmetric because the share of wealth within an agent is symmetric, while that representing the links between agents is not. Corresponding to our programming (in the next section) the effect of adding bi-directional links rather than oriented links simply changes a scale parameter.

To summarize, there is a large number n of financial agents in the economy. Each agent is represented by a dot or group of dots. Financial flows are represented by edges between dots whatever their direction. We define an agent's neighbor as an agent having a direct financial link with the considered agent. The neighbor of agent A is defined as any agent directly connected to agent A by one edge.

The network is first characterized by its interconnectedness (or degree of connectivity) corresponding to the total number p of connections between agents, i.e. the number of edges between the different dots. Second, we introduce a resilience ϕ in the network. The resilience represents the capacity of the network to limit the proliferation of financial distress and is associated to the availability of capital (or capital speed) within the economy. The higher the stock of capital, the easier it is for an agent to resist the possible contagion of financial distress. The exact mechanism

⁵See next section for more details.

is explained later on.

We consider two types of agents. A "junk" agent j is a defaulting agent in capital stock. Their number is: J . By contrast, a "healthy" agent h is in good financial situation (the number of healthy agents is: $n - J$). Among the "junk" agents, we distinguish the sources s that initiate the financial distress, and the other junk agents who are contaminated by the sources. The mechanism of contagion of financial distress, i.e. from a junk to a healthy agent, is as follows. If the proportion of junk neighbors over all the neighbors of a healthy agent exceeds the level of network intrinsic resilience ϕ , the healthy agent becomes junk too. In other words, a h agent adopts state j if at least a threshold fraction larger than ϕ of its neighbors is in state j , else it remains in state h .

The number of junk agents J is therefore a function of the number of sources s , the degree of connectivity p and the network resilience $\phi \in [0, 1]$: $J = f(s, p, \phi)$. Let us add two definitions. The economy is said to be in a stationnary state if (s, J) is constant. Analogously, the economy is said to be in a constant state if (s, J, ϕ) is constant.

For example, in the case where $\phi = 0$, there is systematic contagion: no agent is able to resist a financial problem because it has no share of capital stock. In the case where $\phi = 1$, contagion requires that all connected neighbors be junk. We describe the formation of ϕ in the next sections. We suppose that the state of the economy is a decreasing function of the number of junk agents. We assume that the agents perfectly observe (s, J) .

So the model presents two competing effects: a direct contagion effect from a junk agent to a healthy neighbor and a resilience effect associated with the links a healthy neighbor has to other healthy agents.

3 Simulating the economy

Considering a large number n of agents within the economy, we introduce a symmetric matrix $M : (n \times n)$ to represent the financial links between agents, which values are 0 or 1 depending on the existence of a financial link. If agent i is linked to agent j , then $M(i, j) = M(j, i) = 1$. We analyze the contagion of the fixed number of junk sources s to healthy agents each time we add a financial link between two random agents. We then take an average on a large number of simulations with different random graphs to get a more accurate result. The graph simulation is presented in Figure 2.⁶

⁶Matlab codes are available from the authors upon request.

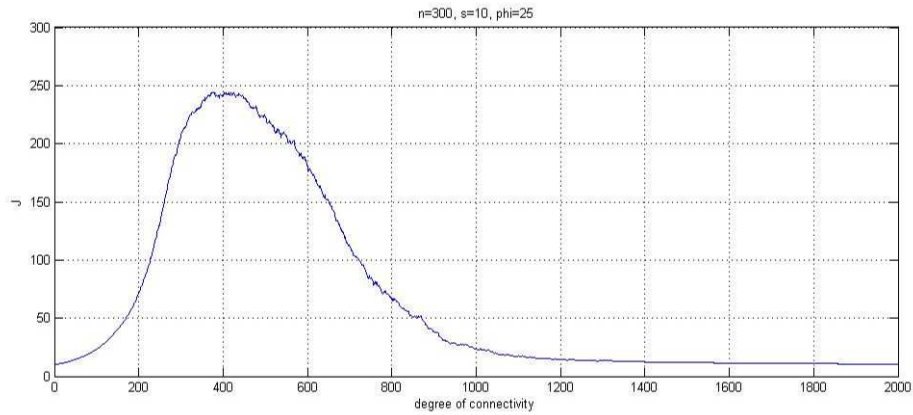


Figure 2: Total number $J = f(s, p, \phi)$ of junk agents in the economy as a function of the degree of connectivity

We can see that for p small enough, we only observe the contamination effect, that represents a percolation phenomenon. This means that when the degree of connectivity in the economy is sufficiently low (securitization tends to zero), the financial network is not resilient: it is unable to resist financial distress by sharing risk. The number of junk agents is growing with respect to p . Once the economy is connected enough, the number of junk agents decreases with respect to p , this is due to the resilience effect. We naturally define the percolation threshold as the maximal growth in the number of junk agents (on the graph, it is the dot at which the slope is the steepest). At this point, the marginal contagion of the network is the highest. We first assume that the number of sources s is a constant.⁷ This assumption enables us to study the effect of different variables separately.

For a fix number J of junk agents, between the number s of junk sources and the total number of agents n , there is an infinity of couples (p, ϕ) that correspond to this number of junk agents. Even for a fixed ϕ , two p give the same number of junk agents (because the curve is unimodal). Economically, this means that agents

⁷This assumption will be relaxed below (see section 4).

observing s and J are not able to determine an exact measure on (p, ϕ) . They will have to form beliefs about this couple of real (p, ϕ) . They have three possible beliefs on the couple (p, ϕ) :

- The economy is in a contagion phase: p is below the percolation threshold P whatever ϕ ;
- The economy is in a resilient phase:
 - ϕ is sufficiently high and p is slightly above the percolation threshold P ;
 - ϕ is relatively low and p is largely above the percolation threshold P .

There are however restrictions about the possible values for ϕ , depending on J . When ϕ is low, economic agents behave selfish because they are unwilling to share capital with their neighbors. Capital becomes somehow costly.

For example, if a healthy agent is connected to say 5 neighbors among which there is one junk neighbor, the 4 remaining healthy neighbors cannot (or are not willing to) help the considered agent to resist the financial distress. The agent becomes junk. By contrast, when ϕ is large, the availability of capital absorbs the risky (contaminating) effect of the junk source. If ϕ is large enough (high network resilience), there is no possibility of large contagion, because the network is precisely resisting. Hence it is impossible to exceed a certain number of junk agents when ϕ is sufficiently high.

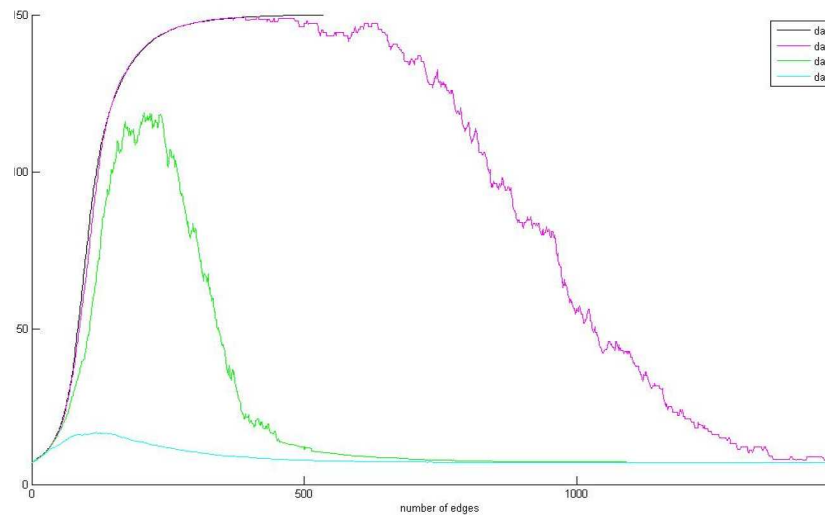


Figure 3: Simulations for different levels of network resilience

This phenomenon is illustrated on Figure 3. For example a resilience of 40 percent considerably limits the possibilities of contagion, whereas a resilience of 20 percent allows full contagion of the economy until the number of edges becomes sufficiently high to absorb the contaminating effect of the initial junk agents.

4 Agent's behaviors

Consider that the economy is in a constant state. Agents have expectations about ϕ . We denote the real value of ϕ by ϕ_r ; so $\phi_r = \frac{1}{n} \sum_i \phi_i$, where ϕ_i is the estimation of agent i on the real value of ϕ . Indeed, agent i shares its capital depending upon what he believes about ϕ_r . The global disponibility of capital is the aggregation of all individual sharings.

Assume that the agents believe on average that the resilience is different from the real resilience, for example $\phi_r < \frac{1}{n} \sum_i \phi_i = \mathbb{E}[\phi_r]$, where $\mathbb{E}[\phi_r]$ is the average expectation of agents on the real value ϕ_r . In this case, agents are willing to share capital more easily than they would if they believed on average that the resilience were ϕ_r . The real effect of such belief is that they move ϕ_r to $\frac{1}{n} \sum_i \phi_i$. This way, we use ϕ as an endogenous parameter. We make no hypothesis about the distribution of ϕ_i .

We suppose that there exists a natural level L of junk agents within the economy. This level economically corresponds to the natural risk in financial markets. Therefore financial agents consider this level of junk agents as natural. However when the number of junk agents exceeds this threshold, financial agents act to decrease the number of junk agents. We will detail the different types of behavior of agents depending on their beliefs.

There are two possible types of agents, optimistic and pessimistic. We will detail these states further (in 4.4). Considering all the agents, we define the market sentiment as the aggregation of all individual sentiments, optimistic and pessimistic. The market sentiment can be optimistic or pessimistic, when every agent is optimistic or pessimistic, but it can also be intermediary, when there are both optimistic and pessimistic agents.

4.1 Optimistic market

An optimistic market sentiment is a situation in which each agent expects the highest network resilience as possible. Indeed the agents expect ϕ being the highest corresponding to the number of junk agents. Therefore we have a unique couple

(p, ϕ) corresponding to the number of junk agents. Since all agents have the same belief on (p, ϕ_i) , it means that $\forall i$ we have $\phi_i = \phi_r$. The aggregation of agents' actions is the same as the action of a unique representative optimistic financial agent.

We study the behavior of this representative agent on Figure 4. When the number of junk agents exceeds the natural threshold of risk, financial agents react to decrease this number of junk agents. We calculate the cost of the different actions

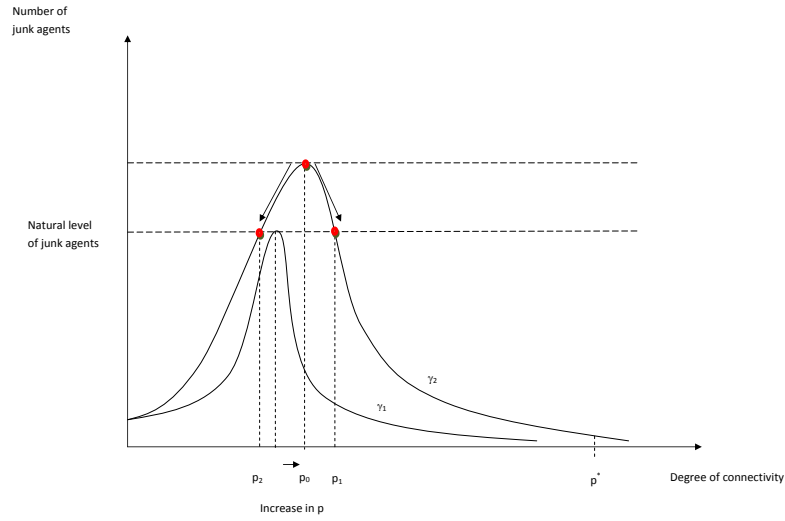


Figure 4: Optimistic reaction to a positive shock on the number of junk agents

for the representative agent to go back to the threshold of natural risk, considering the positive market sentiment. There are two possible actions. An agent can act on p by changing the number and weight of its financial links. In our model based on random graphs, it corresponds to adding or removing one or more edges. We call these two possible actions connecting or disconnecting.

The representative agent can increase its connectivity to move along the curve γ_2 . The cost is :

$$\int_{[p_0, p_1]} c_c(p) d\gamma_2,$$

where c_c represents the cost of connection and γ_2 the curve of the junk agents. The agent can alternatively decrease its connectivity to move along the curve towards

the natural level of junk agents. The cost is:

$$\int_{[p_0, p_2]} c_d(p) d\gamma_2,$$

where c_d represents the cost of disconnection.

We assume that the cost of connection c_c is convex in p . The minimum of the cost of connection is the percolation threshold. Indeed all the connections of an agent represent its funding or investment possibilities. The more connected the agent, the more difficult it becomes to create new financial links, the agent accepting first the most attractive financial links. For a slightly connected economy, the cost of connecting is high due to the absence of financial structure. The cost of connection is minimal around the percolation threshold because it is a degree of connectivity where each agent tries to link to at least one other agent, and a lot of them are available to create financial links.

The cost of disconnection c_d represents the cost of breaking a financial link, such as an investment or loan contract or removing funds. We assume that this cost is constant in p (although it may be an irregular distribution). On average, the cost of disconnection is higher than the cost of connection. There also exists a sufficiently high degree of connectivity p^* , such that $c_c(p^*) = c_d(p^*)$.

When there is a small number of junk agents, agents do not expect such a high degree of connectivity, i.e. $p_0, p_1, p_2 < p^*$, because they expect a relatively good network resilience ϕ according to the optimistic market sentiment. The solution consisting in increasing the connectivity is considerably cheaper than the solution that consists in decreasing the connectivity. We can state our first proposition.

Proposition 4.1 *When the market sentiment is optimistic, agents naturally decrease the level of junk agents by creating financial links.*

The interpretation is the following. For example, a small shock implies risky assets in one agent's balance sheet. The fact to diversify risk (hold diversified products or assets, or diversify funding, etc.) by increasing the degree of connectivity maintains the economy in a sound state and hides risk.

4.2 Pessimistic market

A pessimistic agent is an agent who does not expect ϕ being the highest as possible. In a pessimistic market the expectations of the agents on ϕ are any intermediate expectation, but not the optimistic one. Indeed considering any level of junk agents,

they do not imagine that this level of junk agents corresponds to the maximum level of junk agents obtained for a fixed ϕ making p vary. With this estimated ϕ there are two possible p_0 and p_1 corresponding to the same number of junk agents.

Since agents expect different values of ϕ , they form beliefs on the others' beliefs. The aggregated behavior of agents depends on what they expect being the aggregated behavior. If the agents had exactly the same beliefs on (p, ϕ) , the overall action – the sum of individual actions – , consisting in connecting or disconnecting, would be the same as that of a representative pessimistic agent. However there is a priori no reason why agents would have the same beliefs. Knowing that, an agent naturally tries to benefit from the others' action. The following table is the payoff matrix of the effect of aggregation for an agent acting independently from all the others. For the sake of simplicity we consider that the others – who are say in this example one hundred – have the same beliefs and act the same way. We analyse the following possibilities with $\phi_0 = 0 < \phi_1 < \phi_2$. In column, the payoffs of the isolated agent are presented (right number), while in line the payoffs of any single agent from the group of 100 are given (left number).

Agents	ϕ_0	ϕ_1	ϕ_2
ϕ_0	0,0	-0.01,-1	-0.01,-1
ϕ_1	-0.01,-1	0,0	0.005, -0.5
ϕ_2	-0.01,-1	-0.005,0.5	0,0

Payoff matrix of aggregation for an agent acting independently from all others (acting the same).

Table 1 presents 3 Nash equilibria. Without any coordination, there is no reason why agents' beliefs would converge to a fixed ϕ_1 or ϕ_2 . Therefore the only possible equilibrium is given by $(\phi_0, \phi_0) = (0, 0)$.

Proposition 4.2 *When the market sentiment is pessimistic, there exists an equilibrium belief of agents and it is to expect the real resilience ϕ close to 0.*

The proof is given in Appendix.

Consequently ϕ_r moves to 0. Agents expect the limiting case where $\phi \rightarrow 0$. Since the curve where $\phi \rightarrow 0$ is continuous and strictly increasing, there is only one p corresponding to the number of junk agents.⁸ Consequently, the overall action consists in decreasing the degree of connectivity above the percolation threshold P to decrease the number of junk agents. The overall cost becomes $\int_{[p,p']} c_d(p) d\gamma$.

Proposition 4.3 *With a pessimistic market sentiment, agents largely reduce financial links.*

⁸This directly results from the intermediate value theorem.

This proposition is illustrated on Figure 6.

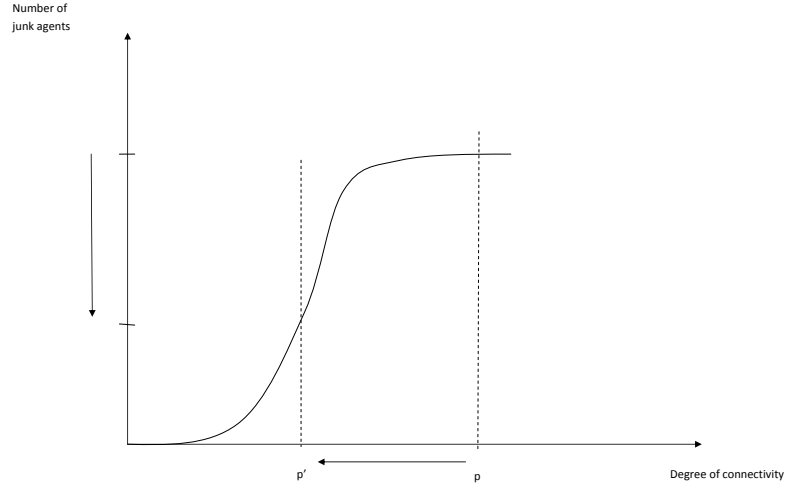


Figure 5: Pessimistic solution - representation of curve γ

4.3 Intermediary cases

We have analyzed the limiting cases where the market sentiment is either optimistic or pessimistic. Considering that each agent has a private sentiment, either pessimistic or optimistic, the union of all agents is a combination of both sentiments.

Let α be the proportion of optimistic agents among the population ($1 - \alpha$, the proportion of pessimistic agents). Optimistic agents behave on the curve where ϕ is the highest possible corresponding to the current level of junk agents. Let us call this point ϕ_O . Pessimistic agents imagine the limiting case where $\phi \rightarrow 0$. Let ϕ_P be this other one. Consequently, the real economy – the sum of all agents – is following the intermediary curve with $\phi_r = \alpha\phi_O + (1 - \alpha)\phi_P = \alpha\phi_O$ since $\phi_P = 0$. To reduce the number of junk agents, pessimistic agents are disconnecting while optimistic ones are connecting.

The global action also depends on the number of optimistic and pessimistic agents. We can identify three possibilities.

1. There are more optimistic than pessimistic agents. The global induced action is a connection. The effect on the economy depends on where the economy

is located on the intermediary curve corresponding to $\phi_r = \alpha\phi_O$. If the economy is not so connected, around the percolation threshold, the slope of the curve of junk agents is positive. The overall action creates connection and also increases the number of junk agents. Conversely if the economy is largely connected, the slope of the curve of junk agents is negative. The overall action reduces the number of junk agents.

2. There are more pessimistic than optimistic agents. The overall action is a disconnection. Again the effect on the economy depends on where the economy stands on the curve. When the economy is slightly connected the overall action reduces the number of junk agents. When the economy is highly connected, the overall action increases the number of junk agents.
3. There is the same number of optimistic and pessimistic agents. The overall action is canceled out. We distinguish two equilibria: the first one when the economy is slightly connected – around the percolation threshold – and the second one when the economy is largely connected. We test the stability of such equilibria. In the first case, a small increase in the number of optimistic agents raises the number of junk agents (see above) and therefore increases the number of pessimistic agents (as explained below). The result is a disconnection, that reduces the number of junk agents. Therefore this equilibrium is stable. In the second case, a small increase in the number of optimistic agents induces an overall connection that reduces the number of junk agents (see Figure 6). This creates more optimistic agents. The connection is somewhat “accelerated”. It is behaving a similar way with pessimistic agents. This second equilibrium is unstable.

Proposition 4.4 *An intermediary market sentiment in a slightly connected economy leads to the existence of an intermediate level of junk agents where the economy can be attracted and converges.*

Note that when the economy converges to this state, some actions (connecting or disconnecting) cancel out and are therefore uselessly costly.

4.4 Agents’ sentiments

We still have to explain the formation of optimistic and pessimistic beliefs. Each agent in the financial market has its own behavior towards risk. For each agent there exists a limit J_0 on J above which he will not think optimistic. Consequently

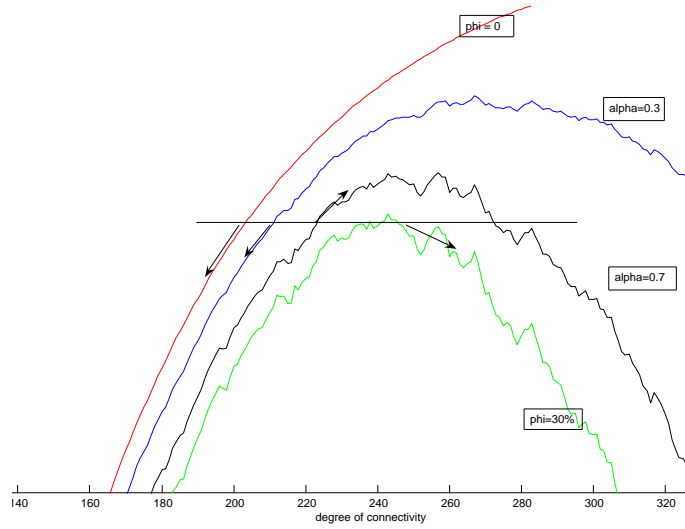


Figure 6: Stable equilibrium

the agent is optimistic until the level of junk agents exceeds his personal limit⁹ and then the agent is pessimistic. There is a priori no reason why agents would have the same threshold between being optimistic and pessimistic. So we can consider that the individual threshold of optimism J_{o_i} follows a continuous¹⁰ distribution. There is no need to detail the form of the distribution, and it may vary. We only need to identify the mean of the distribution: $\mathbb{E}[J_o]$. When the number of junk agents $J < \mathbb{E}[J_o]$, they are more optimistic agents, we are in the first case of the previous paragraph. When $J > \mathbb{E}[J_o]$, we get the second case, and when $J = \mathbb{E}[J_o]$,

⁹We assume here that this limit is constant. However it could depend on the historical behavior of the agent: an optimistic agent is more inclined to keep being optimistic while a pessimistic one is more inclined to remain pessimistic. This creates some hysteresis phenomenon.

¹⁰The continuous aspect is due to the fact that $n \rightarrow \infty$.

we get two equilibria¹¹.

4.5 Variable sources

Up to now, we have considered that there was a natural level of junk sources inside the economy. Since this number of sources is an exogeneous parameter, we now study the effect of variations in this natural level of junk sources s .

Using simulations, we can see that an increase in the level of junk sources systematically induces an increase in the level of junk contaminated agents (Figure 7). This increases the maximum number of contaminated agents, and the economy needs to be more connected to benefit from the effect of network resilience.

We consider that the agents get information on the number of junk sources s as they observe the number of contaminated agents J . The agents react to a shock on sources by increasing or decreasing the connectivity. This only depends on level of J compared to $\mathbb{E}[J_0]$ as explained before.

There are two worsening effects arising from shocks on sources. Since a shock raises the natural number of junk agents, the number of junk agents cannot decrease above the number of junk sources. Especially in case of a large shock on the number of sources, increasing or decreasing the connectivity does not allow reaching the initial state of junk agents. The other effect is that the shock on the number of sources is amplified by connectivity as illustrated on the following figure.

5 Applications

We now apply our model to two recent episodes of crisis (Japan crisis and subprime crisis) in a dynamic progression. This will emphasize the crucial determination of the equilibria, and the way regulators should act to try to improve the state of economy in worsening periods.

5.1 Japanese crisis

The economy of Japan was already highly connected in 1987.¹² This high connection was especially due to the existence of Kereitsu, and to the strong financial links between large banks and big firms. Japan was experiencing a great economic

¹¹Again, the equilibria could be not exactly around the same level if the individual thresholds varied with historical behavior.

¹²See ? for a description; see also Allen and Gale (2007).

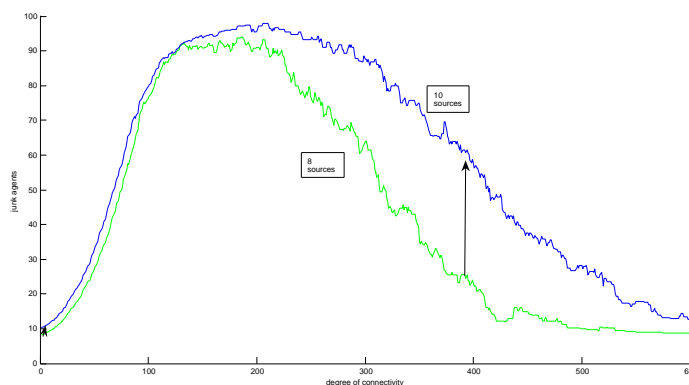


Figure 7: Shocks on the number of sources

growth, illustrating a global movement of investment, reinforcing the already high level of connectivity. The global movement of financial liberalization and the high rate of money availability (excessive liquidity) created underlying conditions for a bubble to be formed. The appreciation of assets and real estate allowed investors to share more capital. This is represented by an increase in ϕ in our model: the demand for credit increases because credits are lent against assets which values also increase. Quickly in Europe inflation and demand pressures have induced restrictive monetary policies and contained the development of a bubble. These pressures did not exist in Japan and allowed for low nominal and real interest rate. The high growth of GDP was sustained by investments. The rise in world interest rates at the end of the 80's provoked the burst of the bubble and weakened the credit market. This feature is represented by a decrease in ϕ . Almost all develop-

ped countries consequently used policies of cleaning up their fundamentals. On the opposite Japan has tried to limit the cost of a recession. The banks have continued to grant insolvent entities credits to avoid an increase in the number of junk agents. At the same time, the level of junk assets increased a lot. The global level of junk agents was maintained relatively low by a global policy of connection. Indeed the economy is around the unstable equilibrium and slightly below $\mathbb{E}[Jo]$. It is represented in our model by a progressive increase in s associated to an increase in p . The situation lasted until 1997 when the government was unable to save the bank Hokkaido Takushoku as well as the prestigious Yamaichi fund from bankruptcy. From these episodes on, a couple of financial panics appeared. Foreign investors removed their investments. Everyone was fully aware of a large systemic risk. This represents a global disconnection. In 1998 a lot of financial entities became bankrupt. The situation was at its the worst. It can be interpreted as the consequence of a global disconnection combined with a decrease in ϕ . The global monetary policy has evolved because the Bank of Japan moved its interest rates to 0 precisely to avoid systemic risk. This measure has allowed long term refunding, which represents a global policy of disconnection. Then progressively the quantity of junk assets decreases and therefore the number of junk agents also decreases to reach a low level, which feature has been observed (only) recently. We illustrate this crisis on the following graph.

5.2 Subprime crisis

Our analysis is based on Figure 8 and starts by considering an economy in relatively good health as was the case at least for industrialized economies in 2005.

The existence and the positions of special entities concentrating credit risks was little known outside expert circles. They contributed to the opaqueness of overall bank risk vulnerability. Due to this uncertainty, a largely local quality problem in the subprime lending market triggered a near-collapse of the Commercial Paper market, as well as the interbank lending market. Indeed, delinquency rates in the subprime market started to rise since 2005 without any significant market response to this development until mid 2007. However, when a small number of funds decided to freeze on redemptions (on 9 August 2007), the turmoil in financial markets came to the boil. Central banks started to undertake extraordinary measures in an attempt to restore order in the interbank market. The subprime crisis suddenly occurred in spite of no change in the fundamental situation of financial institutions and rapidly affected various other sectors (see e.g. IMF (2008), ECB (2008), and BIS

(2008)).

The development of the subprime delinquency is represented by progressive increases in the number of sources (vertical arrow at the bottom left of the graph). Agents' reaction consists in increasing the connectivity to maintain the economy in good health (arrow down towards the right of the graph). This phenomenon has taken place progressively via portfolio diversification of the first financial intermediaries linked to the subprime market, such as investment banks and funds.

The next step is a relatively large positive shock on ϕ , from 36% to 32% (vertical arrow on the right of the graph). Intuitively the diversification induced by securitization has reached its top. No diversification is easy to achieve anymore. Agents suddenly become aware of the global exposure of banks to risky assets. It is again a systemic risk.

Subsequently to the shock on ϕ , the economy is above the threshold $\mathbb{E}[J_0]$ and located around the unstable equilibria. The agents start disconnecting, this situation induces an increase in the number of junk agents. Governments of a few countries have saved a couple of big financial institutions like AIG because they were parts of big connected components, and also were factors of systemic risk. The economy moves along the curve with $\phi = 32\%$. Agents therefore first increase the number of junk agents. This paradoxical effect partly explains the over adjustment that has been observed on equity markets.

The next step consists in decreasing the number of junk sources and disconnecting until the economy reaches the state $\mathbb{E}[J_0]$. It induces a decrease in the number of junk agents (this movement is represented by the arrow towards the bottom left of the graph). Once the economy reaches the intermediary state $\mathbb{E}[J_0]$, the economy converges to the stable equilibrium due to the opposite beliefs of agents. This phenomenon represents a recessionary period. The only way to decrease the number of junk agents in this case consists in reducing the number of sources.

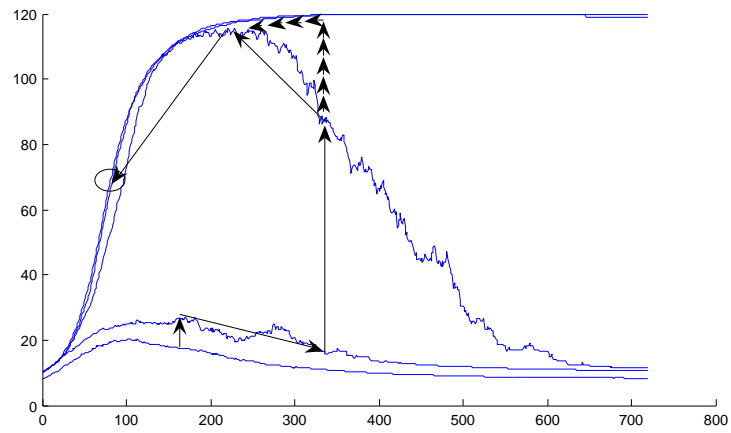


Figure 8: Subprime proliferation

A Proof of 4.2

Without loss of generality, we prove the results of the table for two agents and then extend to any case.

We consider two agents, A_1 and A_2 having different respective beliefs (p_1, ϕ_1) and (p_2, ϕ_2) . Assume that $\phi_1 < \phi_2$ and that both agents believe that the economy is relatively highly connected, above the percolation threshold P of the curve γ as in Figure 5. In the case where they imagine that the economy is below P , the only viable solution is to decrease the connectivity. The last case where one of them believes that the degree of connectivity of the economy is below the threshold P and the other agent believe that the economy is highly connected is dealt with at the end of the proof.

We suppose first that both agents consider that the economy is highly connected. We specify first the cost of the possible actions of agents as if they were acting separately, and then specify the different costs of aggregation. For the first agent, the cost of increasing connectivity is: $\int_{[p_1, p'_1]} c_c(p) d\gamma$; for the second agent, it is: $\int_{[p_2, p'_2]} c_c(p) d\gamma$. We have $p_1 > p_2$ and $p'_1 > p'_2 + (p_1 - p_2)$, because the slope of the curve with the highest ϕ is steeper than that with the smallest ϕ . So $\int_{[p_1, p'_1]} c_c(p) d\gamma > \int_{[p_2, p'_2]} c_c(p) d\gamma$. Economically, it is easy to understand that the agent expecting a good network resilience has a less costly solution.

The aggregated action is the average of both, it means there exist (p_3, ϕ_3) and (p'_3, ϕ_3) such that $p_1 > p_3 > p_2$, $p'_1 > p'_3 > p'_2$ with $\phi_1 < \phi_3 < \phi_2$. The common action must satisfy:

$$\beta \int_{[p_1, p'_1]} c_c(p) d\gamma + (1 - \beta) \int_{[p_2, p'_2]} c_c(p) d\gamma = \int_{[p_3, p'_3]} c_c(p) d\gamma,$$

with $\beta \in]0, 1[$ representing the different financial weights of agents.

Finally for both agents the cost is $\int_{[p_3, p'_3]} c_c(p) d\gamma$. We want to determine the effect of aggregating the behavior of agents. To this aim, we calculate the difference between the cost of being alone and being two inside the economy. For the first agent the benefit of aggregating his action to that of the other is: $\int_{[p_3, p'_3]} c_c(p) d\gamma - \int_{[p_1, p'_1]} c_c(p) d\gamma < 0$; for the second agent it is: $\int_{[p_3, p'_3]} c_c(p) d\gamma - \int_{[p_2, p'_2]} c_c(p) d\gamma > 0$.

To get the different payoffs of section 4.2, we consider that all agents are homogenous in size. The situation is also equivalent to two different agents, one of size 1 and one of size 100. We consider that the action of each agent is proportional to its financial weight. If both agents expect the same ϕ the cost of the aggregation is 0. If they imagine the cases ϕ_1 and ϕ_2 we see that one works for the other. For example

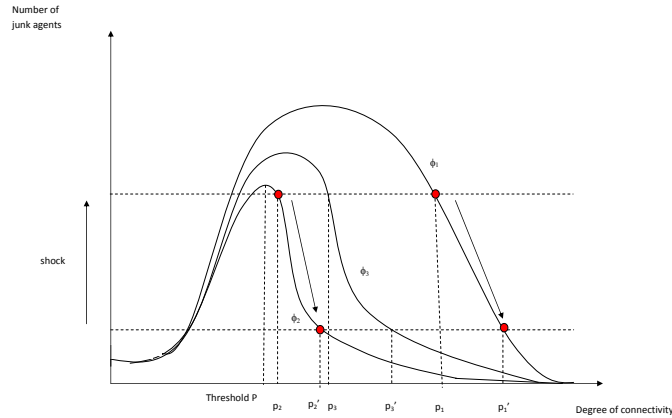


Figure 9: Different pessimistic expectations of agents about network resilience

if the big agent believes that $\phi_r = \phi_1$ and the other one believe $\phi_r = \phi_2 > \phi_1$, the small agent is going to make connections for the big one, doing part of his work. The payoff of the big agent is also 0.5, while the other works 0.5 times more. His payoff is -0.5 . The same way we get the other possible situation with (ϕ_2, ϕ_1) . To conclude, the more an agent expects a resilience lower than the others, the more he benefits from actions of the others. If we consider now that there are a hundred agents acting the same instead of one big agent representing a hundred units, we divide the payoff by a hundred.

The last case deals with the situation where the expectations about the connectivity are different. Suppose for example that the 100 agents imagine a low connectivity. Their common action is a disconnection. On the contrary the 1 agent imagines a high degree of connectivity and his action is a connection. In this case, the global action is a disconnection (sum of all actions). The action of the 1 agent is therefore completely canceled, while the 100 agents must connect for one agent more, so they need to work $1/100$ times more. We therefore get the payoffs, -1 for the 1 agent and -0.01 for the others. We see that no one benefits from the aggregation, and the total cost is negative. It means that they "distroy" wealth. **NEGATIVE SUM GAME.**

Finally when the agents expect a high degree of connectivity, they expect a weak ϕ . When the agents expect a low degree of connectivity, they also behave as if they were imagining $\phi \rightarrow 0$, because the curves for different values of ϕ are equal up to

the percolation threshold. This leads to Proposition 4.2.

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