The Subjective value of Executive Stock Options : A Comparative Study.

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Abstract

Executive compensation plans usually take the form of incentive options that are held by underdiversified risk averse managers. This lack of diversification combined with the risk aversion make the managers discount the market value of their options. In a first time, this paper derives a model for the subjective value of such an instrument. The developed framework relies on the Sharpe ratio technique, and is no more difficult to apply than is the Black-Scholes model. Then, we provide a comparative study of alternative valuing models such as those of Meulbroek (2001), Ingersoll (2002) and Tian (2002). The analysis shows that results of the selected approaches do not always converge especially when it's about out of the money options. According to Meulbroek (2001) and Ingersoll (2002), managers seem to always discount longer maturity options whereas the results of our model and those of Tian (2002), show that the choice of option maturity should depend on the choice of the option exercise price.

1 Introduction

During the two last decades, many companies have made of stock options the central feature of their remuneration programs. Even if Economic theory supports the use of such stock based instrument as an incentive mean for managers to maximize shareholder value, this same theory tempers the use of this instrument by several drawbacks. In fact, linking the executive performance to that of the firm is synonym to compelling the grantee to hold in his wealth a large part of the firm's stock. Because of the undiversified nature of his portfolio, the risk averse constrained manager discounts the market value of his option.

In this paper, we address the problem of estimating the value the manager places in his stock option, also called subjective option value.

This framework relies on the Sharpe ratio technique, but unlike Meulbroek (2001), it incorporates the manager's preference and hence yields the hole value of the "deadweight cost" [1](Meulbroek (2001)) associated with this equity based compensation.

Consistent with prior research (Hall and Murphy (2000), Tian (2002), Ingersoll (2002), Lambert and al (1991), Kulatilaka and Marcus (1994)), our paper analyses the extend the manager's lack of diversification and risk aversion impact the option value. This examination provides an insight into the incentive power of the granted option. Actually, the larger the

[1] According to Meulbroek (2001), the "deadweight cost" refers to the diffrence between the private value and the market value of the stock option.

gap between the subjective value of the option and its market value, the less efficient is the incentive alignment benefit of this instrument.
To better place our work in a proper body research, the analysis is extended to some previous models and further light are shed on the Meulbroek (2001), Tian (2002) and Ingersoll (2002) approaches by comparing their numerical results to those of our applied model.
The purpose of this analysis is to test the homogeneity of the results derived from different valuation models. This comparison delivers significant new insights. It highlights a divergence between the studied approaches and demonstrates that the incentive power of longer maturity deep out of the money options varies depending on wether we consider our model and that of Tian (2002) or the approach of Meulbroek (2001) and that of Ingersoll (2002).
This paper is organized as follows. In section 2, we briefly review the approaches of Meulbroek (2001), Ingersoll (2002) and Tian (2002), section 3 presents our valuation model, section 4 provides numerical results illustrating the behavior of subjective option value derived from the different selected models and section 5 concludes.

2 The Previous Models

The problem of estimating the private option value has been analyzed by previous literature. In this section, we briefly review three approaches : the model of Meulbroek (2001), the model of Tian (2002) and that of Ingersoll (2002).

2.1 The Meulbroek (2001) Model

The approach adopted by Meulbroek (2001) is based on the Sharpe technique and aims to measure the cost generated by the manager's loss in diversification. To estimate this cost, Meulbroek (2001) derives the minimum return r_s^d the executive would require in order to be indifferent between holding a stock of the firm he is managing, and a diversified portfolio. This minimum expected return is given by the following equation:

$$E(r_s^d) = \frac{\sigma_s}{\sigma_m}(\mu_m - r_f) + r_f, \tag{1}$$

Where σ_s is the firm volatility, σ_m is the market volatility, μ_m is the expected return on market portfolio and r_f is the riskless return.
From the extracted expected return premium $r_s^d - \mu_s$, Meulbroek (2001) deduces the private value S^d the manager places in the stock of the firm.

$$S^d(t) = S(t)e^{-(r_s^d - \mu_s)(T-t)}, \tag{2}$$

Where μ_s is the expected rate of return of the security S under CAPM pricing:

$$\mu_s = r_f + \beta_s(\mu_m - r_f), \tag{3}$$

where

$$\beta_s = \frac{\rho_s \sigma_s}{\sigma_m}$$

ρ_s is the instantaneous correlation coefficient between the firm and the market portfolio. and T the date at which the undiversified manager is no more constrained and hence free to sell the stock of his firm.
This value is thus used as an input in the Black-Scholes model ($C(.)$) in order to derive the corresponding subjective value of the executive stock option $\hat{C}$.

$$\hat{C}(t) = C(S^d(t), T-t, \sigma_s, r_f, X). \tag{4}$$

Where X is the exercise price generally equal to the stock price at the grant date of the option.

These two estimates are, however, upper bounds of the real subjective value of a stock/option based compensation. In fact, an exact estimate would take into account the manager's individual preference. We can measure the impact of such preference by using the second category of models which is the Certainty Equivalent approach.

2.2 The Tian (2002) Model

The Certainty Equivalent approach defines the subjective value of the executive stock option as the immediate cash payment that yields the same expected utility as an option does.

Several studies have adopted this approach (e.g., Lambert & al (1991), Hall and Murphy (2000), Kulatilaka and Marcus (1994)) but only Tian (2002) incorporates the executive optimal investment decision while calculating the manager's private value of the option. He supposes, given the option grant, that the executive invests his unrestricted wealth between the market portfolio and the risk free asset in order to maximize the expected utility of his terminal wealth W_T.

$$W_T(\lambda_1) = w_L(\lambda_1 \frac{M(T)}{M(0)} + (1-\lambda_1)\, e^{r_f T}) + nMax(0, S(T) - X), \tag{5}$$

Where w_L is the liquid wealth of the manager, n the number of options held by the executive and $\lambda_1$the percentage of his liquid wealth invested in the portfolio market M.

This optimal investment decision is different from that made by an external unconstrained investor who can freely invest his total wealth between the market portfolio and the riskless asset.

$$\hat{W}_T(\lambda_2) = (w_L + n\hat{C})(\lambda_2 \frac{H(T)}{H(0)} + (1-\lambda_2)\, e^{r_f T}), \tag{6}$$

where $\hat{C}$ is the certainty equivalent of the option, and λ_2 is in this case the percentage of wealth invested in the portfolio market.

Given the geometric Brownian motions of the two risky assets:

$$\frac{\partial S(t)}{S(t)} = \mu_s dt + \sigma_s dZ_s, \tag{7}$$

$$\frac{\partial M(t)}{M(t)} = \mu_m dt + \sigma_m dZ_m, \tag{8}$$

Where, $Z_s(t)$ and $Z_m(t)$ are standard Weiner processes. Returns on the firm's stock and the market portfolio are supposed to be correlated:

$$dZ_m(t) dZ_s(t) = \rho_s dt,$$

and given the power utility function of the manager defined as follows:

$$u(W) = \begin{cases} \frac{W^{1-d}}{1-d} \text{ if } d \neq 1 \\ \ln(W) \text{ otherwise,} \end{cases}, \tag{9}$$

Where d is the coefficient of risk aversion

Tian (2002), solves numerically the following optimization problem:

$$\begin{cases} \underset{\lambda_1}{MaxE} \{u(W(\lambda_1))\} = \underset{\lambda_2}{MaxEu}(\hat{W}(\lambda_2)) \end{cases}. \tag{10}$$

The optimal solution of this problem provides the optimal investment decision in both the constrained and unconstrained cases, λ_1^*, λ_2^* and therefore the certainty equivalent value of the executive stock option $\hat{C}$.
Even if this method incorporates the manager's preference, compared with the Meulbroek (2001) model, it does not propose a closed form solution for the manager's private option value. This is achieved by Ingersoll (2002) model.

2.3 The Ingersoll (2002) Model

Ingersoll (2002) examines the following problem for a manager specified with a power utility function defined over lifetime consumption and bequest:

$$\frac{1}{d-1}\int_0^{T'} e^{-\delta t} C_t^{1-d}\, dt + \frac{b}{1-d} W_{T'}^{1-d} \tag{11}$$

Where T' is the manager's death time, b a multiplier determining the relative contribution to utility from consumption and the bequest of wealth at time T'.
Before retiring, the author assumes that the manager is compelled to hold a certain proportion α of the stock of his firm. His wealth is therefore allocated between the company's stock, the market portfolio and the risk free asset. However, after his retirement, the no longer constrained manager invests his entire wealth between the market portfolio and the riskless asset as predicted by the mean-variance efficient frontier.
Ingersoll (2002) derives with his model the subjective value of any asset by using the marginal utility function of the executive as a martingale pricing process. He recognizes the partial differential equation of the subjective value of the option or other compensation instrument as the Black-Scholes equation with discounting at a subjective interest rate:

$$\widehat{r_f} = r_f - d\alpha^2 \upsilon^2, \tag{12}$$

and with a subjective adjustment to the dividend yield:

$$\widehat{q} = q + d\alpha(1-\alpha)\upsilon^2, \tag{13}$$

where υ^2 is the residual variance[2].
The private value of the option held by the manager is then equal to:

$$\hat{C} = C(S(t)e^{-\widehat{q}(T-t)}, T-t, X, \widehat{r_f}, \sigma_s). \tag{14}$$

3 The valuation Model

The main objective of executive stock options is to align the financial interest of executives with that of shareholders and hence to improve the economic performance of the firm.
Unfortunately, the side effects of such instruments are not only benefits. In deed, as well noticed by Meulbroek (2001) "if the only results were incentives alignments, manager's compensation would be 100% equity based".
So what prevents firms from granting more and more stock options?
An equity based compensation tends to make executive "think like owners" without letting them really behave like owners. Unlike fully diversified shareholders, managers are forced

[2] The total risk of the stock of the firm is $\sigma_s^2 = \beta_s^2\sigma_m^2 + \upsilon^2$.

to hold non traded stock options that can neither be exercised until maturity nor hedged by shorting the firm's stock.
These binding constraints, necessary to produce proper incentives, are responsible for the manager's idiosyncratic risk exposure. In fact, by investing not only their human capital but also a part of their wealth in the firm, executives are obliged to face a high level of specific risk which is completely removed by outside investors.
Even more, because of this additional risk, the executive's portfolio becomes more volatile without any commensurate increase in its expected value. As shown by The CAPM model, the derived expected return only rewards the systematic risk of the firm. As a consequence, risk averse executives discount their option value from its market value. This section presents the model designed to address the important question of how valuable is a stock option to an undiversified risk averse manager?.
Consistent with prior research, the manager is assumed to be risk averse, with constant relative risk aversion d and specified by the same power utility function as defined above.
The shareholder is also taken to be risk averse with constant relative risk aversion a and identified by the following power utility function:

$$u(W) = \begin{cases} \frac{W^{1-a}}{1-a} & \text{if } a \neq 1 \\ \ln(W) & \text{otherwise,} \end{cases} , \tag{15}$$

The argument behind this assumption is that diversification does not cancel out all risks. A positive risk premium is required by the shareholder for bearing the systematic risk component.
In addition, according to Bliss and Panigirtzoglou (2004), a change in volatility can be expected to have a coincident change in market stock prices as follows:

$$\frac{\triangle\, S(t)}{S(t)} \approx -a\triangle\sigma^2(t), \tag{16}$$

By applying this relation to the manager we obtain:

$$\frac{\triangle\, S^d(t)}{S^d(t)} \approx -d\triangle(\sigma^d(t))^2, \tag{17}$$

Where $\sigma^d(t)$ is the risk borne by the manager and $S^d(t)$ the value the manager places in the stock of his firm at date t.
At date T, the binding constraints are taken away and the undiversified manager is free to sell his/her shares in the market. So that, the value of the stock to the manager will equal its expected market value:

$$S^d(T) = S(T), \tag{18}$$

and the risk borne by the executive will equal that borne by the shareholder:

$$(\sigma^d(T))^2 = \sigma^2(T), \tag{19}$$

The manager's private value of the stock is then related to its market value as follows:

$$S^d(t) = \frac{S(T)}{1 + \frac{d}{a}(\frac{S(T)}{S(t)} - 1) + d\,[(\sigma^d(t))^2 - \sigma^2(t)]}, \tag{20}$$

We also know that the discounted expected future value of the stock at time T, $S(T)$ equals today's stock price :

$$S(t) = e^{-\mu_s(T-t)}S(T), \tag{21}$$

Substituting (21) in (20), we have:

$$S^d(t) = \frac{e^{\mu_s \tau} S(t)}{1 + \frac{d}{a}(e^{\mu_s \tau} - 1) + d\left[(\sigma^d(t))^2 - \sigma^2(t)\right]}. \tag{22}$$

Where $\tau = (T - t)$.
The manager's subjective value of the stock is equal to its market value, multiplied by the amount:

$$\frac{e^{\mu_s \tau}}{1 + \frac{d}{a}(e^{\mu_s \tau} - 1) + d\left[(\sigma^d(t))^2 - \sigma^2(t)\right]}.$$

Equation (22) suggests that, the more risk averse the manager, the less worth is the stock to him/her as:

$$\frac{\partial S^d}{\partial d} \prec 0,$$

Similarly, the larger the excess risk the executive faces, the smaller the private value he/she places in the stock of the firm.

$$\frac{\partial S^d}{\partial \sigma^d} \prec 0,$$

Equation (22) should also provide support for the assumption that the longer the manager is forced to hold an undiversified portfolio, the less valuable it is for him/her. This is true only if:

$$\frac{\partial S^d}{\partial \tau} \prec 0,$$

Hence, the choice of (a) is not quite arbitrary and should respect, for positive expected rate of return μ_s,the following condition:

$$a \prec \frac{d}{1 + d\left[(\sigma^d(t))^2 - \sigma^2(t)\right]}. \tag{23}$$

3.1 The case of a completely undiversified manager

Using the Sharpe ratio technique, Meulbroek (2001b) shows that the manager would require a risk return ratio equal to market's risk return one in order to be indifferent between holding the stock of the firm and the market portfolio. this expected return:

$$E(r_s^d) = r_f + \frac{\sigma_s}{\sigma_m}(u_m - r_f),$$

is different from that derived from the CAPM model because of the excess risk the manager should face. To show that this additional risk is effectively the idiosyncratic component, we calculate, from the shareholder point of view, the volatility of a portfolio rewarding the expected return required by the executive.
Consider the self financing portfolio P which is long fraction 1.0 in the stock of the firm, long fraction $\beta_s(\frac{1}{\rho_s} - 1)$ in the market portfolio and short fraction $\beta_s(\frac{1}{\rho_s} - 1)$ in the riskless asset B.

$$\frac{\partial P}{P} = \frac{\partial S}{S} + \beta_s(\frac{1}{\rho_s} - 1)\frac{\partial M}{M} - \beta_s(\frac{1}{\rho_s} - 1)\frac{\partial B}{B} \tag{24}$$

Where

$$\frac{\partial B}{B} = r_f dt$$

This construction creates a portfolio with an expected return equal to that required by the constrained manager (r_s^d) and a volatility of $\sigma_s^2(1-\rho^2)+\sigma_s^2$.
The standard deviation of portfolio P corresponds to the volatility of the stock return plus its specific risk component. Accordingly, the market would reward an unconstrained investor, excess return $E(r_s^d)-\mu_s$ for bearing the idiosyncratic risk.
This can lead us to that, if the outside investor is exposed to risk σ_s^2, the manager is, from the shareholder point of view, facing the risk $\sigma_s^2(1-\rho^2)+\sigma_s^2$.
Table (1) summarizes these results:

Table 1:

	case of unconstrained investor	**case of constrained investor**
expected return	$r_f+\beta_s(u_m-r_f)$	$r_f+\frac{\sigma_s}{\sigma_m}(u_m-r_f)$
volatility	σ_s^2	$\sigma_s^2(1-\rho_s^2)+\sigma_s^2$

One can again confirm that the risk the undiversified manager faces is larger than that borne by the shareholder and is equal to:

$$(\sigma^d(T))^2=\sigma_s^2(1-\rho^2)+\sigma_s^2, \tag{25}$$

The private value of the stock held by an undiversified risk averse executive is then equal to:

$$S^d(t)=\frac{e^{\mu_s\tau}S(t)}{1+\frac{d}{a}(e^{\mu_s\tau}-1)+d\left[(1-\rho^2)\sigma^2(t)\right]}, \tag{26}$$

Consequently his private option value is :

$$\hat{C}=C(S^d(t),T-t,X,r_f,\sigma_s). \tag{27}$$

3.2 The case of a partially diversified manager

The subjective option value calculations outlined above assume that the manager is completely undiversified, so that all his wealth is invested in stock or options of the firm. The previous assumption does not fit to the reality of a large part of executives[3] that can hold partially diversified portfolios. This partial diversification is fulfilled, in this section, by allowing the manager invest his unconstrained wealth in the riskless asset and the market portfolio.
We so can again derive the expected return, the partially diversified risk averse manager require to be indifferent between holding the stock of the firm and the market portfolio. This expected return is equal to r_s^{*d} (In Appendix A, we derive all calculations)
But what kind of risk would the partially diversified manager be compelled to face if he has the opportunity to invest his liquid wealth in the market portfolio and the free risk asset. Appendix B, contains this derivation and demonstrates that the standard deviation $(\sigma^{*d})^2$ is the risk borne, according to the market, by the manager requiring the expected return r_s^{*d}.

[3]Executives in start-up firms could however be completely undiversified.

The subjective private stock/option value calculation for a partially diversified manager parallels that for the case of completely undiversified one. This calculation gives respectively the stock's and option's subjective value:

$$S^{*d}(t) = \frac{e^{\mu_s \tau} S(t)}{1 + \frac{d}{a}(e^{\mu_s \tau} - 1) + d\left[(\sigma^{*d}(t))^2 - \sigma^2(t)\right]}, \tag{28}$$

$$\hat{C} = C(S^{*d}(t), T - t, X, r_f, \sigma_s). \tag{29}$$

4 Numerical results

It's now well understood that the value-cost inefficiency of an executive stock option can be quite large, all previous studies corroborate this result. But none of these studies show that the subjective value of an executive stock option may react differently depending on the model it's based on. In this section we apply the different approaches developed in section 2 and 3 to examine, in each case the discount an undiversified and risk averse manager may assign to the option value and show how the subjective option value may vary with different key factors and selected models.
The first stage of our analysis consists of implementing our model and those of Meulbroek (2001), Tian (2002) and Ingersoll (2002). If Meulbroek (2001) and Ingersoll (2002) approaches are a direct application of the Black and Scholes formula, the certainty equivalent valuation model is not. To solve the optimization problem of Tian (2002), we have used Monte Carlo simulations.
The second stage consists of conducting the numerical simulations and analyzing them. For that purpose, the executive total wealth is normalized at 100$, the risk free rate is assumed to be 5%, the expected return of the market portfolio to 12.5%, its volatility to 20%, the stock's volatility to 30% and the dividend yield to 0%.
We also assume that the coefficient of risk aversion of the shareholder is equivalent to:

$$a = \frac{d}{1 + (T - t)d\left[(\sigma^d(t))^2 - \sigma^2(t)\right]}, \tag{30}$$

respecting by that, for $T - t \succeq 1$, the condition of equation (3):

$$\frac{d}{1 + (T - t)d\left[(\sigma^d(t))^2 - \sigma^2(t)\right]} \preceq \frac{d}{1 + d\left[(\sigma^d(t))^2 - \sigma^2(t)\right]}$$

As long as the executive is compelled to hold the nontradable incentive option ($(T - t) \neq 0$) or/and not allowed to diversiy away the idiosyncratic risk of the firm $(\sigma^d(t))^2 - \sigma^2(t) \neq 0$), his/her relative risk aversion coefficient is diffrent from that of a fully diversified shareholder. When time to maturity $(T - t)$ is however inferior to one year, the constraints on manager's wealth become less binding, we choose for this case a coefficient of risk aversion (a) equal to:

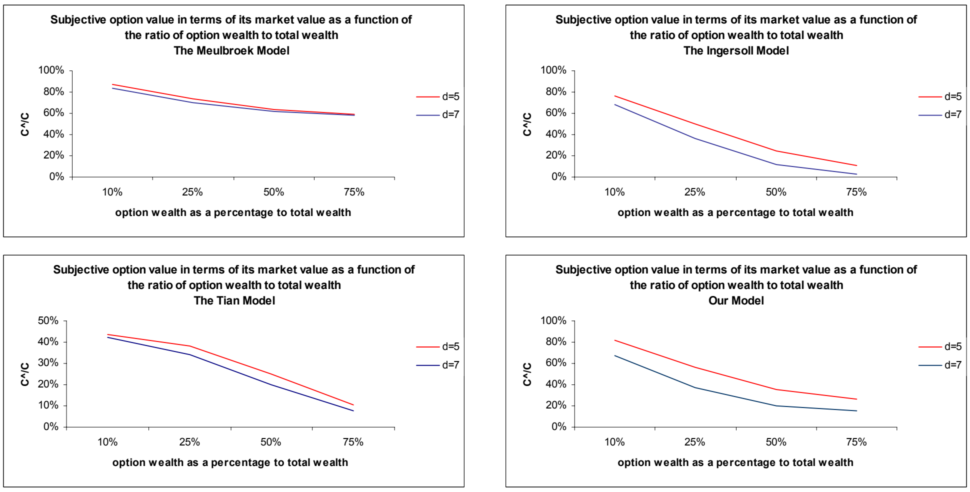
$$a = \frac{d}{1 + d\left[(\sigma^d(t))^2 - \sigma^2(t)\right]} \tag{31}$$

To better examine our results, we divide our framework in two parts, the first one is devoted to at the money options however the second one is devoted to in and out of the money options. We consequently, show for each case, how the subjective value of the option $\hat{C}$ in terms of its market value C varies over a wide range of parameters.

4.1 At the money options

Consistent with previous research, the key parameters, considered in this section, and impacting the subjective value of the option are : the coefficient of relative risk aversion, maturity of the option, the fraction of wealth invested in options, the firm volatility and the correlation between firm returns and market returns.
As expected, all the models considered here, show that the private value of the option in terms of its market value ($\hat{C}/C$) is negatively related to the coefficient of relative risk aversion (d), to the fraction of option wealth (w) and to the firm risk (σ_s).
Figure (1) illustrates the first relationship and shows that the more risk averse the manager is, the lower the subjective value becomes[4].

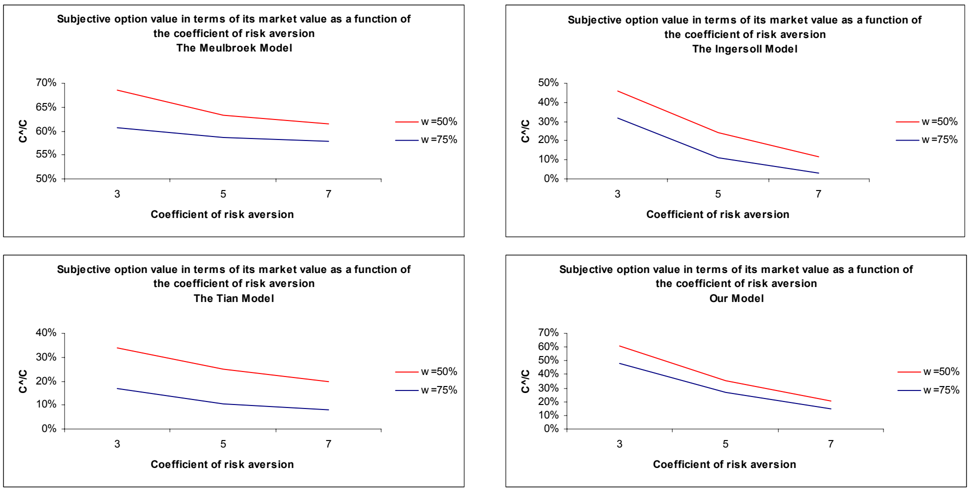
Figure 1: Relation between the subjective option value and the coefficient of risk aversion of the manager



For the following base case ($d = 5$, $\sigma_s = 30\%$, $\sigma_m = 20\%, w = 50\%, \rho_s = 0,25$, $T = 2$) and when the coefficient of relative risk aversion rises from 5 to 7 (all other parameters are kept constants) the $\hat{C}/C$ ratio falls from 24.3% to 11.5% in the Ingersoll (2002) case, and from 25.02% 19.95% for the Tian (2002) model, from 35.32% to 20.38% according to our model and from 63.42% to 61.47% in the Meulbroek (2001) case.
Similarly, if the fraction of option wealth increases from 50% to 75% or the firm volatility rises from 30% to 60%, The $\hat{C}/C$ ratio falls respectively from 24.3% to 10.97% or to 2.48% in the Ingersoll (2002) case, and from 25.02% to 11.52% or to 11.49% for the Tian (2002) model, from 35.32% to 26.65% or to 10.04% according to our model and from 63.42% to 58.67% or to 53.78% in the Meulbroek (2001) case.
Figure (2) illustrates the magnitude of the relation between the subjective option value and the fraction of option wealth.
Figure (3) illustrates the relation between the private option value and the stock volatility. The negative correlation between option value and option wealth or firm volatility is explained by the fact that when one or both of these parameters rise, the terminal wealth

[4]Before applying the Meulbroek (2001) model, we have supposed that the partially diversified manager can invest a part of his wealth in the market portfolio and the riskless asset. Consistent with the other studied models we assume that the executive invests his liquid wealth in the proportion of Merton (1969), which makes the derived option value indirectly slightly dependent on the risk aversion of the manager.

Figure 2: Relation between the subjective option value and the fraction of option wealth



of the executive becomes more riskier without any commensurate increase in its expected value.
In addition, the ratio $\hat{C}/C$ is found to be positively related to the coefficient of correlation between the firm and the market portfolio ρ_s. In fact, for the same previous base-case and when the coefficient of correlation (ρ_s) increases from 0.25 to 0.67 making the firm's beta equal to 1, the $\hat{C}/C$ ratio rises from 24.3% to 46.83% in the Ingersoll (2002) case, from 25.02% to 47.41% for the Tian (2002) model, from 35.32% to 48.25% according to our model and from 63.42% to 82.24% in the Meulbroek (2001) case. Figure (4) illustrates these results.
This positive relationship can be explained by two simultaneous effects:

1 A higher coefficient of correlation (ρ_s) leads to a larger Beta and therefore to a greater expected return (all other variables and especially the firm volatility and portfolio market risk are held constant) which in its turn makes the option more valuable to its holder.

2 A higher correlation between the firm's stock and the market portfolio provides the manager with possibility to hedge his firm's risk exposure by trading the portfolio market.

Numerical simulations display a positive relation between the option maturity and its subjective value. We hence find, when the option maturity rises from 2 years to 5 and for a manager with a coefficient of risk aversion of 7 (all the other parameters are equal to those of the base case), that the ratio $\hat{C}/C$ falls from 11.5% to 2.10% for the Ingersoll (2002) model, and from 19.95% to 12.75% for the Certainty equivalent approach, from 20.38% to 16.29% in our case and from 61.47% to 43.52% for the Meulbroek (2001) model.
The Figure (5) illustrates this correlation.

Figure 3: Relation between the subjective option value and stock volatility

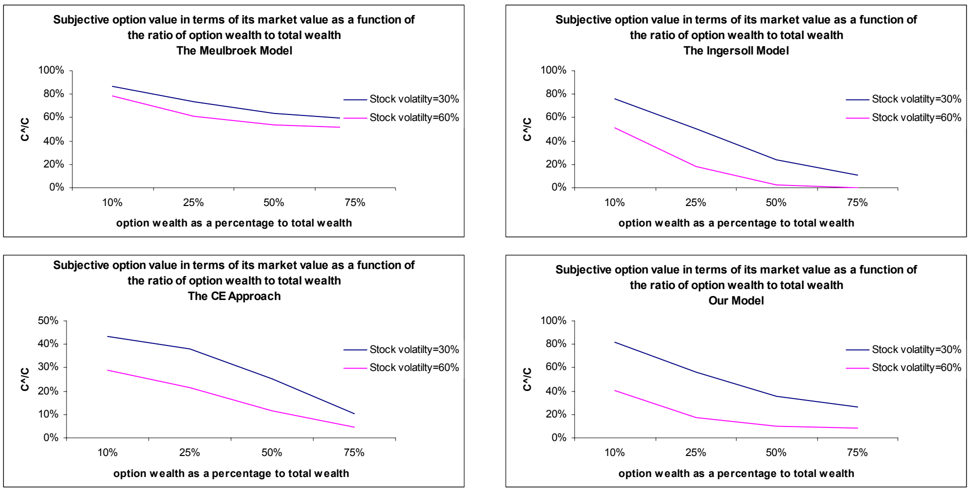


Figure 4: Relation between the subjective option value and the coefficient of correlation ρ_s

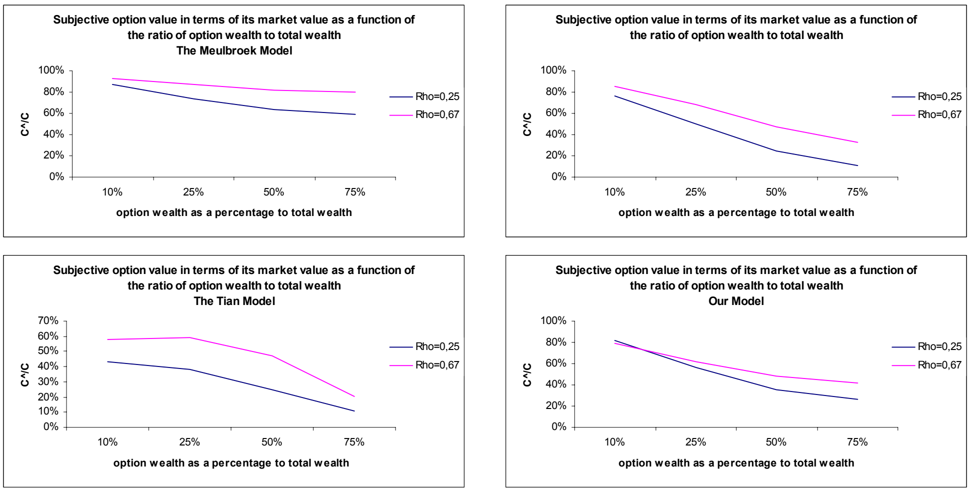
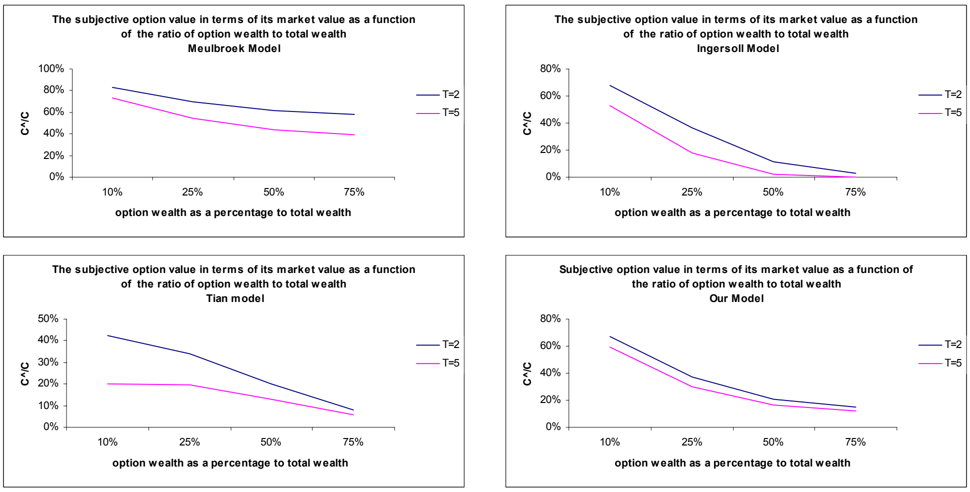


Figure 5: Relation between the subjective option value and the option maturity

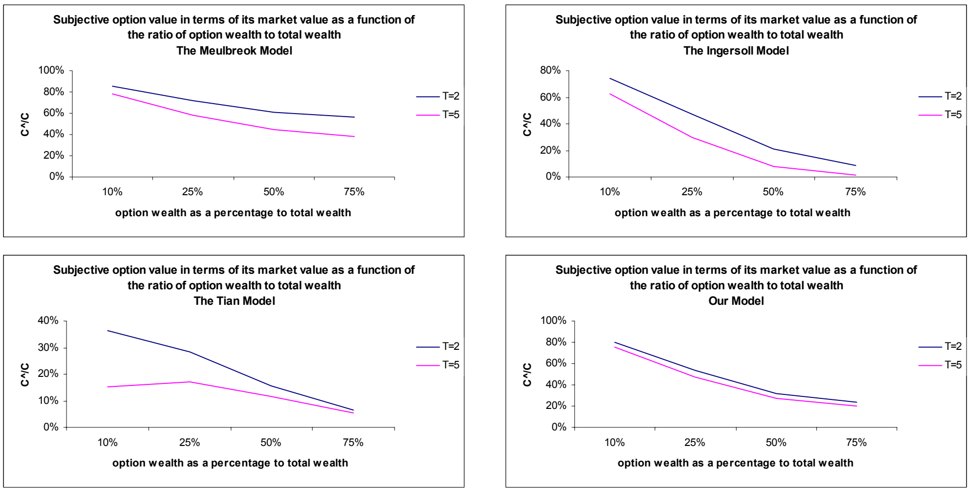


4.2 Out and in the money options

Table (2) reports for the different selected models, the $\hat{C}/C$ ratio for various levels of exercise prices K and shows that the manager discounts more the value of an out of the money option than that of an in the money one. Thus, according to Ingersoll (2002), the $\hat{C}/C$ ratio, for an in the money option (X/S=0.9) and out of the money option (X/S=1.1) is equivalent respectively to 14.25% (9.5%), it reaches 64.26% (58.84) in the Meulbroek (2001) case, 36.71% (11.49%) for the Tian (2002) model and 37.02% (17.78%) for our model.
Accordingly, the higher the executive price, the lower is the probability that the option terminates in the money and the less valuable is this pay instrument for its holder.
Besides, the four selected models reveal that the option moneyness (X/S) does not seem to alter the relationship between the subjective option value and several key parameters such as risk aversion, option wealth and coefficient of correlation.
For example, for an option moneyness of 1.1, we notice that an increase in the coefficient of relative risk aversion of the manager from 5 to 7 or in the firm volatility from 30% to 60% will make the $\hat{C}/C$ ratio decrease respectively from 15.72% to 11.49% or to 9.11% for the Tian (2002) model, from 60.90% to 58.84% or to 32.52% for the Meulbroek (2001) model, from 21.2% to 9.5% or to 1.31% for the Ingersoll (2002) model and from 32.24% to 17.78% or to 5.7% for our model.
In a similar way, when the option wealth rises from 50% to 75% or the coefficient of correlation (ρ_s) rises from 25% to 67%, the $\hat{C}/C$ ratio falls from 15.72% to 6.49% or increases to 32.71% for the Tian (2002) model, from 60.90% to for 55.94% or to 81.03% for the Meulbroek (2001) model, from 21.29% to 8.83% or to 43.33% for the Ingersoll (2002) model and from 32.24% to 23.74% or to 45.26% for our model.
Alternatively, the relationship between the option value and the option maturity is much more ambiguous. The four studied models do not provide homogeneous results. Unlike, the Meulbroek (2001) and Ingersoll (2002) models, our approach with the Certainty equivalent one point out that this relationship does change with option moneyness.
When the option is in or near the money, the impact of extended maturity on the option value is negative for all models. For example, for an option moneyness of 1.1 the $\hat{C}/C$ ratio falls from 15.72% to 11.52% when the option maturity passes from 2 to 5 years for the certainty equivalent approach, it drops from 21.29% to 7.85% in the Ingersoll (2002) case, from 60.90% to 44.20% for the Meulbroek (2001) model and from 32.24% to 27.36% in our case.
Figure (6) illustrates the relation between the private option value and the option maturity (for an option moneyness of 1.1):
In contrast, when the executive option is deep out of the money both certainty equivalent approach and our model show that $\hat{C}/C$ ratio increases with longer option maturities. For example, for an option moneyness of 1.5 and for a coefficient of risk aversion of 7[5], the $\hat{C}/C$ ratio rises from 10.83% to 11.06% for our model and from 1.77% to 2.12% for the Tian (2002) model, when option maturity increases from 2 years to 5. A contrary relation is revealed by both Meulbroek (2001) and Ingersoll (2002) models. These latters still display a drop in the value of $\hat{C}/C$ even when option maturity grows. Hence, the subjective option value in terms of its market value falls from 4.58% to 0.83% in the Ingersoll (2002) case and from 50.34% to 36.22% in the Meulbroek (2001) one.
Figure (7) gives an illustration of the relation between the private option value and the option maturity for deep out of the money option, the option moneyness is equal to 1.5 in this case.

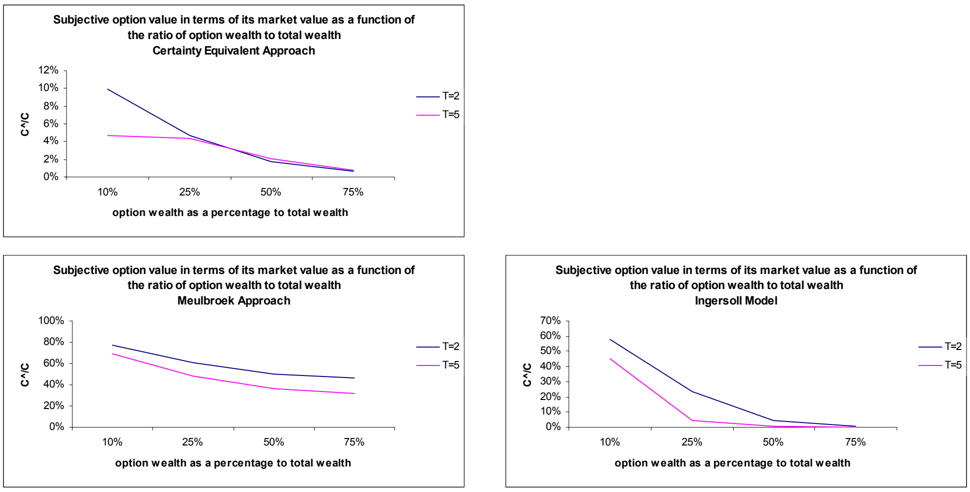
[5] All other parameters are equivalent to those of the base-case.

Figure 6: Relation between the subjective option value and the option maturity : the out of the money option case



These results show that owing to Meulbrek (2001) and Ingersoll (2002) models, longer maturity option are less valuable and hence less attractive to managers whereas for our model and the certainty equivalent method, this assumption is only true when the option is in or at the money. Out of the money options, however, become more desirable with increasing maturities.

Figure 7: Relation between the subjective option value and the option maturity :the deep out of the money option case



5 Conclusion

In this paper we have provided a simple framework to value executive stock options. The approach adopted is no more difficult than the Black-Scholes model with a modified parameter incorporating the risk aversion of the manager and its lack of diversification.
In the absence of a benchmark subjective value, the numerical results of the model developed here are compared to those of Meulbroek (2001), Tian (2002) and Ingersoll (2002). Consistent with these previous studies we find that the constrained risk averse manager can largely discount the market value of his option. His private value is effectively bounded above by the Black-Scholes one.
What is more surprising, is that the results provided by the considered models do no not always converge. According to Meulbroek (2001) and Ingersoll (2002), managers seem to always discount longer maturity options whereas the results of our model and those of Tian (2002), show that the choice of option maturity should depend on that of the exercise price. In fact, they show that it's more efficient to grant shorter maturity at or in the money options however it's more incentive to allocate longer out of the money options. These latter results seem to adhere to the reality of an undiversified risk averse manager trying to maximize the probability that his option pays off.
The model derived can be also easily extended to other features of executive stock options. Actually, in the design of incentive options a growing number of firms is using new techniques such as indexing.

Appendix A

Derivation of the expected return required by the partially diversified manager
Let K represent the portfolio of a partially diversified manager with fraction w of his/her wealth in stock of his own firm , w_λ in the market portfolio and fraction in the riskless asset.
r_K is the expected rate of return of portfolio K under CAPM pricing
r_K^* is the expected rate of return of portfolio K required by a partially diversified manager
r_s^{*d} is the expected rate for return on the firm's stock required by the manager to be indifferent between holding portfolio K an the market portfolio.
by definition of K :

$$r_K = w\mu_s + w_\lambda r_m + (1 - w - w_\lambda)r_f = r_f + \beta_K(r_m - r_f), \quad (32)$$

$$r_K^* = wr_s^{*d} + w_\lambda r_m + (1 - w - w_\lambda)r_f \quad (33)$$

$$\rightarrow r_K - r_K^* = w(r_s^{*d} - \mu_s) \quad (34)$$

In addition, to be indifferent between portfolio K and the market portfolio, the manager requires a reward per unit of risk equal to that of the market. To estimate this return we use the Sharpe ratio:

$$r_K^* = wr_s^{*d} + w_\lambda r_m + (1 - w - w_\lambda)r_f = r_f + \frac{\sigma_K}{\sigma_m}(r_m - r_f) \quad (35)$$

Where σ_K is the volatility of portfolio K and is equal to:

$$\sigma_K = \sqrt{w^2\sigma_s^2 + w_\lambda^2\sigma_m^2 + 2ww_\lambda\sigma_{sm}} \quad (36)$$

where $\sigma_{sm} = \rho_s\sigma_s\sigma_m$
Substracting $w\mu_s$ from each side of equation (5) and collecting terms,

$$w(r_s^{*d} - \mu_s) = (r_m - r_f)(\frac{\sigma_K}{\sigma_m} - w_\lambda) - w(\mu_s - r_f), \quad (37)$$

Using CAPM in expression (5) to substitute for $\mu_s = r_f + \beta_s(r_m - r_f)$ we have:

$$r_s^{*d} = (r_m - r_f)(\frac{\sigma_K}{w\sigma_m} - \frac{w_\lambda}{w}) + r_f. \quad (38)$$

By applying $w_\lambda = 1 - w$, we find the result of Meulbroek (2001).

Appendix B

Derivation of the risk borne by the partially diversified manager
let G represent the portfolio constructed to have the same expected rate of return r_s^{*d} required by the partially diversified manager.

$$G = \begin{cases} \frac{1}{w}K \\ \left[\frac{\beta_K}{w}(\frac{1}{\rho_K} - 1) - \frac{w_\lambda}{w}\right] M \\ \left[1 - \frac{1}{w} - \frac{\beta_K}{w}(\frac{1}{\rho_K} - 1) - \frac{w_\lambda}{w}\right] Bond \end{cases}$$

Where K the portfolio defined in Appendix A.
the volatility of portfolio G, σ^{*d} is:

$$\sigma^{*d} = \sqrt{\frac{1}{w^2}\sigma_K^2 + \left[\frac{\beta_K}{w}(\frac{1}{\rho_K} - 1) - \frac{w_\lambda}{w}\right]^2 \sigma_m^2 + \frac{2}{w}\left[\frac{\beta_K}{w}(\frac{1}{\rho_K} - 1) - \frac{w_\lambda}{w}\right]\rho_K\sigma_K\sigma_m}. \quad (39)$$

σ^{*d} is, according to the market, the risk a partially diversified manager really faces.

Appendix C

Table 2: Private option value in terms of its market value and option moneyness

	Meulbroek (2001)	Ingersoll (2002)	Tian (2002)	Our Model
Exercise price over stock price	$\hat{C}/C$			
0.9	66.12%	28.10%	43.8%	51.57%
1	63.42%	24.30%	25.02%	35.32%
1.1	60.90%	21.29%	15.72%	32.24%
1.5	52.83%	12.73%	2.73%	23.09%

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