Constant-Maturity Default Swaps

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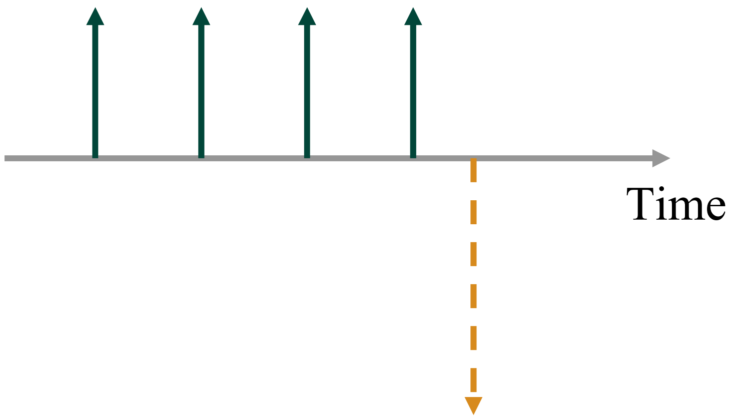
Product Description

Constant-Maturity Default Swaps (CMDS)

- Evolved as a variant of standard credit default swaps (CDS)
- Innovation is the introduction of a floating premium leg
- Premium leg pays a floating coupon proportional to the observed market spread on a standard CDS with a fixed time to maturity.

Standard CDS

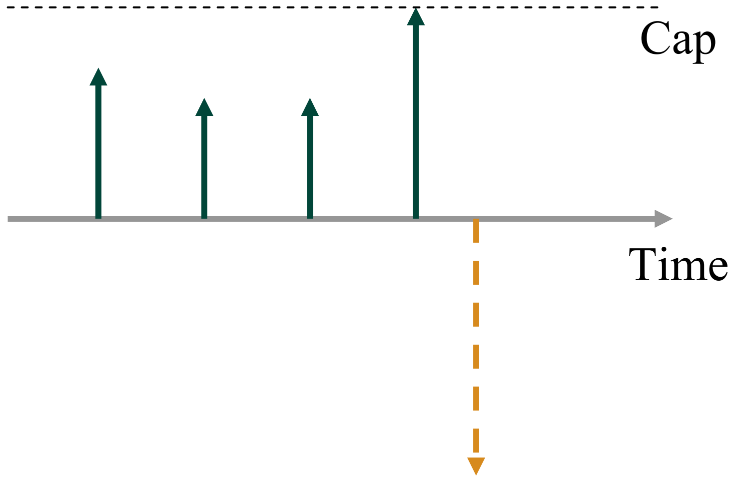
Fixed premium leg cashflows



Protection following credit event: 1-R

CMDS

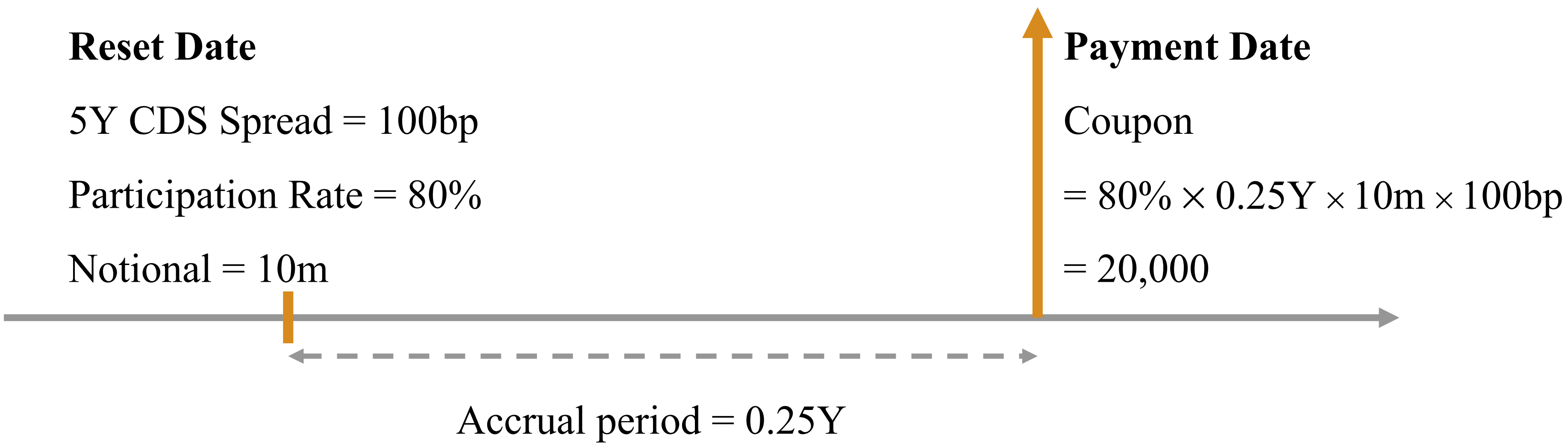
Floating premium leg cashflows



Protection following credit event: 1-R

The CMDS Floating Premium Leg

- Floating payments are linked to the prevalent market spread of a standard CDS with constant time to maturity – this is called the tenor of the CMDS
- The constant maturity spread sets in avance and pays in arrears
- The spread paid is a contractually fixed fraction of the reset spread – this is called the participation rate and we denote it by α
- A CMDS is quoted by a bid-offer on the participation rate.
- The floating spread is capped to avoid circumstances in which the spread may become unobservable if the credit goes to high yield.



Standardised Inter-Dealer CMDS Contract

- Standardization of the contract is in process but not complete.
- The reset mechanism is probably the biggest hurdle in making CMDS a more liquid product
  - Party receiving the floating premium makes a bid for CDS protection at a spread that will become the reset spread.
  - The required notional to bid is determined so that the floating receiver has incentive to bid at current market level.
  - If no bid is made and the reset spread will be determined by a dealer poll.

Variations on CMDS

- Floating-for-Protection:
  - Reduced mark-to-market volatility
  - For investors concerned about volatility in their reported earnings.
  - For investors bullish about the credit fundamentals of an issuer but concerned about current market valuations.
- Fixed-for-Floating:
  - Take a view on spread widening or narrowing
  - No recovery rate risk following a credit event
- Floating-for-Floating:
  - Take a view on the shape of the curve
  - No recovery rate risk
- CMDS can be structured into tranched products giving investors diversification and the option to choose leverage.

Valuation

Valuation of the Floating Leg

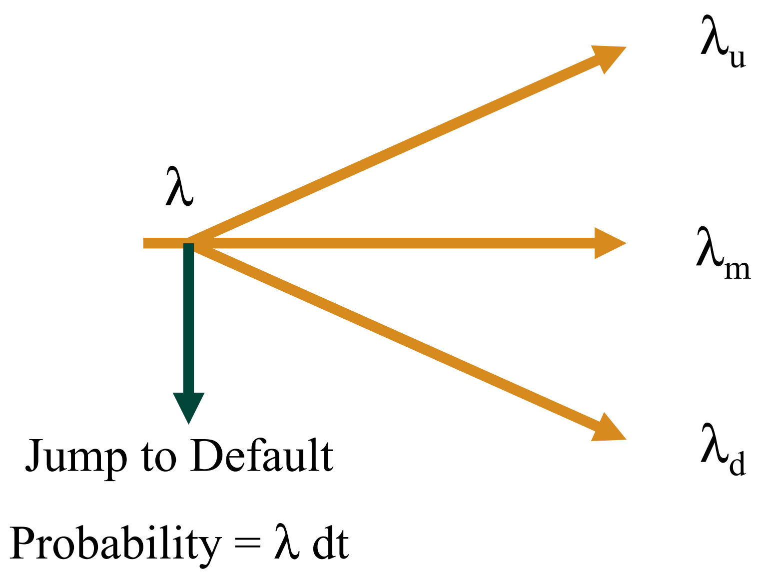
Reset Spread = Forward (Main) + Convexity Adjustment (Secondary)

- Forward spreads are the main determinant of reset spreads
- A convexity adjustment is needed if spreads are volatile
- Intuitively, this is because mark-to-market depends on the forward PV01, which is a convex function of spreads
- Mathematically, the PV of a future coupon is its discounted expected value
- Expectation must be computed under the appropriate numeraire
- The forward spread is the expected spread under the risky PV01 numeraire
- A convexity correction is therefore required in order to use a risky discount bond as numeraire

Stochastic Hazard Rate Models

Integrated framework for pricing embedded options

- Assume, say, lognormal dynamics for the hazard rate
- Implies almost lognormal spread dynamics
- Can be implemented on a trinomial tree with jump-to-default at each node
- Integrated pricing of embedded options
- Lognormal is not perfect, but preferable to, say, a Gaussian model
- Can incorporate features like mean-reverting hazard rates



Change of Numeraire Technique

Compute E[S] and use Black Model for pricing embedded options

- The “correct” expected reset spread has to be computed using a risky discount bond as numeraire
- The forward spread is the expected reset spread under the risky PV01 numeraire
- The Black formula can be used to price embedded options
- Change of numeraire:

$$E^D\left[S_T\right] = \frac{PV01(t,T,T')}{D(t,T+\Delta)} E^{PV01}\left[S_T \frac{D(T,T+\Delta)}{PV01(T,T,T')}\right]$$

$F\left(S_T\right)$

Taylor Series

$$E^D\left[S_T\right] = S_t + \frac{1}{2}\frac{PV01(t,T,T')}{D(t,T+\Delta)} F''\left(S_t\right) Var\left(S_T\right)$$

Semi-Analytic Valuation

Exponential-affine survival probabilities

- Similar to affine term structure models in interest rates
- Assume some dynamics for the hazard rate, e.g.:

$$d\lambda_t = \kappa\left(\theta_t - \lambda_t\right)dt + \sigma dW_t$$

- Leads to closed-form survival probabilities

$$Q\left(t,T\right) = A\left(t,T\right)\exp\left\{-\lambda_t B\left(t,T\right)\right\}$$

- If the distribution of hazard rate is tractable, then expected future cashflows can be computed using numerical integration
- Model can be calibrated to match the initial term structure

Volatility Modelling

- Embedded options are typically deep out of the money
- For this reason, it is important to model the volatility smile carefully
- To correctly price a strip of cashflows, we also need information about the term structure of volatility
- Models can incorporate volatility surfaces, but calibration data is still sparse
- Single-name default swaptions are becoming more liquid
- Otherwise, we have to rely on historical estimates

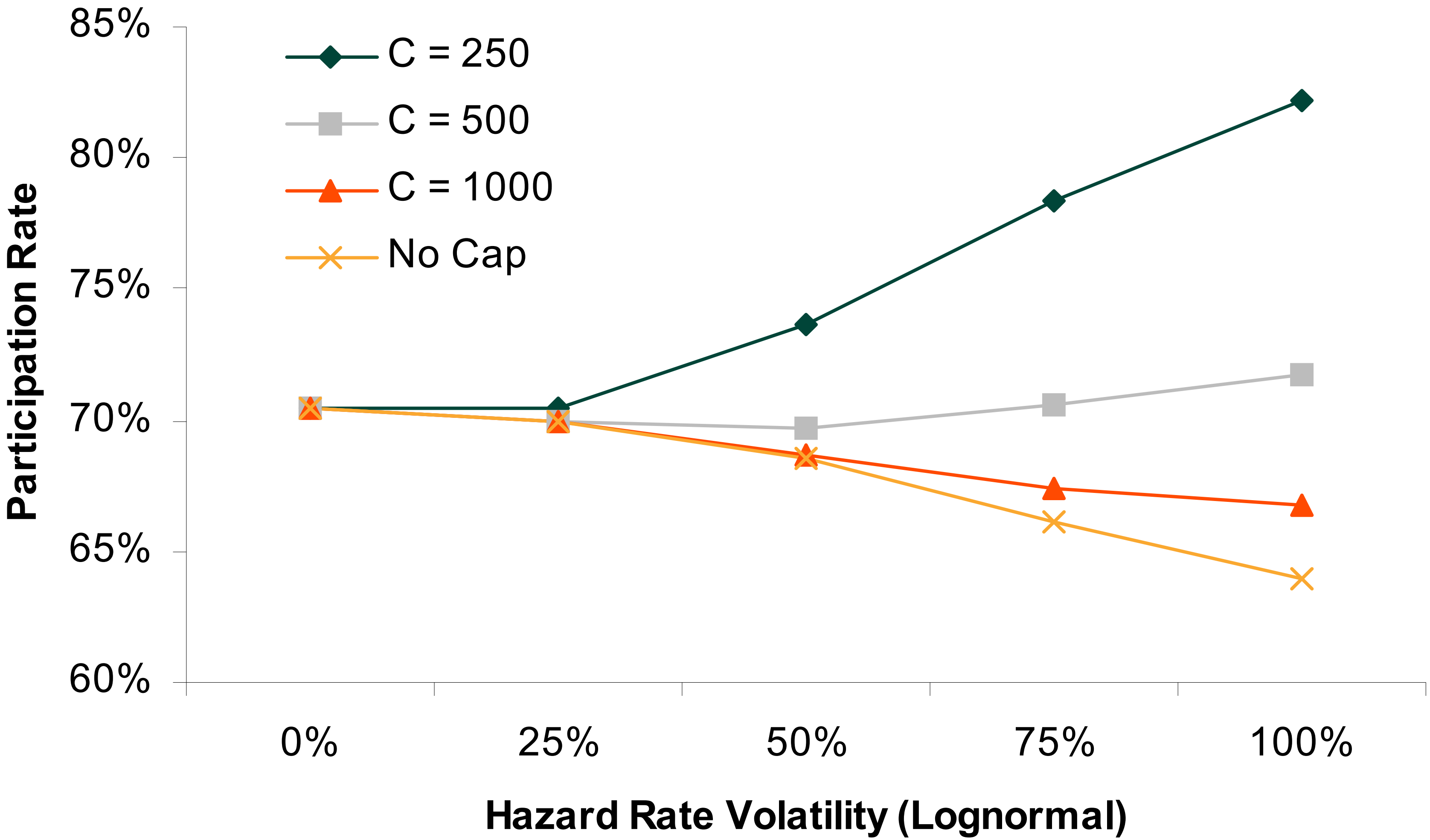
Numerical Examples

Size of the Convexity Adjustment

Forwards are the main determinant of reset spreads

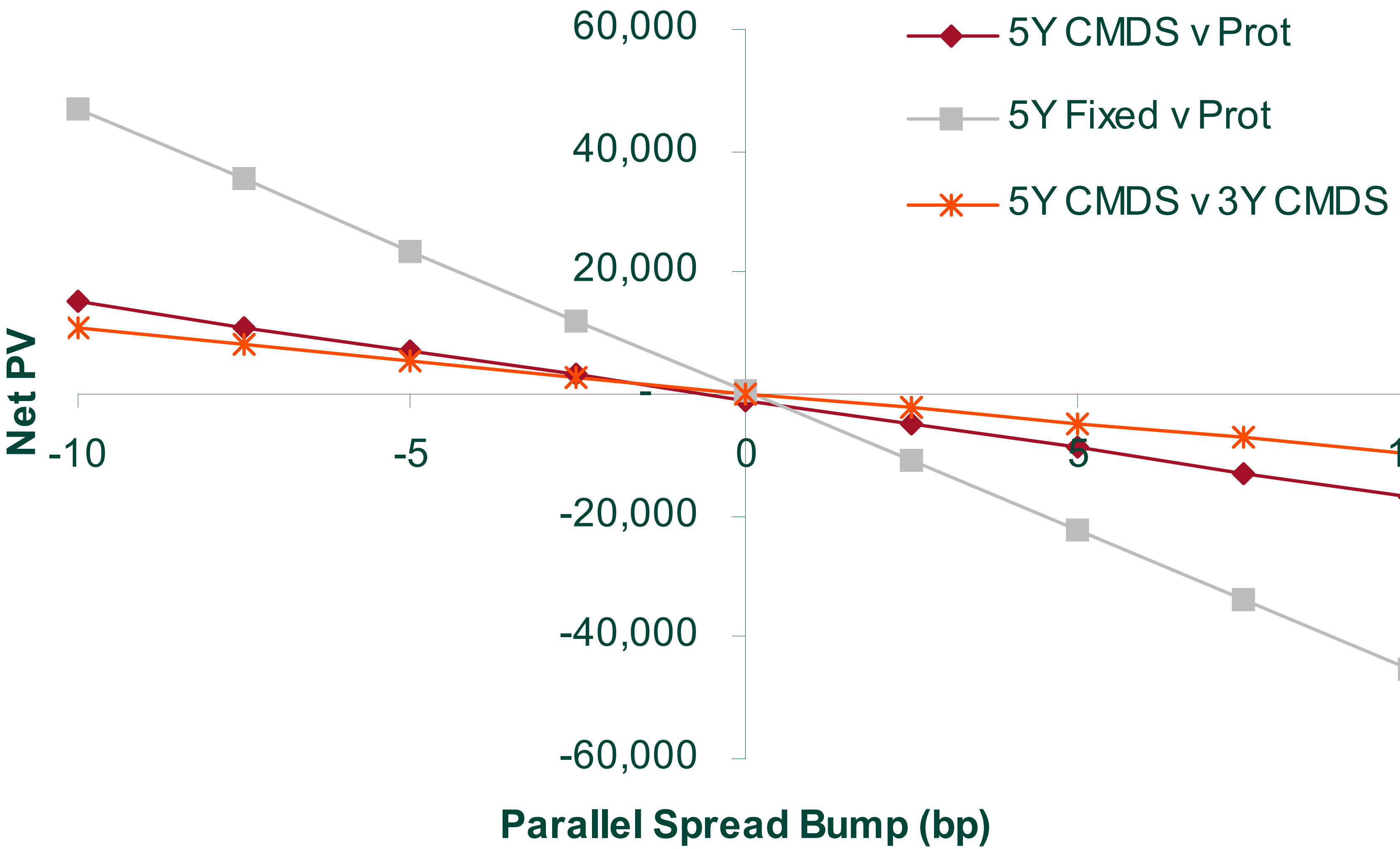
Participation Rate as a Function of Volatility

Moneyness of the embedded options affects the participation rate



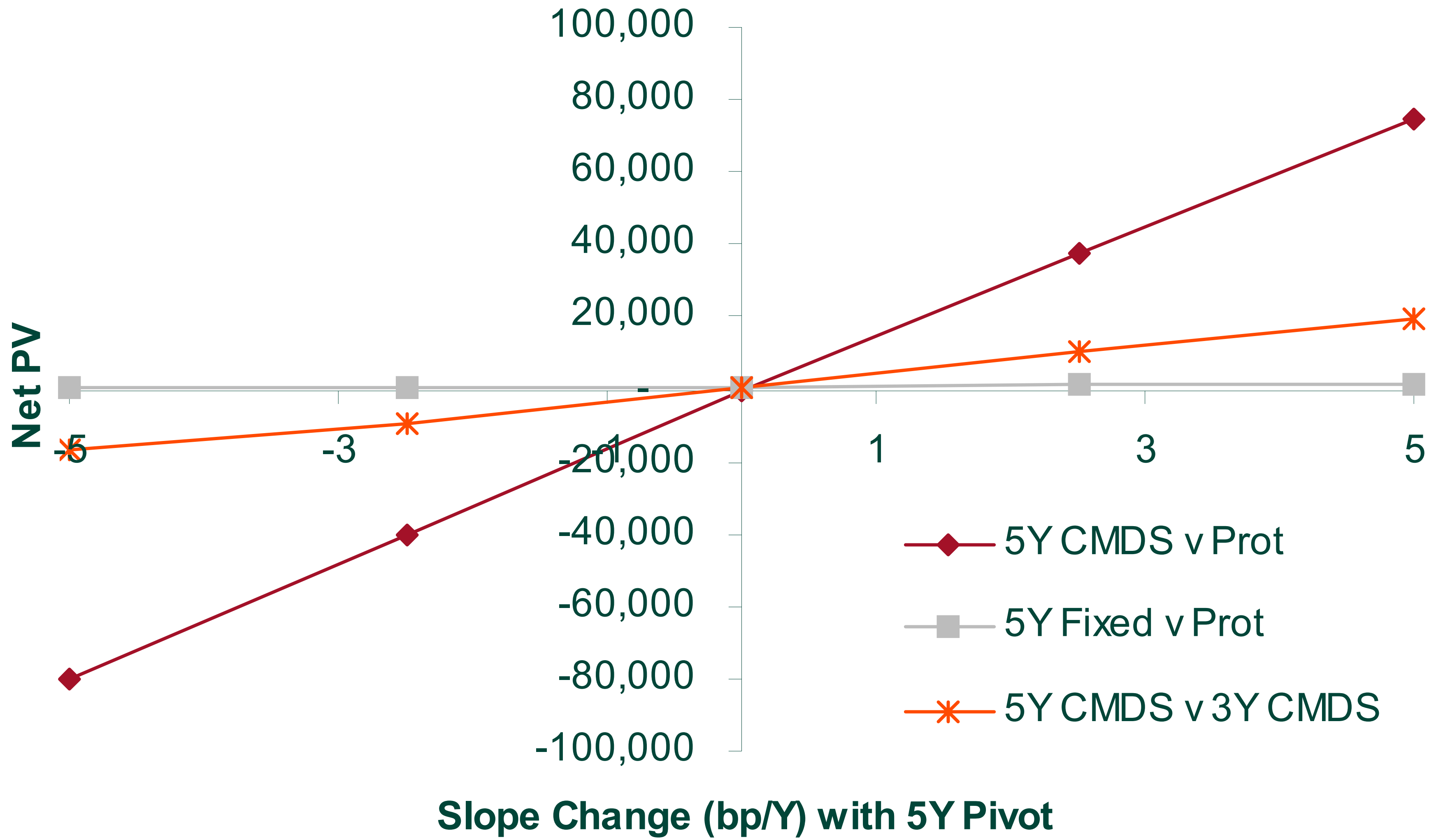
Sensitivity to Parallel Curve Movements

CMDS are much less sensitive to parallel curve shifts than CDS



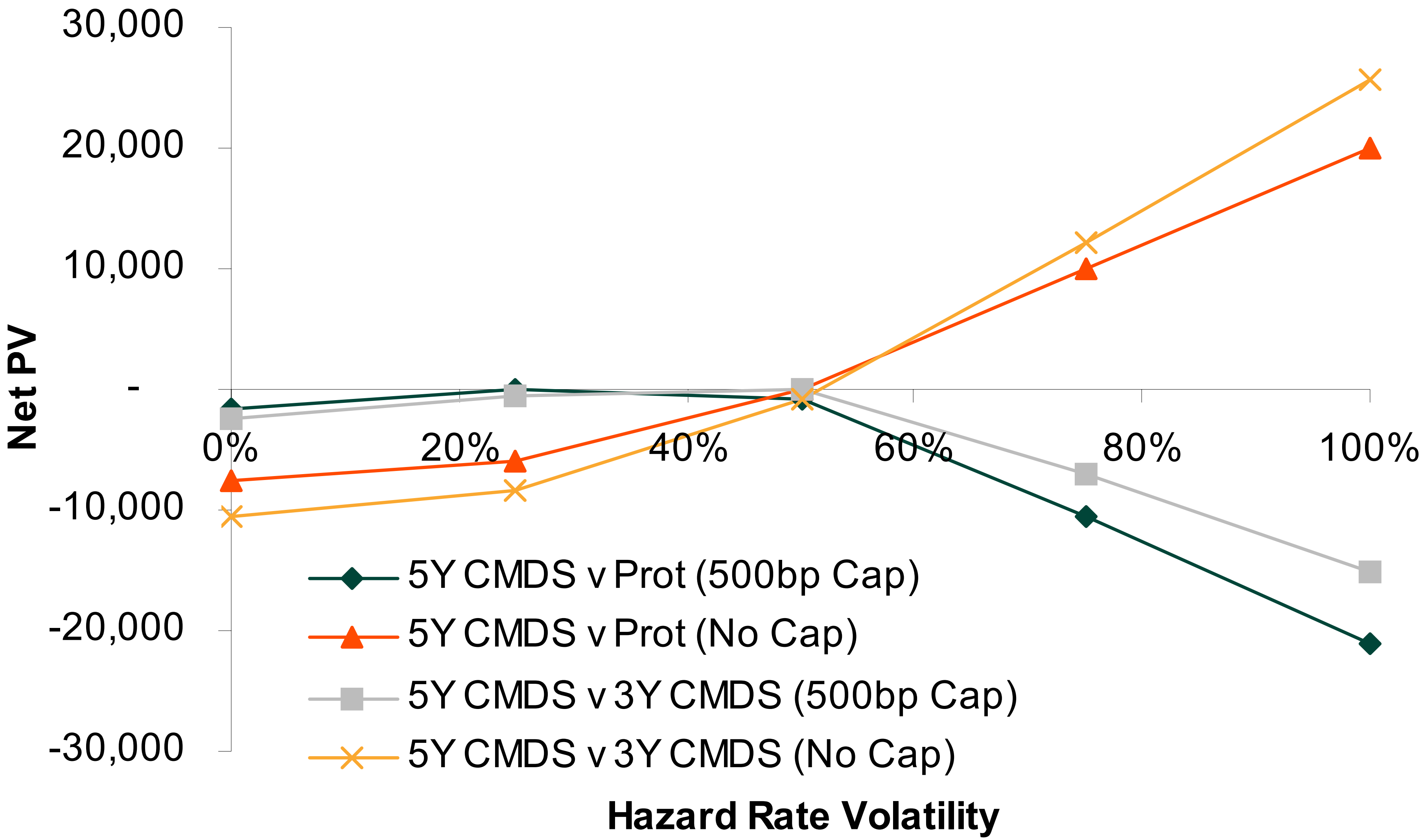
Sensitivity to Curve Slope Changes

CMDS allow investors to take a view on the CDS curve slope



Effect of Volatility

Embedded caps can have a significant effect on the vega of CMDS



Mark-to-Market

Mark-to-Market of CMDS

$$MTM_{CMDS} = \left(\frac{\alpha_{Contract}}{\alpha_{Market}} - 1\right) \times S_{Market} \times PV01$$

- MTM = Floating Premium Leg minus Protection Leg

- The value of the protection leg is:

$$PVP_{Market} = S_{Market} \times PV01$$

- The breakeven participation ratio satisfies, by definition:

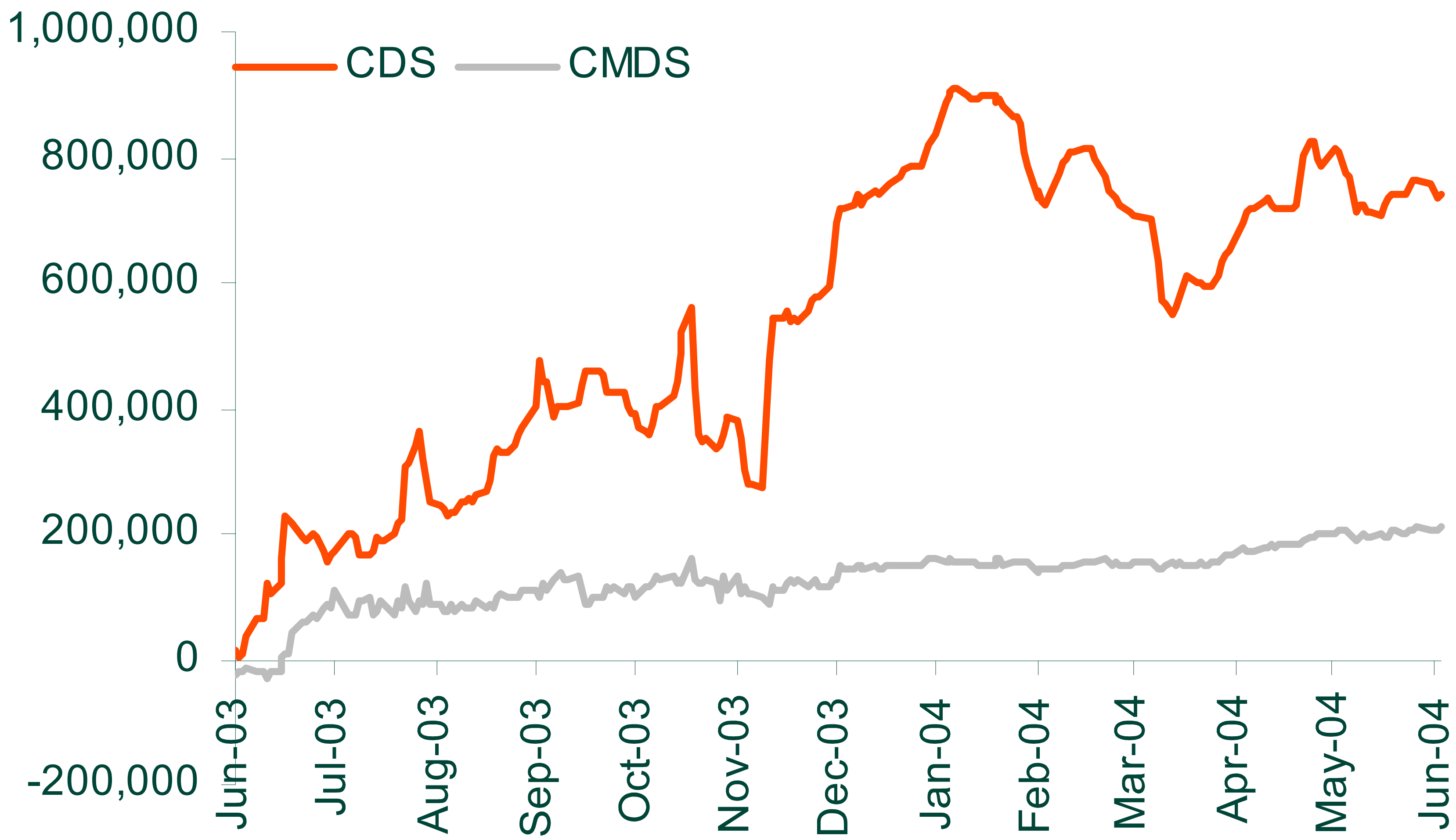
$$\alpha_{Market} \times FL_{Market} = PVP_{Market}$$

- MTM is given by:

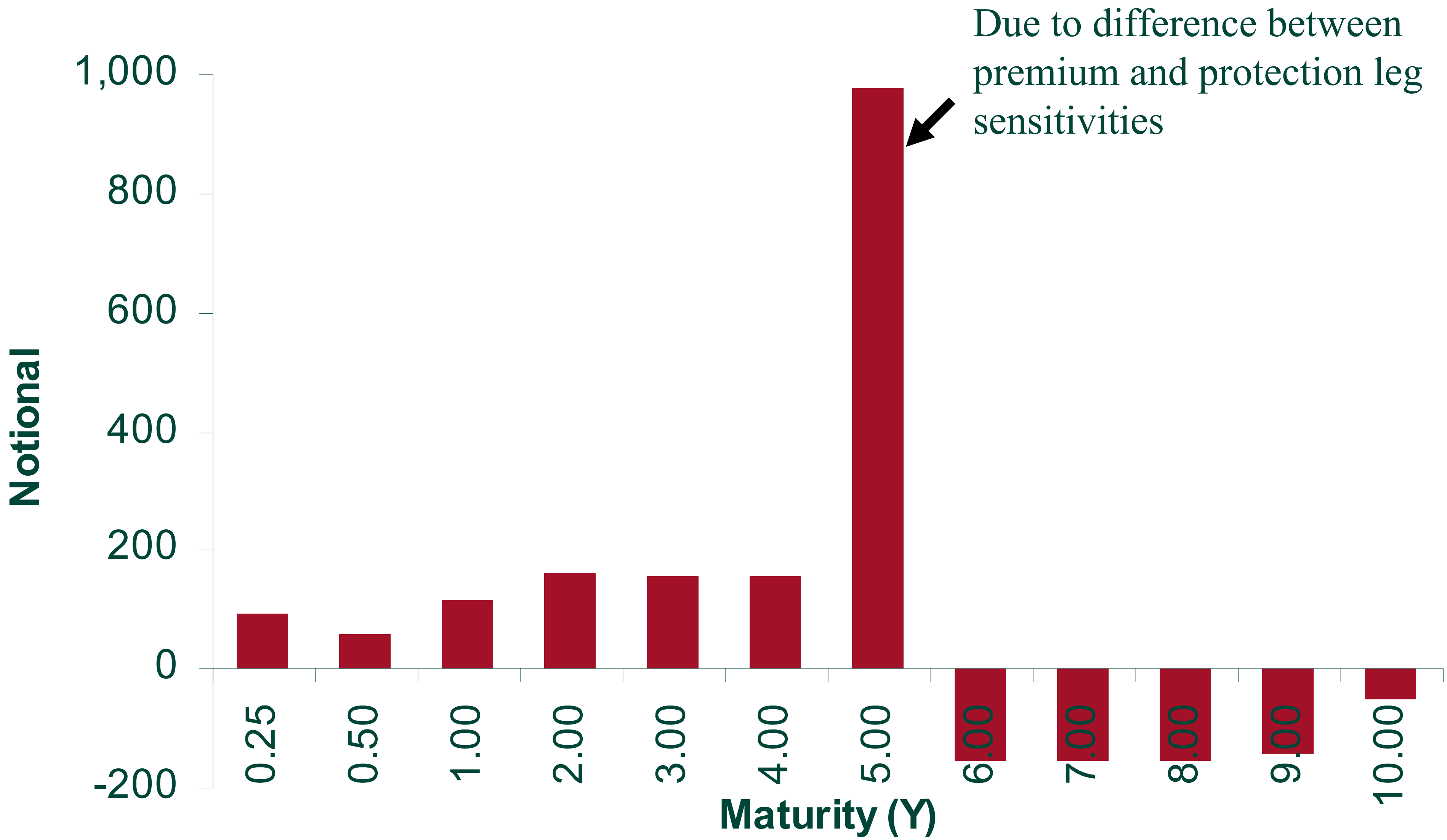
$$\alpha_{Contract} \times FL_{Market} - PVP_{Market}$$

Example: Historical MTM Volatility

Historically, spreads tend to move more parallel and so CMDS have lower MTM volatility (the example is for Ford Motor Credit)



Hedging CMDS with CDS



Summary

- CMDSs represent an innovation in the credit derivatives market
- Allow investors to take a view on the shape of the credit curve
- Volatility becomes important when embedded options are near-the-money
- Some outstanding issues concerning the reset mechanism
- But of potential interest to a wide class of investors

References

- Pedersen, C. M., and S. Sen: “Valuation of Constant-Maturity Default Swaps”, Lehman Brothers Quantitative Credit Research Quarterly, Vol. 2004-Q2.

- Naldi, M., D. O’Kane et. al.: *The Lehman Brothers Guide to Exotic Credit Derivatives*, special supplement to *Risk*, October 2003.

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