Does Conditional Asset Allocation Truly Outperform? Analysis under Real-Time Uncertainty

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Abstract

This paper examines the impact of real-time uncertainty on the superior performance of mean-variance conditional asset allocation documented in the literature. Real-time uncertainty is modeled by implementing a large number of conditional strategies that an investor could reasonably choose. This analysis based on 12 international equity markets between January 1990 and September 2004 reveals that the performance of conditional asset allocation is currently overstated. Consistently across different levels of transaction costs and investor's risk aversion, we mainly show that (i) conditional strategies present important downside risk (ii) conditional strategies are dominated by buy and hold strategies (iii) performance is very sensitive to minor specification changes.

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1 Introduction

A significant part of academic research initiated in the mid 80's has been devoted to the analysis of variables that may have some predictive power on the expected returns of various asset classes and more specifically equity indices. These widely reported results have conducted both academics and practitioners to implement strategies based on this predictability in order to beat the market and other unconditional models using historical data. While the mere existence of a statistical relation between returns and lagged instruments is still hotly debated by academics (e.g. Bossaerts and Hillion, 1999; Stambaugh, 1999; Ferson, Sarkissian and Simin, 2003), conditional allocations based on mean-variance optimization provide an intuitive way to assess the economic importance of return predictability. This is simply done by evaluating the performance of these strategies from the view point of an uninformed investor considering that the multivariate asset return distribution is unpredictable. It turns out that most studies implementing these allocations conclude that they outperform passive or unconditional strategies[1]. As an illustration, Solnik (1993) considers stock and bond indices of developed countries and uses a model based on local predictive variables such as the dividend yield and the short term interest rate. Harvey (1994) and Cavaglia & al. (1997) add emerging markets to the analysis and select global predictive instruments related to currency rates or the MSCI world index. In a domestic setting, Klemkosky and Bharati (1995) introduce transaction costs for the US market and use a selection criterion to choose the optimal number of variables, while Robertsson (2000) measures the performance of this approach applied to the Swedish market.

However, we think that the positive performance documented so far in the literature is largely called into question because of a lack of realism. First, the diversity encountered in all of these studies clearly shows that conditional approaches can be implemented with many different predictive variables, estimation window lengths or financial assets. This diversity due to the lack of solid theoretical grounds behind these choices and especially behind the notion of predicability leads most reseachers to focus on exogenously defined specifications (Cooper and Gulen, 2004). As a result, a given specification of conditional asset allocation may have obtained outstanding out-of-sample results but since this specification is unknown ex-ante, one can reasonably wonder if an investor would have been able to replicate it in real-time. Second, none of them jointly explain three essential aspects underlying the concrete application of mean-variance conditional asset allocation. The first one consists in defining a consistent way to implement these tactical approaches characterized by substantial portfolio turnover. The second concerns the performance sensitivity of both conditional and unconditional asset allocations to different levels of transaction costs. Finally, since different risk aversion coefficients greatly modify the bets taken by a given investor (Chopra and Ziemba, 1993), variations of this parameter may greatly impact the results of the various strategies.

[1] To our knowledge, the only study finding that mean-variance conditional methods yield disappointing results is proposed by Handa and Tiwari (2002). Other papers come to the same conclusion by using conditional asset pricing models. But the restrictive structure imposed by these models greatly reduces performance (e.g. Hamelink, 2000; Fletcher & Hillier, 2003).

The contributions of this paper to the existing literature are threefold. First, we propose a new assessment of the performance generated by conditional asset allocation by taking into account the real-time uncertainty faced by an investor willing to implement this strategy. Since it is not sufficient to only examine a few exogenous specifications, we rather consider a large set of 168 conditional strategies among which a real-time investor could reasonably make his choice. Each strategy varies according to three sets of exogenous specifications, which are the predictive variables, the estimation window length and the portfolio selection constraints. Then, the overall performance of conditional asset allocation is measured against 24 unconditional asset allocations as well as 3 buy and hold strategies. Second, this paper models a complete framework which allows a practical and concrete implementation of conditional asset allocation. Indeed, we explicitly show how to use developed stock market index futures contracts in order to reduce transaction costs and make the excess return of the conditional asset allocation independent of the investor's country of origin. Moreover, a broad analysis of the performance sensitivity of conditional asset allocation to both changes in the level of transaction costs and investor's risk aversion is provided. Finally, we propose a rationale for the use of the Cornell measure and the Sharpe ratio to assess the performance of conditional asset allocation and propose an innovative use of the bootstrap method developed by White (2000) to obtain significance p-values related to these two performance measures.

The results of our empirical tests applied to 12 developed market equity indices between January 1990 and September 2004 clearly indicate that the economic gains generated by conditional asset allocation are currently overstated and imply that various conditional strategies that could be reasonably chosen by a real-time investor lead to important capital losses. This strong conclusion, which is consistent across different levels of investor's risk aversion, is based on three different procedures which highlight important limitations of conditional asset allocation. First, the comparative analysis of conditional and unconditional asset allocations indicates that the standard deviation of conditional strategies is always significantly higher (at the 10% level) whereas its excess mean is not significantly superior except when the risk aversion coefficient is low. Moreover, the final wealth distribution faced by an investor who randomly picks up one of the 168 possible conditional strategies is prone to substantial downside risk. Second, performance analysis shows that the percentage of strategies having a positive Cornell measure is very low, implying that none of the conditional strategies are based on superior information. Examination of the Sharpe ratio reveals that conditional asset allocation is able to outperform unconditional asset allocation but is beaten by buy and hold strategies especially when transaction costs are accounted for. Besides, the important performance variation across the different sets of exogenous specifications suggests that modelling choices greatly affect the profits generated by conditional strategies. Third, we prove that the best conditional models do not share common characteristics that could be detected ex-ante and are very sensitive to minor specification changes, thus confirming the important impact of real-time uncertainty.

The remainder of the paper is as follows. The next section defines the conditional as well as the unconditional asset allocations and explains how they can be implemented in concrete terms. Section 3 defines the three sets of exogenous specifications characterizing real-time uncertainty. The following section describes the investment universe and the empirical modelling of the various predictive models, estimation window lenghts and portfolio constraints. Section 5 contains a comparative analysis of conditional and unconditional strategies. Section 6 examines performance measures of all conditional strategies based on the Cornell measure and the Sharpe ratio. Section 7 investigates the sensitivity of the best conditional strategies to small specification changes and the final section concludes.

2 Description of the Conditional and Unconditional Asset Allocation

2.1 Conditional asset allocation

2.1.1 Implementation

Let us assume that an investor forms his portfolio from a universe of N risky assets and a riskless asset with constant return r_f. At the beginning of each period, the investor receives predictive information about the distribution of asset returns for the next period. If we denote by I_t the investor's information set, the excess return of the i^{th} risky asset $\widetilde{r}_{it+1}$ $(i = 1, ..., N)$ over the riskfree rate between t and $t + 1$ may be written as:

$$\widetilde{r}_{it+1} = E\left(\widetilde{r}_{it+1} \middle| I_t\right) + \widetilde{\varepsilon}_{it+1} = \widetilde{\pi}_{it} + \widetilde{\varepsilon}_{it+1} \tag{1}$$

where $\widetilde{\pi}_{it}$ represents the conditional expected excess return with respect to I_t and $\widetilde{\varepsilon}_{it+1}$ is the unpredictable residual term. Following the literature, the portfolio is rebalanced at each time t $(t = 1, ..., T)$ according to the two following principles. First, the investor is myopic as he only considers the return distribution between t and $t + 1$ to compute the portfolio weights. Second, these weights are defined by the mean-variance criterion applied to the first and second conditional moments. After inserting the constraint that the weights sum up to one directly in the optimization function, the investor with a risk aversion coefficient A maximizes the following mean-variance trade-off at each time t $(t = 1, ..., T)$:

$$\begin{aligned} & \max E\left(\widetilde{r}_{pt+1} \middle| I_t\right) - \frac{1}{2} A \cdot Var\left(\widetilde{r}_{pt+1} \middle| I_t\right) \\ \Rightarrow \quad & \max_{w_t} w_t' \Pi_t - \frac{1}{2} A \cdot w_t' \Sigma_t w_t \end{aligned} \tag{2}$$

where w_t represents the $N \times 1$ vector of risky asset weights set at time t, Π_t the $N \times 1$ conditional expected excess return vector and Σ_t the $N \times N$ conditional covariance matrix of excess returns. The modelling of the conditional covariance matrix is based on the following assumption:

Assumption 1 *The conditional covariance matrix is constant, i.e.* $\Sigma_t = \Sigma$.

This assumption is commonly made because the dynamics of the second moment is difficult to capture as the number of assets increases (for further discussion see Solnik, 1993; Harvey, 1994). The validity of this assumption is an empirical matter which depends on the type of assets and the time horizon chosen by the investor[2].

Since Σ is supposed to be constant, the presence of superior information is only characterized by time-variation of the first moment of the multivariate excess return distribution. The general estimation procedure of $\widetilde{\pi}_{it}$ for each asset i $(i = 1, ..., N)$ is based on the following linear specification estimated by OLS technique:

$$\widetilde{r}_{it+1} = \widehat{\alpha}_i + \sum_{k=1}^{K} \widehat{\phi}_{ik} \cdot \widetilde{Z}_{kt} + \widetilde{\xi}_{it+1} \tag{3}$$

where $\widetilde{Z}_{kt}$ $(k = 1, ..., K)$ represent selected predictive variables known at time t and $\widetilde{\xi}_{it+1}$ the regression residual. The coefficients $\widehat{\alpha}_i$ and $\widehat{\phi}_{ik}$ $(k = 1, ..., K)$ estimated up to time $t-1$ are used together with the value of the predictive variables at time t to obtain an out-of-sample prediction for the i^{th} asset excess return between t and $t+1$. Each of these forecasts are stacked in a $N \times 1$ vector, thus forming an estimator of $\widetilde{\Pi}_t$. Moreover, the estimator of the conditional covariance matrix Σ is simply based on the empirical variances and covariances of the country regression residuals $\widetilde{\xi}_{it+1}$ $(i = 1, ..., N)$.

2.1.2 Return computation

Previous studies confirm the important level of turnover and induced transaction costs generated by conditional asset allocation. In order to mitigate this drawback, it is assumed that the international conditional strategies considered in this paper are implemented by means of equity index futures contracts. Using futures implies that all positions must be rolled over at each rebalancing date but transaction costs are greatly reduced compared to the ones charged in the equity market. As an illustration, the one-way cost estimates for developed equity markets usually range between 30 and 50 basis points relative to the transaction value (Solnik, 1999) whereas cost estimates in index futures markets documented by Sutcliffe (1993) are ten times lower.

To understand how the excess return of the conditional asset allocation between t and $t+1$ is computed consider an example where an investor is willing to invest in a foreign stock market.

[2] The empirical part of the paper is based on developed market equity indices over a monthly horizon. In this case, we show later that this assumption is reasonable. It would certainly not be the case if more volatile assets such as emerging markets were considered over a shorter time horizon. But it is worth noting that covariance misspecification has a far lesser impact on the investor utility than the one related to the expected return vector (Chopra and Ziemba, 1993).

It is assumed that there is no opportunity cost associated with the initial margin[3] and that the expiration date of the futures contract corresponds to the rebalancing interval chosen by the investor. Let us denote by P_t^f the price at time t of the foreign stock index expressed in foreign currency f and by S_{ft} the price at time t of one foreign currency f in terms of the investor's domestic currency. Instead of buying the securities forming the index, the investor can choose to be long one stock index futures contract and invest the initial amount of money $S_{ft} \cdot P_t^f$ in the domestic riskless asset. Since the excess return of the domestic riskless asset is equal to zero, the excess return of the portfolio only depends on the stock index futures return:

$$\widetilde{r}_{pt+1} = \frac{\widetilde{S}_{ft+1}\left(\widetilde{P}_{t+1}^f - F_{t,t+1}^f\right)}{S_{ft} \cdot P_t^f} = (1 + \widetilde{s}_{ft+1})\left(\widetilde{R}_{t+1}^f - r_{ft}^f\right) \approx \widetilde{r}_{t+1}^f \tag{4}$$

where $F_{t,t+1}^f$denotes the stock index futures price (in foreign currency) defined at time t. The terms r_{ft}^f and $\widetilde{s}_{ft+1}$ respectively stand for the foreign riskfree rate set at time t and the currency rate of return between t and $t+1$. Moreover, $\widetilde{R}_{t+1}^f$ and $\widetilde{r}_{t+1}^f$are the stock index total return and excess return (both expressed in foreign currency) between t and $t+1$. The simplification proposed in Equation (4) implies that the cross-product term $\widetilde{s}_{ft+1} \cdot \left(\widetilde{R}_{t+1}^f - r_{ft}^f\right)$ is neglected. This hypothesis is very convenient since the excess return of each market is independent of the investor's country of origin and is equal to $\widetilde{r}_{t+1}^f$. Therefore, it is unnecessary to both adopt the point of view of an investor located in a specific country and to model forecasts of the currency component, which is particularly difficult to predict accurately (Solnik, 1999).

2.2 Unconditional asset allocation

2.2.1 Implementation

A natural benchmark to assess the performance of conditional asset allocation is to compare it with an unconditional asset allocation based on the assumption that the investor does not receive any predictive information. As a result, the only difference with the conditional approach lies in the fact that the myopic mean-variance optimization is based on the unconditional moments. After inserting the constraint that the weights sum up to one directly in the optimization function, the investor with a risk aversion coefficient A maximizes the following mean-variance trade-off at each time t $(t = 1, ..., T)$:

$$\begin{aligned} & \max E\left(\widetilde{r}_{pt+1}\right) - \frac{1}{2} A \cdot Var\left(\widetilde{r}_{pt+1}\right) \\ \Rightarrow \quad & \max_w w'\mu - \frac{1}{2} A \cdot w'Vw \end{aligned} \tag{5}$$

[3] In the worst case where the initial margin does not yield any income, the effect on the portfolio return is weak since in developed markets the initial deposit generally ranges between 5 and 7% of the principal value of the index futures contract.

where w is the $N \times 1$ vector of risky asset weights, μ the $N \times 1$ unconditional expected excess return vector and V the $N \times N$ unconditional covariance matrix of excess returns. In order to implement this unconditional approach, μ and V are simply replaced by the empirical first and second moments computed with past data.

2.2.2 Return computation

Since the uninformed investor does not perceive any time-variation of the asset return distribution, the portfolio is not heavily rebalanced and the turnover is thus low. For this reason, a direct investment in stocks is more appropriate. Considering again the situation described above where an investor wants to invest in a foreign stock market, we assume that he simply buys the securities forming the index in the appropriate proportions. In order to obtain fair comparisons with conditional methods from which the currency part has been removed, the unconditional strategies have to be hedged against currency risk with currency futures contracts. Using the covered interest rate parity and neglecting the cross-product $\widetilde{s}_{ft+1} \cdot \left(\widetilde{R}^f_{t+1}\right)$, the excess return of this portfolio is equal to $\widetilde{r}^f_{t+1}$ similarly to Equation (4). As a result, the excess return of the unconditional asset allocation is also independent of the investor's country of origin.

3 Exogenous Specification and Real-Time Uncertainty

As it is stressed by Cooper and Gulen (2004), the vast majority of researchers analysing strategies based on asset return predictability specify the portfolio selection process in an exogenous way. This exoenous specification covers different aspects: the set of predictive variables, the estimation window length, the trading rule or the period during which the strategies are implemented. Of course, some of these choices can be defended with plausible arguments. For instance, it can be argued that the trading rule based on the mean-variance framework permits to take advantage of predictability while keeping the intuitive results of this well-known optimization technique. Another example applies to the choice of the asset universe: an investor may want to discard emerging countries in order to avoid the specific risks of these markets. Unfortunately, these arguments are rather seldom. The lack of theoretical basis to justify most of the modelling choices casts doubt on the performance robustness of the conditional strategies as the most performing models remain unknown ex-ante. In order to understand the role of real-time uncertainty in the performance of conditional asset allocation, we explicitly model three sets of exogenous specifications described below. The objective of this approach is not to reproduce the complex and almost infinite choices faced by a real-time investor but to create a rich set of conditional strategies based on different characteristics among which the investor could reasonably make his choice. Assessing the performance of conditional asset allocation across this set of conditional strategies permits to test the robustness of the empirical results documented in the literature to the presence of real-time uncertainty.

The first arbitrary specification is related to the choice of predictive variables. The number of papers documenting the presence of predictability in stock returns has grown very rapidly since the late 80's. Early evidence of predictability in US stock returns is provided among others by Keim and Stambaugh (1986) or Fama and French (1988, 1989). They show that variables such as the dividend yield, the term and default spread, the short-term interest rate or the lagged stock index return have some predictive power. Other predictors are represented by the dividend payout ratio (Lamont, 1998) or the stock market volatility (see French et al., 1987). Unfortunately, choosing the optimal set of predictive variables is complicated because theoretical arguments are weak. The most common explanation to the predictive power of some variables rests on their relationships with the business cycle (Harvey, 1989; Chen, 1991). But even if one believes in these empirically tenuous links, the set of eligible predictive variables remain large. For instance, both the term spread and the short term interest rate can be perceived as predictors of future growth. Moreover, all variables related to asset prices (the earning ratio, the dividend yield or the book-to-market ratio) can be interpreted as proxies for the current state of the economy through the discount factor mecanism. In an international setting, the choice of efficient variables becomes even more intricate because of the distinction between global and local predictive variables. Not only the relative importance of local and global variables is likely to depend on the degree of integration of the local markets to the world capital market (Bekaert and Harvey, 1995), but even in integrated developed markets, Harvey (1991) shows that local predictive variables still add some additional explanatory power over global ones. In light of these comments, many different combinations of the mentioned predictive instruments could be reasonably formed to predict the various assets making up the investment universe.

The second exogenous choice that a real-time investor has to make concerns the estimation window length. This decision is far from straightforward because of the following well-known trade-off. On the one hand, it is necessary to take the longest possible time-series in order to reduce the bias of the regression coefficients present in small samples (Stambaugh, 1999). On the other hand, the latter procedure may not be appropriate if economic changes occur within the estimation window. For instance, if the relation between returns and predictive variables is subject to structural breaks (Paye and Timmermann, 2002), it would be better to estimate a model whose coefficients rapidly reflect these changes. For these reasons, the best window length is a priori unknown by a real-time investor. Contrary to the choice of predictive variables, the selection of the estimation window length does not only apply to conditional asset allocation: in order to implement unconditional strategies one needs estimates of the unconditional expected return vector and covariance matrix. As these estimates depend on past data, they are also subject to structural changes in the economy.

Finally, a real-time investor has to decide if portfolio selection constraints should be put on the asset allocation weights. Because of the weak predictive power usually observed in the data and the presence of both estimation bias and structural breaks, the estimator of the conditional expected excess return vector $\widetilde{\Pi}_t$ is subject to estimation risk. It therefore makes the optimization process of Equation (2) prone to the same critics as the ones addressed to unconditional asset allocation based on historical means. For instance, Jobson and Korkie (1981) or Jorion (1985) indicate that estimation risk has disastrous consequences on the out-of-sample performance of unconditional mean-variance strategies. It is especially the case for expected returns, as estimation errors on expected returns have a negative impact on the investor utility function which is 10 to 22 times higher than the one related to covariance matrix estimation errors (Chopra and Ziemba, 1993). These conclusions are also supported by Best and Grauer (1991) who show that portfolio weights are extremely sensitive to small changes in expected returns. Being aware of estimation risk, a real-time investor may want to put various constraints to the optimization process in order to reduce this risk. Frost and Savarino (1988) explain that bounds on the minimum and maximum weights reduce estimation errors and yield better out-of-sample performance. Besides, as it is stressed by Jagannathan and Ma (2003), imposing no-shortsale and upper-bound constraints is equivalent to reducing extreme values present in the expected return vector. Some of the previous studies prevent short-selling (e.g. Hamelink, 2000; Handa and Tiwari, 2000) or add upper-bound constraints (Harvey, 1994), but once again the choice of the optimal constraint remains unknown ex-ante and mainly depends on the quality of the predictions. In fact, these constraints may even be wrong in population and induce specification errors but if the reduction of estimation risk obtained in small sample is sufficient, it can offset the previous drawback.

4 Empirical Specification

4.1 Description of the Data

The investment universe is composed of 12 developed markets which are Australia, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Netherlands, Switzerland, United Kingdom and the United States. The time horizon is set to one month, which defines a reasonable portfolio rebalancing frequency and both reduces the noise in the predictive regressions and the heteroscedasticity of the residuals when shorter periods are considered. Returns in local currency, $\widetilde{R}^{f}_{it+1}$ $(i = 1, ..., 12)$, are computed with Datastream country indices (price and gross dividends reinvested). They correspond to value-weighted portfolios of the larger firms traded in each market. In order to compute country index excess returns in local currency, $\widetilde{r}^{f}_{it+1}$ $(i = 1, ..., 12)$, the riskless asset of each country is proxied by the one-month euro-market rate.

At the time when Datastream indices were created in 1973, many academics thought that stock returns were unpredictable (Fama, 1970). As a result, it is questionable that an investor could have been able to implement conditional methods from the early 70's on. For this reason, all strategies considered in this paper start in January 1990. Since it is assumed that the conditional asset allocation is implemented with futures contracts, another important question is to know whether Datastream indices give a fair representation of the position taken in the country stock index futures markets. To answer this question, we examine the excess return time-series of the Datastream indices and the country indices on which commonly traded futures contracts are based[4]. Unreported results (computed between January 1990 and September 2004) indicate that the average pairwise excess mean difference absolute value between the two types of indices is equal to 0.7% per year and the average correlation amounts to 0.98. As a result, the excess return of the conditional asset allocation can be confidently measured with the Datastream indices. Some descriptive statistics based on the excess return time-series between January 1990 and September 2004 are presented in the Appendix.

Finally to check the consistency of Assumption 1, we test for heteroscedasticity in the predictive regression residuals of each country index. Using different sets of predictive instruments (see the description proposed in the next section), we compute the time-series of residuals for each index excess return between January 1990 and September 2004 and examine autocorrelograms and partial autocorrelograms of the squared residuals. In unreported results, we analyse the various coefficients computed for all sets of instruments. The figures do not reveal the presence of heteroscedasticity: the vast majority of coefficients are not statistically different from zero and the few significant ones appear at distant lags. In light of these results, the homoscedasticity assumption is reasonable in the empirical framework adopted here.

4.2 Definition of the three sets of exogenous specifications

4.2.1 Set of predictive variables

In order to define predictive variables that a real-time investor could reasonably choose, we largely build on past studies. The first step consists in classifying each instrument according to its nature (related to interest rates or asset prices) and its area of influence (global or local influence on capital markets), which allows to take into account the international framework of our paper. This procedure yields four groups of variables shown in Table 1.

[INSERT TABLE 1]

The lagged stock index excess return in local currency and the relative dividend yield for each country are provided by Datastream. The dividend yield at time t is computed as the ratio of the previous 12-month dividends paid by the firms included in the index on their market capitalisation at time t. Short term interest rates are proxied by one-month euro-market interest rates. The term spread is defined for each country as the difference between the yield of long-term government bonds provided by International Financial Statistics and the one-month euro-market interest rate. For the majority of countries, the long-term yield corresponds to the value-weighted average yield of government bonds with a minimal remaining maturity of 5 years. Turning to the description of global variables, the lagged world index excess return and its related dividend yield are computed from the Datastream world index expressed in US dollars. Global information related to interest rates are proxied by US variables. Finally, the

[4] These are the following: S&P/ASX 200 (Aus), BEL 20 (Bel), S&P/TSX (Can), KFX (Den), CAC40 (Fra), DAX (Ger), MIB30 (Ita), S&P Topix (Jap), AE-Index (Net), SMI (Swi), FT-SE 100 (UK), S&P 500 (USA).

US default spread downloaded from the website of the Federal Reserve Bank of Saint Louis is computed by Moody's as the difference between the average yield of long term (superior to 10 years) Baa-rated corporate bonds and the average yield of long term (superior to 10 years) Aaa-rated corporate bonds.

We combine the 4 different groups shown in Table 1 in order to obtain 7 different predictive models used to determine the conditional expected excess returns of all country indices. The 7 specifications are the following: Local (groups 1 and 2), Global (groups 3 and 4), Asset (groups 1 and 3), Interest (groups 2 and 4), GassetLinterest (groups 2 and 3), LassetGinterest (groups 1 and 4) and All (groups 1, 2, 3 and 4).

The estimator of the conditional expected country excess return $\widetilde{\pi}_{it}$ $(i = 1, ..., 12)$ is based on the following linear specification:

$$\widetilde{r}_{it+1} = \widehat{\alpha}_i + \sum_{j=1}^{N_k^1} \widehat{\phi}_{ij} \cdot \widetilde{Z}_{jt} + \sum_{l=1}^{N_k^2} \widehat{\phi}_{il} \cdot \widetilde{Z}_{lt}^i + \widetilde{\xi}_{it+1} \tag{6}$$

where $\widetilde{r}_{it+1}$ is the excess return for the i^{th} country, $\widetilde{Z}_{jt}$ $\left(j = 1, ..., N_k^1\right)$ represent the global variables of the k^{th} model specification $(k = 1, ..., 7)$ and $\widetilde{Z}_{lt}^i$ $\left(l = 1, ..., N_k^2\right)$ are the local variables of the k^{th} model specification $(k = 1, ..., 7)$, which of course depend on i. Finally, $\widetilde{\xi}_{it+1}$ denotes the i^{th} country regression residual. The coefficients of Equation (6) estimated up to time $t-1$ are used together with the value of the predictive variables at time t to obtain an out-of-sample prediction for each country index excess return between t and $t+1$.

4.2.2 Set of estimation window lengths

The uncertainty related to the choice of the estimation window length concerns both conditional and unconditional asset allocations through the estimation of the the conditional moments (Π_t and Σ) and the unconditional ones (μ and V). In order to estimate these parameters for the two different allocation methods, we propose two estimation procedures. The first one consists in taking an expanding window. The initial length of the estimation window is respectively set to 60, 80 and 100 datapoints. The second estimation procedure is based on a rolling window with three different fixed window lengths respectively equal to 60, 80 and 100 datapoints.

4.2.3 Set of portfolio selection constraints

For both conditional and unconditional strategies, four different sets of constraints presented in Table 2 are modelled in order to reduce estimation risk.

[INSERT TABLE 2]

The first strategy (called Free) consists in a free mean-variance optimization. The strategy called Positivity disallows to sell short both risky assets and the riskless one. Following Frost and Savarino (1988), the strategy Diversification imposes diversification as it prohibits weights greater than D. Finally, the strategy Variability forces the weights at two consecutive rebalancing dates t and $t+1$ to be close to one another, implying that bets on various assets remain more stable through time. The results described in the following sections are based on D equal to 20% (implying a maximum 5-asset diversification) and L equal to 30% (implying a maximum turnover of 180%). In unreported results, we find that the performance of the strategies Diversification and Variability remains virtually unchanged if D or L are fixed to 10%, 20% or 30%. For instance, if we compare the two sets of strategies Variability obtained with L equal to 10 and 30%, the average pairwise correlation amounts to 0.82 and the average pairwise excess mean difference absolute value is equal to 1.05% per year. As a result, the performance analysis documented hereafter is robust to changes in D and L. In order to be sure that all these strategies generate weights that can be reasonably taken by a real-time investor, it is assumed that the sum of the squared weights are inferior to 4. It prevents situations where estimation error generates extreme positions that could never be taken by any investor. For instance, a free optimization done with totally wrong parameters could easily produce weights with absolute values between 2000 and 3000 percent especially when the risk aversion coefficient A is low.

To summarize the procedure, the conditional strategies are generated by means of 7 sets of instruments, 6 estimation techniques and 4 types of constraints, which amounts to the estimation of 168 models for a given risk-averse investor. For the unconditional approach, considering 6 estimation techniques and 4 types of portfolio constraints yields a total of 24 models.

4.3 Levels of risk aversion and transaction costs

The results are examined for three types of investors characterized by different risk aversion coefficients A. The impact of this parameter on the portfolio allocation process is important because of the following trade-off. On the one hand, a low risk aversion parameter increases the portfolio exposure to estimation risk as the investor plunges into securities with high expected returns, low variance or covariances. Indeed, Chopra and Ziemba (1993) show that the smaller the risk aversion parameter, the greater impact estimation errors produce on the investor's expected utility. On the other hand, if the estimated expected return vector is sufficiently accurate a low risk aversion parameter allows to take fully advantage of the predictions by inducing more aggressive bets. The relative importance of these two factors is an empirical matter which needs to be investigated. The three different values taken by A are respectively equal to 2, 5 and 10.

Another important aspect concerns the impact of the transaction costs on the profits generated by the various strategies. Three different levels of transaction costs are modelled (low, medium and high) for both the stock index futures contracts (for the conditional strategies) and the underlying securities (for the unconditional strategies). The net excess return of each strategy r_{pt+1}^{net} is based on the following formula:

$$r_{pt+1}^{net} = r_{pt+1} - t_c \cdot w_{turn} \left(1 + r_{pt+1}\right) \tag{7}$$

where t_c is the roundtrip transaction cost and w_{turn} is the portfolio turnover at each rebalancing date. It is computed as:

$$w_{turn} = \frac{1}{2} \sum_{i=1}^{N} \left| w_{it+1} - w_{it}^{B} \right| \tag{8}$$

where $w_{it}^{B} = \frac{w_{it}(1+r_{it+1})}{(1+r_{pt+1})}$. Since the positions in the futures market are taken for one month, w_{turn} is equal to one for conditional strategies. In the first case, the roundtrip transaction costs t_c amount to 10 basis points (bp) of principal value in the index futures market and to 50 bp in the stock market. In the second case, they are equal to 20 and 80 bp and in the third cases they are set to 40 and 120 bp.

5 Comparative Analysis of Conditional and Unconditional Strategies

5.1 Descriptive statistics

We begin this section by showing characteristics of the investment process at each porfolio rebalancing date induced by conditional and unconditional asset allocations. As there exists 168 conditional and 24 unconditional strategies, all figures documented in Table 3 represent averages over all possible specifications. For instance, the column "average country weight mean" is computed as follows: we first evaluate the average country weight at each date t for each strategy (average across countries having an non zero weight). Then, we compute the mean of the average portfolio weight for each strategy (average across time). Finally, we collect this figure for each strategy and calculate the grand mean (average across strategies). The other columns are computed in a similar way. On average both unconditional and conditional strategies are fairly well diversified since around 8 countries are targeted on average at each rebalancing date. We can notice that the average country weight mean is higher for conditional strategies, thus implying greater bets on the various assets. Consistently with the intuition, an investor with a lower risk aversion parameter A takes higher bets since the average country weight mean is 10% higher than the one obtained for $A = 10$. The standard deviation of the average country weight is nearly two times higher for the conditional strategies, indicating that the latter are prone to substantial weight variability. This feature is confirmed by the portfolio turnover. On average, more than 80% of the conditional portfolio is rebalanced for all three values of A whereas it is much lower for the unconditional approach. This comment gives credit to the approach used in this paper consisting in implementing conditional asset allocation by means of stock index futures contracts.

[INSERT TABLE 3]

Turning to the description of the annualized excess mean, standard deviation, Sharpe ratio and final wealth computed as averages over the 168 conditional and 24 unconditional specifications, Table 4 examines the case where no transaction costs are considered. Comparing conditional and unconditional strategies, we first notice that both the excess mean and standard deviation of the conditional strategies are higher independently of A. When A is low, the increase in risk (33.7 against 32.0%) is compensated with a substantial return premium over the unconditional strategies (10.3 against 4.9%). On the contrary, when A is high, the conditional asset allocation produces a much higher risk (20.3 against 13.5%) for a low premium (5.9 against 4.0%). To examine if all of these differences are significant, we bootstrap the excess return time-series of all conditional and conditional strategies in order to construct the distribution of the excess mean and standard deviation differences under the null assumption of no difference[5]. The excess mean of the conditional strategies is significantly higher at the 10% level for $A = 2$ but it is not significantly different from the unconditional strategies for $A = 5$ and 10. Moreover, the standard deviation of the conditional strategies is significantly higher for all values of A, meaning that conditional asset allocation is always riskier. Second, examination of the Sharpe ratio and the final wealth generated if one currency unit is invested in January 1990 suggests that on average conditional strategies generate higher risk-return trade-off and capital gains than unconditional strategies. Nevertheless, using the bootstrap procedure we find that the differences are never significant because of the important standard deviation of the strategies. It implies that the striking differences documented for $A = 2$ (0.14 for the Sharpe ratio and 1.02 for the final wealth) are frequent under the null assumption of no differences. It is therefore difficult to draw strong conlcusions based on the Sharpe ratio and the final wealth.

For comparative purpose, Table 4 also contains summary statistics of 3 buy and hold strategies: the weights are defined at the beginning of the period in January 1990 and the portfolio is never rebalanced during the whole period. The first portfolio is value-weighted (Market), the second one equally-weighted (Equal) and the third one GDP-weighted (GDP)[6]. Because of the sharp decline of the Japanese market in the 90's, holding the market portfolio leads to an excess mean which is always significantly lower than the conditional strategies at the 10% percent level. On the contrary, Equal and GDP represent interesting alternatives to both unconditional and conditional approaches in terms of risk-return trade-off and final wealth. For example, the bootstrap analysis shows that the standard deviation of Equal is significantly lower that the

[5] The bootstrap is often more accurate in finite samples than first-order asymptotic approximations (Horowitz, 2001). In unreported results, we also implemented a blockbootstrap methodology with block length equal to $n^{\frac{1}{4}}$ (proposed by Hall et al., 1995) but the difference between bootstrap and blockbootstrap is negligible in our tests.

[6] In the early 90's, the market capitalization of Japan was very important (49.3% in our sample). For these reasons, some institutions decided at that time to reduce their exposures to the Japanese market by creating global indices based on GDP weights.

standard deviation of conditional strategies whereas its excess mean is significantly lower in only one situation (for $A = 2$).

[INSERT TABLE 4]

When transaction costs are taken into account, the average standard deviation remains constant for both unconditional and conditional strategies, but the average excess mean is reduced, thus implying smaller Sharpe ratios and final wealth. Table 5 illustrates these comments in the intermediate case of medium transaction costs (20 bp in the futures and 80 bp in the stock market). The diminution of the average excess mean, Sharpe ratio and final wealth are fairly homogenous in all cases and are approximately equal to 2%, 0.07 and 0.40 respectively. Since the statistics of the buy and hold strategies are not influenced by the introduction of transaction costs, Equal and GDP become the most performing strategies in terms of Sharpe ratio and final wealth. This is supported by the $p-$values related to the excess mean and standard deviation differences between the conditional strategies and Equal: the difference is never significant for the excess mean but is always significant for the standard deviation.

[INSERT TABLE 5]

To summarize this descriptive analysis, we notice that on average the conditional asset allocation provides better results in terms of excess mean return, Sharpe ratio and final wealth than the market portfolio and unconditional strategies. But the bootstrap analysis shows that most of these differences are not statistically significant except for the case where A is low. Moreover, the introduction of transaction costs makes the conditional allocation less attractive than GDP and equally-weighted portfolios for the three different levels of risk aversion.

5.2 Final wealth distribution

An interesting approach to assess the economic significance as well as the risks associated with conditional asset allocation consists in examining the distribution of final wealth generated by all conditional specifications. More precisely, the latter represents the distribution of final wealth faced by an investor who randomly picks up one specification among the 168 conditional strategies and invests one currency unit. As an illustration, Figures 1 shows these distributions for the intermediate case where $A = 5$ under the four levels of transaction costs.

[INSERT FIGURE 1]

These figures deliver an important message concerning the risks related to the two approaches: the final wealth distribution of conditional strategies is widely spread and covers a large range of outcomes, thus implying substantial risks. The most disturbing feature comes from the important downside risk related to the fat left-tails of the distribution. Without transaction costs, the 5%-VaR of the conditional strategies is equal to 0.44 (compared to 0 for the unconditional methods). In the case of medium transaction costs, the 5%-VaR amounts to 0.67 (compared to 0.17 for the unconditional methods) and jumps to 0.84 when transaction costs are high. On the contrary, the final wealth distribution of the 24 unconditional strategies is peaked around one, yielding a more homogeneous overall performance. The gains generated by these strategies are never spectacular and the probability of a loss is similar to the one related to conditional strategies. But if a loss occurs, it is not severe.

Considering the case of a more risk-averse individual ($A = 10$), the previous comments are strenghtened since the implementation of the 24 unconditional strategies under the 4 levels of transaction costs never cause capital losses, whereas the 5%-VaR of the conditional strategies repectively amount to 0.29 (no transaction costs) and 0.57 (medium transaction costs). Finally, if A is low, the large bets taken by the investor induce important capital losses for a substantial number of conditional strategies and procuce the highest Values-at-Risk. In fact, examination of the final wealth distribution explains why the high average final wealth documented in Tables 4 and 5 for $A = 2$ is high: it is simply due to a couple of extreme strategies, which produce impressive final wealth and compensate the general bad results obtained by the other specifications. The main conclusion that can be drawn from this analysis is that the conditional asset allocation is subject to important downside risk independently of the level of risk aversion. Besides, this risk increases when transaction costs are taken into account.

6 Performance Analysis across the Three Different Exogenous Specifications

6.1 Methodology

In order to assess the performance of conditional asset allocation, we use two different measures: the Cornell measure and the Sharpe ratio. Contrary to other measures based on asset pricing models, they are benchmarkfree. This important property makes them more adapted to an international context as reasonable sources of systematic risk are very difficult to define (Solnik, 1993). The Cornell measure, denoted by C, (first proposed by Cornell, 1979), is defined as the sum of the N covariances between the i^{th} risky asset weight $\widetilde{w}_{it}$ defined at time t and its excess return $\widetilde{r}_{it+1}$ between t and $t+1$:

$$C = \sum_{i=1}^{N} cov(\widetilde{w}_{it}, \widetilde{r}_{it+1}) \tag{9}$$

The general objective of the Cornell measure is to determine if an active strategy is based on superior information. The intuition behind this measure is straightforward: if an investor thinks that the excess return of the i^{th} security is higher than usual, he will decide to overweight this asset in the portfolio, thus inducing a positive value for $cov(\widetilde{w}_{it}, \widetilde{r}_{it+1})$. In our context where it is assumed that $\Sigma_t = \Sigma$ (see Assumption 1), the investor's superior information is only defined with respect to the predictive ability on the first moment of the excess return distribution. Using this relation, it can be shown (see the proof in the Appendix) that the Cornell measure of the conditional strategy is positive if and only if there is superior information. As a result, the Cornell measure is capable of detecting superior information and can be seen as a test of global predictability over all assets. To estimate the Cornell measure, we use the following unbiased estimator proposed by Grinblatt and Titman (1993):

$$\widetilde{C}_{cond} = \sum_{t=k+1}^{T} \sum_{i=1}^{N} \widetilde{r}_{it+1} \left(\widetilde{w}_{it} - \widetilde{w}_{it-k} \right) = \sum_{t=k+1}^{T} \widetilde{r}_{0t+1} \tag{10}$$

where $\widetilde{r}_{0t+1}$ is the return of a zero-cost portfolio formed with a long position $\widetilde{w}_{it}$ (defined at time t) and a short position $\widetilde{w}_{it-k}$ (defined at time $t-k$) taken in the i^{th} asset ($i = 1, ..., N$). Following the arguments of Grinblatt and Titman (1993), k is based on a one quarter lag.

The second performance measure based on the Sharpe ratio has been widely used in previous studies. It is defined as the differential between the Sharpe ratio of the conditional strategy and the one of an uninformed strategy. If this differential is positive, it is then argued that the conditional strategy is preferable. It is certain that the conditional optimization rule defined in Equation (2) may affect higher moments of the excess return distribution which are not captured by the Sharpe ratio (Dybvig and Ross, 1985). To anwser this question, Barras (2004) simulates the excess return distribution of conditional asset allocation based on reasonable levels of predictability (i.e. it is assumed that the predictive variables explain 1% or 3% percents of the asset excess return variance). He shows that both the skewness and kurtosis of the conditional asset allocation are low, thus implying that the differential Sharpe ratio is always positive. In light of this result, it is reasonable to use the Sharpe ratio as a preference indicator between conditional and unconditional strategies.

The objective of this section is to assess the global performance of conditional asset allocation considering the 168 strategies and to know if the performance varies according to the different exogenous specifications that may be reasonably chosen by any investor. If the performance remains constant, it means that real-time uncertainty is not an issue since the investor could take any conditional strategies and still achieve superior performance. The approach is based on the percentage of specifications yielding a Cornell measure and a Sharpe ratio differential superior to zero. The p-value of this percentage under the null hypothesis that all conditional strategies have in turn a zero Cornell measure and a zero Sharpe ratio differential is computed with the bootstrap method developed by White (2000). More precisely, the p-value under the null is computed by bootstrapping simultaneously the excess return time-series of all strategies 1'000 times. As a result, this test does not destroy the contemporaneous links between the strategies contrary to the approach used by Cooper and Gulen (2004). A brief description of the methodology adapted to our context is presented in the Appendix.

6.2 The Cornell measure

The percentages of conditional strategies with a positive Cornell measure are displayed in Table 6. For the three different risk coefficients A, the results are disappointing. The percentages of strategies having a positive Cornell measure are low since they are equal to 40.5, 36.9 and 30.4 for the three respective values of A. More importantly, they are not statistically different from zero at the 10% percent level as it shown by the White's p-value in parenthesis. It means that if all conditional strategies are assumed to have a Cornell measure equal to zero, it is frequent to observe the percentage values shown in Table 6. This bad performance is consistent with the average explanatory power of the predictions: if we run out-of-sample regressions of excess returns on the forecasts, the average R^2 across the 12 countries is always equal to zero for all sets of predictive variables, thus confirming the results obtained by Bossaerts and Hillion (1999). Looking at the different percentages across the three specification sets (predictive variables, selection constraints and estimation techniques), we notice that none of them is statistically different from zero. It therefore prevents us from drawing strong conclusions based on percentage comparisons. Concerning the predictive variables, we can see that strategies based on Local, All or Asset generate low percentages in all cases: it is particularly striking for $A = 2$ since the highest percentage related to Global or Interest is substantially higher. Similarly, the percentages associated with the optimization Free is in all cases lower than those obtained with constrained optimizations, indicating that constraints may have a positive impact on performance. Finally, strategies based on rolling windows display a positive Cornell measure more often but there is no clear pattern for the size of the window: for instance, increasing the window size has a negative impact with an Expanding technique whereas it is positive under a Rolling specification for $A = 5$ and 10. The main conclusion from this performance analysis is that the Cornell measure is unable to detect the presence of superior information across the different specifications on which conditional strategies are based.

[INSERT TABLE 6]

6.3 The Sharpe ratio

In order to compute the Sharpe ratio differential we must select uninformed strategies serving as benchmarks. 4 different benchmarks are selected: 2 unconditional and 2 buy and hold strategies. In order to choose 2 unconditional strategies among the 24 existing ones, we use the following method: all unconditional strategies are ranked in increasing order according to their Sharpe ratios and the specifications in the 50 and 75% quantiles are chosen (they are hereafter denoted by 50 and 75%)[7]. Concerning the buy and hold strategies, we use the market portfolio (denoted by Market) and the equally-weighted portfolio (denoted by Equal). Since the results

[7]Even if the implementation of the unconditional asset allocation is also prone to real-time uncertainty, we argue that these two benchmarks give a fair representation of the performance achievable with the unconditional approach for two reasons. First, we purposely select strategies which are not at the top of the ranking. Second, as it is explained in the previous section the performance of the unconditional asset allocation is quite homogeneous across the 24 specifications.

are similar for the three aversion parameters, we only discuss the intermediate case (i.e. $A = 5$). Table 7 contains the percentage of strategies having a positive Sharpe ratio differential without transaction costs. The global percentages related to the 50, 75% benchmarks and the market portfolio respectively equal to 79.8, 69.0 and 94.0% are strongly significant at the 10% level. This result is positive for the conditional asset allocation since a significant proportion of the 168 conditional specifications are able to beat three among the four uninformed benchmarks. But this superior performance is not systematically present across the different specifications (predictive variables, selection constraints and estimation techniques) that could be chosen by a real-time investor. First, if we examine more precisely the percentage variation across the predictive variables, there is a wide dispersion demonstrating that the choice of the correct information set has a crucial impact. For instance, the percentages related to Asset are extremely low, while the ones associated with All are always superior to 75%. Among the set of portfolio constraints, Diversification and Variability obtain the best results since their percentage of positive Sharpe ratio differential is significantly different from zero for all benchmarks. The performance is very disappointing for Free: it only beats Market, which is not surprising in light of the disastrous performance of the market portfolio during the period. This result highlights the need for portfolio constraints in the optimization process in order to reduce estimation risk. Finally, the examination of the performance across the set of estimation window does not reveal a clear pattern: in the case of Expanding window, the smallest window obtains the best results whereas the medium window size is more appropriate for Rolling specifications.

[INSERT TABLE 7]

In order to analyze the impact of transaction costs on the performance of conditional asset allocation, we present in Table 8 the percentage of positive Sharpe ratio differentials considering medium transaction costs (20 bp in the futures and 80 bp in the stock market). This modification does not alter the main comments previously developed. First, the various percentages related to the two unconditional benchmarks (50% and 75%) remain fairly constant. This result is not surprising since the unconditional strategies are also affected by the introduction of transaction costs. Second, the percentage variation across the different specifications (predictive variables, selection constraints and estimation window) is also present. The most important modification comes from the performance related to the buy and hold strategies. Indeed, we notice that the percentages related to the equally-weighted portfolio drop substantially: since the reduction ranges from 20 to 30% in all cases, only one type of specifications (Diversification) yields a percentage significantly different from zero. Since the same outcome is obtained with the GDP-weighted porfolio, it is likely that the conditional asset allocation is not able to outperform a large range of passive strategies (except those which are heavily invested in the Japanese stock market in January 1990).

[INSERT TABLE 8]

To conclude this section, let us insist on the two important results stemming from this analysis. First, the overall performance in terms of Sharpe ratio is mitigated: on the one hand, conditional asset allocation are able to beat unconditional strategies for different levels of transaction costs and risk aversion parameters. But on the other hand, they have difficulty in outperforming buy and hold strategies especially when realistic transaction costs are accounted for. Second, the percentage variation across the various specifications clearly indicates that real-time uncertainty has an important impact. For instance, an investor who would use GassetLinterest variables would have a high probability of beating all uninformed benchmarks, which is not the case if he had selected Asset variables. The same comment applies to Free and Diversification portfolio selection constraints. This remark raises an interesting issue: concrete applications of conditional asset allocation could still generate high economic profits if the real-time investor was able to select performing models. It would be the case if the best models shared common characteristics that could be detected from an ex-ante perspective. This analysis is the object of the next section.

7 Characteristics of the Conditional Strategies

In order to know if the best strategies share common characteristics, our approach consists in comparing some of the best and worst strategies and assessing their sources of differences. For all levels of risk aversion coefficients and transaction costs, we rank the 168 conditional strategies according to their final wealth and compare the composition of the 30 worst and best strategies across the different specifications (predictive variables, selection constraints and estimation window). Table 9 shows these results for the three levels of risk aversion without transaction costs (the results considering transaction costs are similar and thus not displayed). Notice that the proportion of worst models inducing a capital loss (i.e. a final wealth inferior to one) is respectively equal to 100% for $A = 2$, 93.4% for $A = 5$ and 66.6% for $A = 10$. The most striking feature lies in the fact that some of the exogenous variables among the different specification sets are largely spread among the best and worst models. For instance, the predictive model Interest or LassetGinterest both serve as a basis for the implementation of the best and worst strategies independently of A. Moreover, it may be argued that Global constitutes an ex-ante proper predictive model for developed markets since they are likely to be fully integrated but it is not the case: 20% of the worst models rest on this specification for $A = 2$ and 5. This conclusion also holds for the estimation windows: if we examine the distribution of expanding and rolling windows, the worst and the best models use all of the different window lengths with similar frequencies. The dispersion of the selection constraints between the worst and best models is even more disturbing. Since there are good reasons to believe that the estimation of the conditional expected excess return vector $\widetilde{\Pi}_t$ is subject to estimation risk, putting constraints represents one of the only actions that may be rationally taken ex-ante by a real-time investor. Unfortunately, the results provided in Table 9 are not encouraging. First, the unconstrained optimization Free forms 30% of the best models, even though its application leads most of time to disastrous results. Second, it is not rare to observe specifications based on portfolio constraints which yield disappointing performance. This is the case for Positivity (consider the case where $A = 5$ and 10) and Variability for all levels of A.

[INSERT TABLE 9]

This wide parameter dispersion implies that minor specification changes substantially reduce the generated profits. Several examples documented below illustrate this fact. First, let us first modify the set of predictive variables. For $A = 2$ and no transaction costs, the conditional strategy based on All, Variability and an Expanding window of 80 datapoints obtains a final wealth and a Sharpe ratio respectively equal to 6.3 and 0.55. If we select Asset instead of All, the final wealth and the Sharpe ratio fall to 0.62 and 0.05. Second, if the portfolio constraints are changed, the performance reduction is also substantial: for $A = 10$ and no transaction costs, if an investor adopts the strategy based on Interest, Positivity and Expanding 100, he obtains a final wealth equal to 1.51. If he had selected Variability instead of Positivity, he would have lost half of his initial wealth. Finally, consider changes of the estimation windows. For $A = 5$ and medium transaction costs, let us modify the strategy Interest, Variability and Rolling 80 to the strategy Interest, Variability and Expanding 80: the final wealth falls from 3.21 to 0.41 and the Sharpe ratio from 0.41 to 0.08. The main conclusion of our analysis is that there are no common characteristics that could be used ex-ante to select performing conditional asset allocations. Since the results are very sensitive to minor changes inside the sets of exogenous specifications, we conclude that the performance of the best conditional models is difficult to replicate in real-time. In fact, the lack of solid theory behind the choice of the predictive models represents a crucial point. Even if, at a precise point in time, the investor realizes that the majority of the successful models use similar predictive variables and estimation techniques, these specifications cannot confidently be used in the future: it is almost sure that these relations only hold during the investigated period and is likely to be modified in the future. Since it is extremely difficult to justify their use with theoretical arguments, information obtained from past data are unlikely to be useful for a real-time investor.

Of course it is possible that the real-time investor tests several conditional specifications during the investigated period. For instance, he may want to modify the conditional strategy if the latter has performed poorly in the recent past. But this behaviour may not be optimal: first, it is possible that the initial model will generate substantial profits in the future and second his new selected model may be as bad as the first one. Another possibility consists in combining a large set of predictive models in order to obtain more precise asset excess return predictions (see Aiolfi and Favero (2002) for an application to the US market). Indeed, Yang (2004) shows that when a single variable is predicted, a convex combination of all the model predictions yields a lower square error loss function[8] than the best individual model. Unfortunately, an extension of these results to our multi-asset framework is far from straightforward. Indeed, the loss of utility

[8] This function is defined as $\frac{1}{T}\sum_{t=1}^{T} E\left(\pi_t - \widetilde{\pi}_t\right)^2$, where T is the number of forecasts, π_t and $\widetilde{\pi}_t$ respectively denote the true and estimated conditional expectation of the dependent variable.

implied by bad predictions rests on the mean-variance trade-off expressed in Equation (2). As it depends on the simultaneous prediction errors made for the N assets, the results obtained by Yang for a single asset can not be used. Moreover, the theory behind the use of thick modelling only focuses on predictive models and does not indicate how different portfolio selection constraints can be combined together. Finally, the required time to combine all predictions across all predictive variables and countries is extremely long. Therefore, as long as coherent model aggregation rules for both predictive models and portfolio constraints are not available, it is conceivable that this approach also leads to disappointing performance.

8 Conclusion

Previous studies show that mean-variance conditional asset allocation based on predictability systematically outperforms uninformed strategies. We test the robustness of these documented results to the presence of real-time uncertainty. Once real-time uncertainty is explicitly taken into account, the performance of these strategies turns out to be poor across different levels of transaction costs and investor's risk aversion. First, the average excess return, Sharpe ratio and final wealth over the 168 conditional strategies are not statistitically different than the ones documented for unconditional and buy and hold strategies in the vast majority of cases. Moreover, the distribution of final wealth generated by conditional asset allocation is subject to important downside risk. Second, the performance measured by the percentage of positive Cornell measure indicates that none of the conditional strategies is based on superior information. Examination of the Sharpe ratio differential reveals that conditional asset allocation is able to outperform unconditional asset allocation but is beaten by buy and hold strategies especially when transaction costs are accounted for. Finally, the best models do not share common characteristics liable to help the real-investor to select performing models ex-ante. Since there is no theoretical grounds behind predictability, an unlucky investor could perfectly pick up a bad model inducing important capital losses. Our results can be related to other papers such as Cooper, Gutierrez and Marcum (2001) or Cooper and Gulen (2004) which clearly show the important impact of real-time uncertainty. For instance, Cooper, Gutierrez and Marcum (2001) indicate that popular measures of systematic risk such as the book-to-market ratio or the firm size are not useful ex-ante to form strategies with higher returns. The difficulty in determining the best specifications through time therefore casts an important doubt on both cross-sectional and time-series predictability. Our paper unambiguously calls for a more efficient implementation of conditional asset allocation, which unfortunately would come from a better understanding of very complex relations. For instance, its improvement crucially depends on a more precise definition of the sources of predictability and the causes of their time-variation. It also rests on the analysis of the trade-off between estimation risk and specification risk in small samples in order to specify adequate portfolio constraints likely to improve out-of-sample performance.

9 Appendix

9.1 Descriptive statistics of the Datastream country indices

Table 10 presents some distribution characteristics of the 12 developed market monthly excess returns in local currency between January 1990 and September 2004 (176 datapoints). The computation is based on the country Datastream indices (price and gross dividends reinvested) and the one-month euro-market interest rates. One can notice the high average excess returns for the Switzerland and the USA as well as the sharp decline of the Japanese stock market. The normality assumption is rejected in 7 markets at the 10% level. The correlation matrix reproduced in Table 11 shows that the potential for diversification is limited as 78% of the correlations are superior to 0.50. This result is not surprising since these countries are likely to be integrated to the world capital market. Both Australia and Japan display low correlations with other countries but in light of the Japanese negative excess mean, only Australia offers an interesting potential for diversification during the 90's.

[INSERT TABLE 10 AND TABLE 11]

9.2 Relation between the Cornell measure and superior information

The analytical form of the Cornell measure defined in Equation (9) for the conditional strategy, denoted by C_{cond}, can be derived as follows. From the first-order conditions of the optimization program defined in Equation (2), the $N \times 1$ portfolio weight vector $\widetilde{w}_t$ is equal to $\widetilde{w}_t = \frac{1}{A}\Sigma^{-1}\widetilde{\Pi}_t$. Using this expression, the $N \times N$ covariance matrix between the $N \times 1$ portfolio weight vector $\widetilde{w}_t$ and the $N \times 1$ conditional expected excess return $\widetilde{\Pi}_t$ is equal to:

$$cov\left(\widetilde{w}_t, \widetilde{\Pi}_t\right) = \frac{1}{A}\Sigma^{-1}\Omega \tag{11}$$

where Ω denotes the covariance matrix of the conditional expected excess return vector $\widetilde{\Pi}_t$. Taking the sum of its diagonal elements gives the Cornell measure:

$$C_{cond} = \frac{1}{A}Tr\left(\Sigma^{-1}\Omega\right) \tag{12}$$

We assume that the investor has superior information if the $N \times 1$ conditional expected excess return $\widetilde{\Pi}_t$ is different from the $N \times 1$ unconditional expected excess return μ for at least one realization of I_t. It thus implies that the covariance matrix of the prediction Ω is different from zero. The following proposition examines the relation between the Cornell measure and the notion of superior information.

Proposition 9.1 *Assuming that Σ is invertible, the Cornell measure of the conditional portfolio is positive if and only if the investor has superior information.*

Proof. If part:
Assume that the investor has superior information (i.e. $\Omega \neq 0$). Ω is symmetric and, by the property of the covariance matrices, is at least non-negative definite. As a result, it can be written as $\Omega = \Omega^{\frac{1}{2}}\Omega^{\frac{1}{2}}$, where $\Omega^{\frac{1}{2}}$ is also a symmetric and non-negative matrix. We have $x'\Omega^{\frac{1}{2}}\Sigma^{-1}\Omega^{\frac{1}{2}}x = y'\Sigma^{-1}y$, where $y = \Omega^{\frac{1}{2}}x$: this expression is always stricly positive except for cases where $y = 0$. Therefore, $\Omega^{\frac{1}{2}}\Sigma^{-1}\Omega^{\frac{1}{2}}$ is non-negative definite.

Then we use the following property: if a matrix A is nonnegative definite, its eigenvalues are nonnegative. Indeed, for each eigenvalue λ_i and eigenvector x_i, we can write the following expressions: $Ax_i = \lambda_i x_i \Leftrightarrow x_i' A x_i = \lambda_i x_i' x_i$. Since $x_i \neq 0$, $\lambda_i \geq 0$. Applying this property to $\Omega^{\frac{1}{2}}\Sigma^{-1}\Omega^{\frac{1}{2}}$ implies that at least one eigenvalue is different from zero because otherwise it would contradict the assumption that $\Omega \neq 0$. Therefore, $Tr(\Omega^{\frac{1}{2}}\Sigma^{-1}\Omega^{\frac{1}{2}}) = Tr(\Sigma^{-1}\Omega) > 0$, which proves that the Cornell measure is positive.

Only if part:
Suppose that the Cornell measure is stricly positive. It implies that $Tr(\Sigma^{-1}\Omega) > 0$ and that $\Omega \neq 0$. Therefore, Π_t is different from μ for at least one realization of I_t, which corresponds to the definition of superior information. $\blacksquare$

9.3 Description of the reality check p-value

Let us follow White's notations and denote by $f_{kt+1}\left(Z_t, \widehat{\beta}_t\right)$ the performance measure computed at time $t+1$ for the k^{th} model ($k = 1, ..., l$). Z_t is a matrix which contains the vectors of stock index excess returns as well as predictive variables up to time t and satisfies the assumptions stated in White (2000). $\widehat{\beta}_t$ is the set of estimated coefficients of Equation (6). The performance statistic for the k^{th} strategy ($k = 1, ..., l$) computed over n periods is equal to:

$$\overline{f_k} = \frac{1}{n} \sum_{t=R}^{T} f_{kt+1}\left(Z_t, \widehat{\beta}_t\right) \tag{13}$$

Stacking each element $\overline{f_k}$ we obtain a $l \times 1$ vector $\overline{f}$ with a mean equal to $E(f)$. The approach adopted by White (2000) consists in knowing whether the best strategy is significantly better than a benchmark. Since we are rather interested in the percentage of positive performance statistics among all strategies, let us define the following variables:

$$\begin{aligned} \overline{q}_k &= 1 \text{ if } \overline{f_k} \geq 0 \quad (k = 1, ..., l) \\ &= 0 \text{ otherwise} \end{aligned} \tag{14}$$

$$\overline{P} = \frac{1}{l} \sum_{k=1}^{l} \overline{q}_k \tag{15}$$

$\overline{P}$ is the proportion of positive performance statistics among the l tested strategies. In order to determine the $p-$value of $\overline{P}$ under the null of no superior performance, we need to construct its distribution under the null assumption that all performance statistics are equal to zero (i.e. $E(f) = 0$). One important result proved by White (2000) is that the distribution $n^{\frac{1}{2}}\left(\overline{f} - E(f)\right)$ can be approximated by $n^{\frac{1}{2}}\left(f_i^* - \overline{f}\right)$ where f_i^* is a $l \times 1$ vector of performance statistics (whose single elements are denoted by f_{ki}^*) recomputed from the i^{th} bootstrap ($i = 1, ..., N$) of all l conditional excess return time-series together in order to account for their contemporaneous link[9]. For each bootstrap ($i = 1, ..., N$), we compute the following statistics:

$$\begin{aligned} q_{ki}^* &= 1 \text{ if } f_{ki}^* - \overline{f_k} \geq 0 \quad (k = 1, ..., l) \\ &= 0 \text{ otherwise} \end{aligned} \tag{16}$$

$$P_i^* = \frac{1}{l} \sum_{k=1}^{l} q_{ki}^* \tag{17}$$

[9] Series can also be blockbootstrapped in order to keep the autocorrelation of excess returns. We implemented a blockbootstrap methodology with block length equal to $n^{\frac{1}{4}}$ (proposed by Hall et al., 1995) but the difference between bootstrap and blockbootstrap is negligible in our tests.

Collecting all P_i^* $(i = 1, ..., N)$ and sorting them in ascending order yields an approximation of the distribution of $\overline{P}$ under the null. Then, we find M such that $P_M^* < \overline{P} < P_{M+1}^*$, and define the reality check p-value (under the null hypothesis that the expected performance measure of all strategies is equal to zero, i.e. $E(f) = 0$) as follows:

$$p - value = 1 - \frac{M}{N} \tag{18}$$

To conclude, let us define the form of f_{kt+1} for our two performance measure based on the Cornell measure and the Sharpe ratio. For the Cornell measure, we have:

$$f_{kt+1}\left(Z_t, \widehat{\beta}_t\right) = \widetilde{r}_{0kt+1}\left(Z_t, \widehat{\beta}_t\right) \tag{19}$$

where $\widetilde{r}_{0kt+1}$ is the zero-cost portfolio of the k^{th} strategy defined in Equation (10). Performance statistics based on the Sharpe ratio are computed by comparing the Sharpe ratio of the k^{th} strategy with the Sharpe ratio of an uninformed benchmark strategy denoted by 0. Following Sullivan, Timmermann and White (1999) we denote by h_{t+1} a 2×1 vector containing the following elements:

$$h_{t+1} = \begin{pmatrix} h_{t+1}^1 \\ h_{t+1}^2 \end{pmatrix} = \begin{pmatrix} \widetilde{r}_{kt+1} \\ \left(\widetilde{r}_{kt+1}\right)^2 \end{pmatrix} \tag{20}$$

Let us then define a function $g\left(h_{t+1}\right)$ as follows:

$$g = \frac{h_{t+1}^1}{\left(h_{t+1}^2 - \left(h_{t+1}^1\right)^2\right)^{\frac{1}{2}}} \tag{21}$$

Therefore the performance statistic computed at time $t+1$ takes the following form:

$$f_{kt+1}\left(Z_t, \widehat{\beta}_t\right) = g\left(h_{kt+1}\left(Z_t, \widehat{\beta}_t\right) - h_{0t+1}\left(Z_t, \widehat{\beta}_t\right)\right) \tag{22}$$

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Table 1

Grouping of the predictive variables

Characteristics		Related to asset prices		Related to interest rates
Local variables	1	Dividend yield Lagged stock index excess return	2	Short term interest rate Term spread
Global variables	3	World index dividend yield Lagged world index excess return	4	US short-term interest rate US term spread US default spread

Each instrument is classified according to its nature (related to asset prices or interest rates) and its area of influence (global or local influence on capital markets). This procedure leads to different groups numbered from 1 to 4.

Table 2

Description of the four different sets of portfolio selection constraints

Name of the strategy	Constraints
Free	None
Positivity	$0 \leqslant w_{it} \leqslant 1$ and $\sum_{i=1}^{12} w_{it} \leqslant 1$
Diversification	$0 \leqslant w_{it} \leqslant D$
Variability	$\lvert w_{it} - w_{it-1} \rvert \leqslant L$

Free consists in a free optimization, Positivity disallows short-selling. Diversification imposes diversification on the various country weights and Variability forces the weights at two consecutive rebalancing dates to be close to one another. D is a scalar fixing the weight upper-bound at each rebalancing dates and L is a scalar defining the upper-bound of the weight difference absolute value at two consecutive rebalancing dates.

Table 3

Characteristics of conditional and unconditional strategies for different values of A

		N° of countries	Average country weight mean	Average country weight std dev.	Turnover
$A = 2$	Uncond	8.7	40.5%	7.0%	25.4%
	Cond	8.3	46.2%	9.9%	90.5%
$A = 5$	Uncond	8.2	34.6%	5.8%	18.6%
	Cond	8.1	41.8%	9.9%	88.5%
$A = 10$	Uncond	8.0	26.3%	6.5%	14.3%
	Cond	7.9	36.6%	9.2%	84.6%

All figures represent grand means over all conditional (168 possibilities) and unconditional strategies (24 possibilities). For a given strategy, the number of countries and the average country weight mean are computed as averages across countries and across time. The average country weight standard deviation is computed as the standard deviation of the average country weight computed at each rebalancing date t. Turnover represents the average fraction of the portfolio modified at each rebalancing date t.

Table 4

Comparison of conditional and unconditional strategies for different values of A and no transaction costs

		Excess mean	Std deviation	Sharpe ratio	Final wealth
$A = 2$	Uncond	4.9%	32.0%	0.15	0.93
	Cond	10.3%	33.7%	0.29	1.95
	p-values				
	Cond vs Uncond	0.09*	0.06*	0.15	0.44
	Cond vs Market	0.02*	0.00*	0.08*	0.50
	Cond vs Equal	0.06*	0.00*	0.45	0.57
$A = 5$	Uncond	4.4%	20.2%	0.21	1.42
	Cond	7.6%	25.9%	0.30	1.92
	p-values				
	Cond vs Uncond	0.17	0.00*	0.32	0.49
	Cond vs Market	0.03*	0.00*	0.11	0.42
	Cond vs Equal	0.13	0.00*	0.48	0.54
$A = 10$	Uncond	4.0%	13.5%	0.27	1.61
	Cond	5.9%	20.3%	0.30	1.78
	p-values				
	Cond vs Uncond	0.24	0.00*	0.52	0.53
	Cond vs Market	0.08*	0.00*	0.16	0.32
	Cond vs Equal	0.23	0.00*	0.47	0.50
B&H	Market	0.6%	14.5%	0.04	0.93
	Equal	3.9%	13.9%	0.28	1.54
	GDP	3.3%	13.5%	0.25	1.43

All figures represent averages over all conditional (168 possibilities) and unconditional strategies (24 possibilities). The excess return, standard deviation and Sharpe ratio are annualized. The final wealth of each strategy is computed by assuming that one currency unit is invested at the beginning of the period. For comparative purpose, three buy and hold strategies are presented: Market is the value-weighted portfolio of the 12 countries, Equal the equally-weighted portfolio and GDP the GDP-weighted portfolio. The p-values under the null assumption that the statistics differences between the conditional and unconditional (as well as buy and hold) strategies are equal to zero are obtained by boostrapping the excess return time-series of all strategies 1'000 times. Asterisk denotes 10% significance.

Table 5

Comparison of conditional and unconditional strategies for different values of A and medium transaction costs

		Excess mean	Std deviation	Sharpe ratio	Final wealth
$A = 2$	Uncond	2.5%	32.1%	0.08	0.70
	Cond	8.2%	33.7%	0.23	1.46
	p-values				
	Cond vs Uncond	0.10*	0.06*	0.15	0.42
	Cond vs Market	0.08*	0.00*	0.16	0.54
	Cond vs Equal	0.16	0.00*	0.58	0.71
$A = 5$	Uncond	2.7%	20.3%	0.13	1.08
	Cond	5.5%	25.9%	0.22	1.45
	p-values				
	Cond vs Uncond	0.19	0.00*	0.31	0.48
	Cond vs Market	0.10*	0.00*	0.21	0.43
	Cond vs Equal	0.31	0.00*	0.64	0.76
$A = 10$	Uncond	2.6%	13.6%	0.18	1.27
	Cond	3.9%	20.3%	0.21	1.35
	p-values				
	Cond vs Uncond	0.32	0.00*	0.53	0.57
	Cond vs Market	0.17	0.00*	0.25	0.37
	Cond vs Equal	0.49	0.00*	0.66	0.76
B&H	Market	0.6%	14.5%	0.04	0.93
	Equal	3.9%	13.9%	0.28	1.54
	GDP	3.3%	13.5%	0.25	1.47

All figures represent averages over all conditional (168 possibilities) and unconditional strategies (24 possibilities). The excess return, standard deviation and Sharpe ratio are annualized. The final wealth of each strategy is computed by assuming that one currency unit is invested at the beginning of the period. For comparative purpose, three buy and hold strategies are presented: Market is the value-weighted portfolio of the 12 countries, Equal the equally-weighted portfolio and GDP the GDP-weighted portfolio. The p-values under the null assumption that the statistics differences between the conditional and unconditional (as well as buy and hold) strategies are equal to zero are obtained by boostrapping the excess return time-series of all strategies 1'000 times. Asterisk denotes 10% significance.

Table 6

Percentage of positive Cornell measure for different values of A

Grouping	N°	Risk aversion 2	Risk aversion 5	Risk aversion 10
Total				
	168	40.5% (0.62)	36.9% (0.65)	30.4% (0.74)
Predictive variables				
Local	24	20.8% (0.76)	12.5% (0.85)	4.2% (0.94)
Global	24	62.5% (0.36)	45.8% (0.50)	37.5% (0.60)
Asset	24	29.2% (0.62)	25.0% (0.68)	37.5% (0.55)
GassetLinterest	24	37.5% (0.60)	54.2% (0.43)	33.3% (0.64)
LassetGinterest	24	45.8% (0.50)	41.7% (0.56)	29.2% (0.65)
Interest	24	83.3% (0.14)	75.0% (0.25)	58.3% (0.42)
All	24	4.2% (0.96)	4.2% (0.94)	12.5% (0.86)
Selection constraints				
Free	42	14.3% (0.82)	9.5% (0.85)	2.4% (0.94)
Positivity	42	30.9% (0.87)	30.9% (0.84)	35.7% (0.74)
Diversification	42	57.1% (0.39)	64.3% (0.32)	52.4% (0.47)
Variability	42	59.5% (0.37)	42.9% (0.55)	31.0% (0.68)
Estimation window				
Expanding 60	28	53.6% (0.69)	50.0% (0.70)	28.6% (0.86)
Expanding 80	28	39.3% (0.70)	28.6% (0.70)	28.6% (0.84)
Expanding 100	28	28.6% (0.67)	28.6% (0.64)	14.3% (0.80)
Rolling 60	28	46.4% (0.68)	35.7% (0.62)	21.4% (0.52)
Rolling 80	28	39.3% (0.66)	39.3% (0.60)	42.9% (0.53)
Rolling 100	28	35.7% (0.64)	39.3% (0.60)	46.4% (0.52)

The 168 strategies are classified according to the three sets of exogenous specifications (predictive variables, selection constraints and estimation window). Figures into parentheses represent the White's p-value under the null assumption that all conditional strategies have a zero Cornell measure. They are computed by bootstrapping the excess return time-series of all strategies 1'000 times.

Table 7

Percentage of positive Sharpe ratio differential for A equal to 5 and no transaction costs

Grouping	N°	Uncond		B&H	
		50%	75%	Market	Equal
Total					
	168	79.8% (0.00)*	69.0% (0.05)*	94.0% (0.00)*	60.1% (0.12)
Predictive variables					
Local	24	87.5% (0.00)*	70.8% (0.08)*	95.8% (0.00)*	62.5% (0.15)
Global	24	83.3% (0.00)*	66.7% (0.03)*	100.0% (0.00)*	58.3% (0.14)
Asset	24	58.3% (0.20)	41.7% (0.75)	75.0% (0.00)*	25.0% (0.98)
GassetLinterest	24	100.0% (0.00)*	91.7% (0.00)*	100.0% (0.00)*	91.7% (0.00)*
LassetGinterest	24	79.2% (0.00)*	79.2% (0.02)*	100.0% (0.00)*	58.3% (0.15)
Interest	24	62.5% (0.02)*	58.3% (0.14)	87.5% (0.00)*	50.0% (0.48)
All	24	87.5% (0.00)*	75.0% (0.03)*	100.0% (0.00)*	75.0% (0.02)*
Selection constraints					
Free	42	61.9% (0.14)	45.2% (0.58)	85.7% (0.00)*	38.1% (0.81)
Positivity	42	76.2% (0.00)*	64.3% (0.17)	97.6% (0.00)*	47.6% (0.57)
Diversification	42	97.6% (0.00)*	90.5% (0.00)*	100.0% (0.00)*	83.3% (0.00)*
Variability	42	83.3% (0.00)*	76.2% (0.02)*	92.8% (0.00)*	71.4% (0.06)*
Estimation window					
Expanding 60	28	75.0% (0.00)*	71.4% (0.04)*	89.3% (0.00)*	67.8% (0.09)*
Expanding 80	28	71.4% (0.00)*	60.7% (0.07)*	89.3% (0.00)*	60.7% (0.18)
Expanding 100	28	75.0% (0.00)*	60.7% (0.16)	92.9% (0.00)*	53.6% (0.28)
Rolling 60	28	89.3% (0.00)*	67.8% (0.08)*	100.0% (0.00)*	50.0% (0.25)
Rolling 80	28	89.3% (0.00)*	82.1% (0.03)*	96.4% (0.00)*	67.9% (0.09)*
Rolling 100	28	78.6% (0.00)*	71.4% (0.06)*	96.4% (0.00)*	60.7% (0.18)

The 168 specifications are classified according to the three sets of exogenous specifications (predictive variables, selection constraints and estimation window). Four uninformed benchmarks are used. The two unconditional benchmarks are located at 50 and 75% top quantiles of the ranking based on the Sharpe ratios of all unconditional strategies. The two buy and hold benchmarks are the value-weighted (Market) and equally-weighted portfolios (Equal). Figures into parentheses represent the White's p-value under the null assumption that all conditional strategies have a zero Sharpe ratio differential. They are computed by bootstrapping the excess return time-series of all strategies 1'000 times. An asterisk denotes 10% significance.

Table 8

Percentage of positive Sharpe ratio differential for A equal to 5 and medium transaction costs

Grouping	N°	Uncond 50%	Uncond 75%	B&H Market	B&H Equal
Total					
	168	75.0% (0.00)*	69.6% (0.04)*	85.2% (0.00)*	38.1% (0.92)
Predictive variables					
Local	24	79.2% (0.00)*	75.0% (0.08)*	91.7% (0.00)*	33.3% (0.91)
Global	24	79.2% (0.00)*	66.7% (0.02)*	91.7% (0.00)*	41.7% (0.74)
Asset	24	50.0% (0.39)	45.8% (0.69)	62.5% (0.02)*	8.3% (1.00)
GassetLinterest	24	95.8% (0.00)*	91.6% (0.00)*	100.0% (0.00)*	66.6% (0.11)
LassetGinterest	24	79.2% (0.00)*	75.0% (0.04)*	79.2% (0.00)*	41.7% (0.79)
Interest	24	62.5% (0.00)*	58.3% (0.12)	79.2% (0.00)*	33.3% (0.98)
All	24	79.2% (0.00)*	75.0% (0.02)*	91.7% (0.00)*	41.7% (0.60)
Selection constraints					
Free	42	50.0% (0.42)	45.2% (0.72)	61.9% (0.08)*	14.3% (1.00)
Positivity	42	73.8% (0.00)*	61.9% (0.19)	90.5% (0.00)*	26.2% (1.00)
Diversification	42	97.6% (0.00)*	95.2% (0.00)*	100.0% (0.00)*	66.7% (0.01)*
Variability	42	78.6% (0.00)*	76.2% (0.00)*	88.1% (0.00)*	45.2% (0.58)
Estimation window					
Expanding 60	28	71.4% (0.00)*	71.4% (0.03)*	78.6% (0.00)*	50.0% (0.89)
Expanding 80	28	67.8% (0.00)*	64.2% (0.11)	82.2% (0.00)*	35.7% (0.97)
Expanding 100	28	67.8% (0.00)*	60.7% (0.16)	82.2% (0.00)*	32.2% (0.99)
Rolling 60	28	82.2% (0.00)*	64.2% (0.11)	89.3% (0.00)*	28.6% (0.94)
Rolling 80	28	85.7% (0.00)*	85.7% (0.03)*	89.3% (0.00)*	46.4% (0.77)
Rolling 100	28	75.0% (0.00)*	71.4% (0.06)*	89.3% (0.00)*	35.7% (0.90)

The 168 specifications are classified according to the three sets of exogenous specifications (predictive variables, selection constraints and estimation window). Four uninformed benchmarks are used. The two unconditional benchmarks are located at 50 and 75% top quantiles of the ranking based on the Sharpe ratios of all unconditional strategies. The two buy and hold benchmarks are the value-weighted (Market) and equally-weighted portfolios (Equal). Figures into parentheses represent the White's p-value under the null assumption that all conditional strategies have a zero Sharpe ratio differential. They are computed by bootstrapping the excess return time-series of all strategies 1'000 times. An asterisk denotes 10% significance.

Table 9

Composition of the 30 worst and best conditional models across the different specifications for different values of A and no transaction costs

Grouping	Risk aversion					
	2		5		10	
	Worst	Best	Worst	Best	Worst	Best
Predictive variables						
Local	3.3%	13.3%	6.6%	10.0%	13.3%	6.6%
Global	20.0%	10.0%	20.0%	13.3%	3.3%	13.3%
Asset	30.0%	10.0%	30.0%	0.0%	26.6%	0.0%
GassetLinterest	6.6%	20.0%	3.3%	30.0%	3.3%	30.0%
LassetGinterest	16.6%	3.3%	6.6%	13.3%	6.6%	20.0%
Interest	20.0%	20.0%	20.0%	16.6%	33.3%	13.3%
All	3.3%	26.0%	13.3%	16.6%	13.3%	16.6%
Selection constraints						
Free	70.0%	30.0%	56.6%	30.0%	50.0%	30.0%
Positivity	0.0%	6.6%	13.3%	0.0%	16.6%	3.3%
Diversification	0.0%	10.0%	0.0%	13.3%	6.6%	6.6%
Variability	30.0%	53.0%	30.0%	56.6%	26.6%	56.6%
Estimation window						
Expanding 60	16.6%	23.3%	30.0%	26.6%	23.3%	26.6%
Expanding 80	16.6%	16.6%	13.3%	16.6%	16.6%	20.0%
Expanding 100	16.6%	16.6%	13.3%	16.6%	20.0%	20.0%
Rolling 60	23.3%	13.3%	23.3%	10.0%	20.0%	6.6%
Rolling 80	13.3%	10.0%	13.3%	16.6%	6.6%	16.6%
Rolling 100	13.3%	20.0%	16.6%	13.3%	13.3%	10.0%

The 168 conditional strategies are ranked according to their final wealth and the 30 worst and best strategies are selected. For each of these two groups, we compute the percentage of strategies which use the different exogenous specifications across the three sets (predictive variables, selection constraints and portfolio selection).

Table 10

Descriptive statistics of 12 Datastream monthly excess returns

	Excess mean	Std deviation	Skewness	Kurtosis	Normality test (p-value)
Australia	4.7%	12.9%	-0.06	2.70	0.63
Belgium	3.7%	16.3%	-0.34	4.17	0.00*
Canada	5.5%	14.3%	-0.45	4.58	0.00*
Denmark	5.1%	18.3%	-0.23	2.95	0.44
France	5.0%	19.1%	-0.18	3.36	0.43
Germany	2.3%	19.6%	-0.44	3.61	0.02*
Italy	2.0%	23.9%	0.53	3.65	0.00*
Japan	-4.4%	20.7%	0.26	4.12	0.00*
Netherlands	6.0%	17.4%	-0.50	5.06	0.00*
Switzerland	7.9%	17.6%	-0.32	7.48	0.00*
UK	2.8%	15.0%	-0.15	3.70	0.14
USA	7.2%	14.6%	-0.34	3.34	0.13

Descriptive statistics of 12 Datastream country index monthly excess returns (in local currency) between January 1990 and September 2004 (176 datapoints). The excess mean and standard deviation are annualized. The last column shows the p-value under the null assumption of normality based on the Bera-Jarque test. An asterisk indicates 10% significance.

Table 11

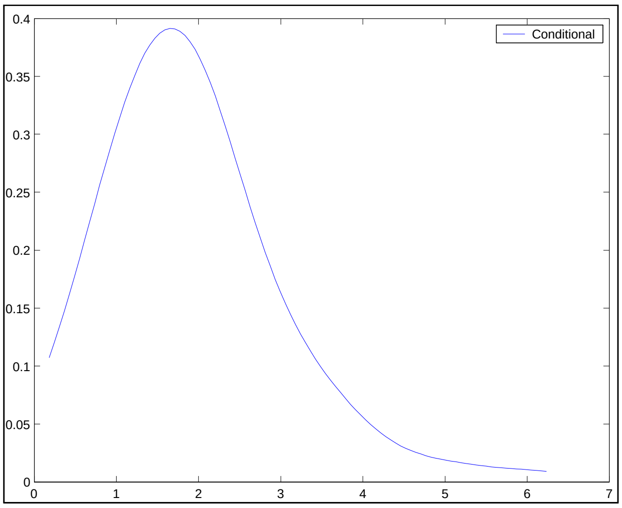
Correlation matrix of 12 Datastream monthly excess return

	Aus.	Bel.	Can.	Den.	Fra.	Ger.	Ita.	Jap.	Net.	Swi.	UK	USA
Australia	1.00											
Belgium	0.50	1.00										
Canada	0.58	0.50	1.00									
Denmark	0.55	0.67	0.58	1.00								
France	0.55	0.72	0.66	0.66	1.00							
Germany	0.57	0.69	0.64	0.73	0.85	1.00						
Italy	0.36	0.57	0.54	0.58	0.72	0.66	1.00					
Japan	0.40	0.27	0.39	0.25	0.37	0.31	0.25	1.00				
Netherlands	0.61	0.79	0.69	0.72	0.84	0.84	0.63	0.38	1.00			
Switzerland	0.57	0.76	0.66	0.66	0.74	0.76	0.58	0.41	0.83	1.00		
UK	0.65	0.69	0.66	0.65	0.76	0.70	0.57	0.36	0.80	0.78	1.00	
USA	0.56	0.57	0.74	0.56	0.65	0.63	0.46	0.37	0.69	0.65	0.72	1.00

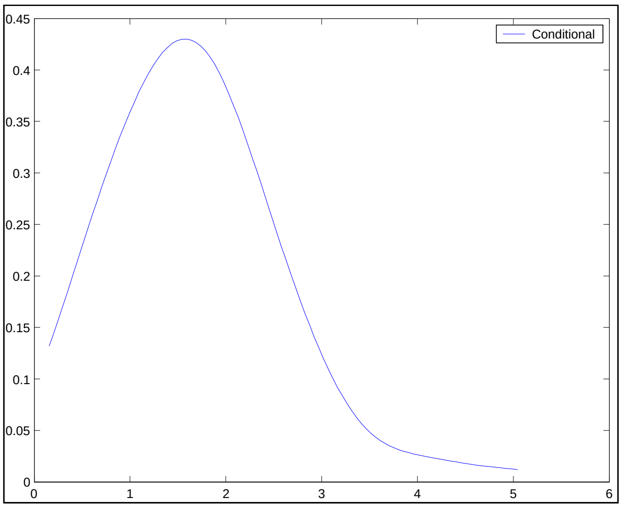
Correlation matrix of 12 national Datastream country index monthly excess returns (in local currency) between January 1990 and September 2004 (176 datapoints).

Figure 1

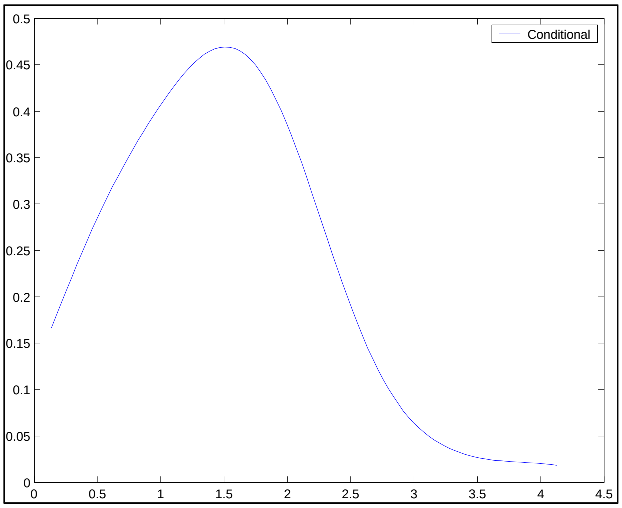
Distribution of the final wealth of conditional strategies for A equal to 5



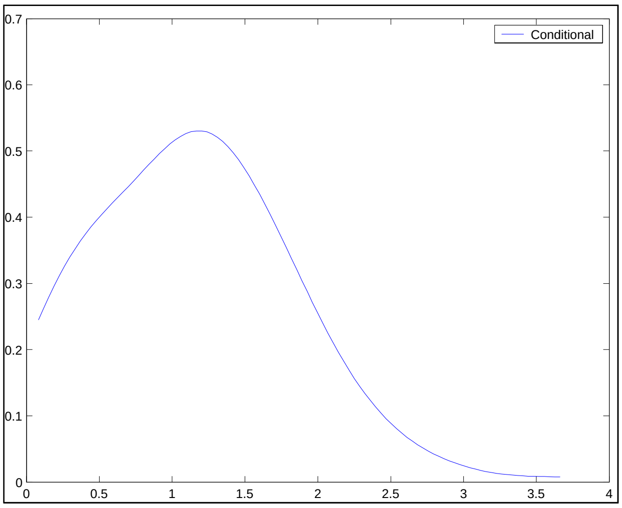
(a) No transaction costs



(b) Low transaction costs



(c) Medium transaction costs



(d) High transaction costs

Distribution of final wealth faced by an investor who randomly picks up one conditional strategy (out of the 168 possibilities) and invests one currency unit at the beginning of the period in January 1990. The distributions are computed using a standard Gaussian Kernel.