The Credit Risk Premium in a Disaster-Prone World

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Abstract

The seminal Barro (2006) closed-economy model of the equity risk premium in the presence of extreme events ("disasters") allowed for leverage in the form of risky corporate debt which defaulted only in states when the Government defaulted on its debt. The probability of default was therefore exogenous and independent of the degree of leverage. In this paper, we take the model a step closer reality by assuming that, on the one hand, the Government never defaults, and on the the other hand, that the "corporate sector" in the form of the Lucas tree owner pays its debts in full if and only if its asset value is sufficient, which is always the case in non-crisis states. Otherwise, in exceptionally severe crises, it defaults and hands over the whole "firm" to its creditors. The probability of default by the tree owner is thus endogenous, dependent both on the volume of debt issued (taken as exogenous) and on the uncertain value of output. We show, using data from both Barro (2006) and Barro and Ursua (2008), that the model can generate values of the riskless rate, equity risk premium and credit risk spread broadly consistent with those typically observed in the data.

JEL Classification: F3, G1

Keywords: equity risk premium, default risk, credit spread, leverage, corporate debt

1 Introduction

For practitioners and academics alike, the size of the credit risk spread is a long-standing puzzle, whose solution seems more urgent than ever against the background of the crisis in global financial markets which started in mid-2007. In this paper, we concur with the view taken in Bhamra et al (2007), Chen et al (2008) and Huang and Huang (2003) among others, that the issue needs to be addressed alongside the equity premium puzzle, and moreover in a setting which takes explicit account of the vulnerability of all economies to occasional extreme, invariably negative shocks. In this spirit, we follow the macrofinance approach pioneered by Rietz (1988) and extended in the seminal work by Barro (2006), whose model of the equity risk premium in the presence of extreme events has already generated a new literature on crises and their implications for asset pricing. Most of the papers modify the basic framework in one way or another in order to generalise the model and make its assumptions more realistic, with varying conclusions regarding the robustness of Barro (2006)'s claim to have resolved the famous equity premium puzzle.

Thus, Gourio (2007) examines the evidence on how quickly countries typically recover from catastrophic falls in the level of economic activity and finds that incorporating a recovery probability into the model makes it again impossible to explain the stylized facts. Starting from a different point of view, Copeland and Zhu (2007) show that if we allow for a second tree (a "foreign country"), then anything less than perfect correlation between the output of the two trees implies the existence of diversification opportunities, making it almost impossible to reconcile the observed 6% risk premium with the parameter values derived in Barro (2006) from a survey of twentieth century experience.

On the other hand, a number of developments on the empirical research front make the problem somewhat more tractable. First, Dimson et al (2006) suggest that, viewed from a global perspective, the equity premium is not quite as large as for the original US dataset of Mehra and Prescott (1985). Secondly, after substantially broadening the scope of their analysis of the historical data, Barro and Ursua (2008) conclude that the probability of a crisis is in fact double the figure given in Barro (2006), which would be likely to boost estimated risk premia for most calibrations, other things being equal.[1] On the theoretical front, Gabaix (2008) introduces a time-varying intensity of disasters into the model and claims to explain the risk premium along with a number of other stylized facts in the asset pricing literature.

The contribution made by this paper relates to the submodel of the debt market. Barro (2006) allows for leverage in the form of risky corporate debt which defaults only in states when the Government also defaults on its debt. The latter event occurs with exogenously given probability in some but not all crisis states. In this setting, the probability of default by the tree-owner is therefore exogenous and independent of the degree of leverage.

Here, we take the model a step closer to reality by assuming that, on the one hand, the Government never defaults,[2] and on the other hand, that the "corporate sector" in the form of the Lucas (1978) tree owner pays its debts in full if and only if its exogenous harvest is

[1] See also Brown, Goetzmann and Ross (1995), Jorion and Goetzmann (1999) who make essentially the same point in drawing attention to the survivorship bias implicit in ignoring the losses inflicted on investors by the wars and revolutions of the Twentieth Century, especially when these disasters resulted in stock market closure, expropriation or wipe-out.

[2] Government defaults on domestic currency unindexed debt are quite rare, at least if we exclude developing countries, though not apparently as rare as we thought prior to reading Reinhart and Rogoff (2008).

sufficient, which is always the case in non-crisis states. In some, but not all crisis states, however, output is inadequate to cover debt service, in which case it hands over the whole harvest to its creditors. The probability of default by the tree-owner is thus endogenous, dependent both on the volume of debt issued (taken as exogenous) and on the uncertain value of output. By deriving a closed-form solution for the critical value of output below which the firm is unable to repay its borrowing, we are able to find expressions for the riskless rate, equity risk premium and credit risk spread as well as the (endogenous) probability of default. Calibrations with parameter values based on data from both Barro (2006) and Barro and Ursua (2008), show that the model specified in this fashion can generate results consistent with the broad outline of the facts, even with reasonably low levels of risk aversion.

In the next section, we give a brief introduction to the original Barro (2006) crisis model. We then proceed to set out the submodel of corporate debt which is to be embedded in the crisis model (Section 3). Solving the model for the rates of return on the three types of security (equity, private sector debt and Government debt) in Section 4, we are able to derive expressions for the credit risk spread and equity premium (Section 5). The results of our calibration are discussed in Section 6.

2 The Barro (2006) Setting

The broad framework of our analysis follows Barro (2006), insofar as we start from an economy populated by a representative agent with time-additive utility and an initial endowed income in the form of the fruit of a Lucas-tree, with equity claims on the time $t+1$ stochastic endowment ("dividends") traded at time t. Specifically, agents maximise a power utility function[3] of the standard form:

$$E_t(U) = U(C_t) + \sum_{s=1}^{\infty} e^{-\rho s} E_t\left[U(C_{t+s})\right] \tag{1}$$

where:

$$U(C_{t+s}) = \frac{C_{t+s}^{1-\theta}}{1-\theta} \tag{2}$$

while the (log of the) endowment process, A_t, is a random walk with drift γ, and subject to two types of disturbance at any time, $t+1$:

$$g_{t+1} = \ln A_{t+1} - \ln A_t = \gamma + u_{t+1} + v_{t+1} \tag{3}$$

The first disturbance, u_{t+1}, is normally distributed with zero mean and constant variance, σ^2. The crucial component is the nonnormal shock v_{t+1} which takes the value zero (i.e no crisis) with probability e^{-p} and the value $\ln(1-b)$ with probability $(1-e^{-p})$, where p is approximately the probability of a crisis (a "disaster"), and $0 < b < 1$ is a random variable representing the proportion by which output falls in a disaster scenario. The distribution of b is approximated empirically by the frequency found in Barro's research on the economic history of the past century. Note that we assume u_{t+1} and v_{t+1} are independent identically-distributed shocks.

[3] Barro and Ursua (2008), Gourio (2007), Chen et al (2008) use the Epstein-Zinn utility function, which has the advantage of incorporating separate parameters for the marginal rate of intertemporal substitution and coefficient of risk aversion, but has the disadvantage of making it even harder to arrive at closed-form solutions. It could also be argued that using a nonseparable utility function, as we do here, makes the task of matching the observed data more of a challenge.

3 Debt Markets

We deviate from Barro (2006) in our specification of the market for the two types of fixed-income instrument traded in the model. First, we assume the Government issues a bill or bond in the form of a claim paying a fixed return, R. Government borrowing is completely riskless, in the sense that in this model the Government never defaults in any state of the world. This specification is in contrast to Barro (2006), who assumes that in extreme crisis states, the Government and private sector both default, an event which occurs with an exogenously given probability conditional on an output disaster.

Unlike Government debt, the privately-issued fixed interest security is subject to default risk. The firm which owns the tree issues a one-period security[4] promising a face return, fixed in advance at time t for payment at $t + 1$. In noncrisis situations, output is always adequate to cover the cost of paying the face return, R^F, leaving the residue as a dividend to the equityholders, but in the event of a severe crisis, it may fall short, so that the firm is forced to default on its debt. In this extreme scenario, the bondholder receives the whole output, and the equity gets nothing. However, in the aftermath of a default, the firm is reconstituted in the next period with the same degree of leverage, an assumption required in order to preserve the IID property of cashflows.

These changes to the model have far-reaching implications. First, instead of being an event occuring with an exogenously fixed probability, default happens whenever output falls below the endogenously-determined critical level, b^*, for which we solve below. For the

[4] For the sake of consistency, we can think of the equityholder issuing the debt in order to finance the purchase of shares. In a sense, therefore, the tree is not part of the initial endowment, but needs to be purchased at the outset with the proceeds of the bond sale, as in Barro (2006).

moment, note that b^* is determined by a number of factors, most importantly the leverage ratio since, other things being equal, the more debt the tree-owner issues, the greater the burden of coupon payments and therefore the smaller the output contraction sufficient to cause a default. Associated with this critical level of b is a probability which, for convenience in writing out the model equations, we denote π:

$$\pi = \Pr(b > b^*)$$

Note that, in spite of the shorthand, π is not an exogenous constant, but a function of a number of the model parameters, particularly the leverage ratio.

4 The Model Solution

As a first step to solving the model, we compute the stochastic discount factor (SDF) in each of the three states of the world with which we are concerned here. In the no-disaster scenario, when output is subject only to normally-distributed shocks, the SDF is derived straightforwardly from the first-order conditions as:

$$M_{t+1} = e^{-\rho-\gamma\theta-u_{t+1}\theta} \tag{4}$$

Now consider the two disaster states. In the first, output falls disastrously, but not enough to cause a default i.e. $0 < b \leq b^*$. In the second, the negative shock is so great that output falls below the critical value at which the firm goes into default. However, since the SDF depends on the level of output and hence consumption, and not on its distribution between equity and bondholders, it is the same function of b whether the outcome is above or below the critical value. The crisis SDF is therefore simply:

$$(1-b)^{-\theta} M_{t+1} \tag{5}$$

Note that since $(1-b)$ is the level of output in the crisis state as a proportion of its noncrisis level, and it is usually assumed that $\theta >> 1$, it follows that the discount factor in these states is a multiple of its normal size, a factor which will be critical in explaining why the model generates substantial risk premia, since it implies that consumption is valued most highly when it is lowest. It follows that assets which either deliver reduced payoffs (like corporate debt) or possibly no payoff at all (equity) in disaster scenarios are valued far lower, other things being equal, than those which give a return in all possible states (Government debt).

As far as the payoff on the corporate bill is concerned, in the two nondefault states it pays the face return originally promised, R^F, whereas when $b > b^*$, the situation is more complicated. In this case, bondholders receive the total output, so that the relationship between the face return and b^* depends on the degree of leverage:

$$R^F = \Lambda_t e^{\gamma + u_{t+1}} (1 - b^*) \tag{6}$$

where:

$$\Lambda_t \equiv \frac{P_t}{\delta_t}$$

is the asset-to-debt ratio at t.[5] and δ_t is the face value of the debt issued at t. However, in terms of time $t+1$ variables, this amounts to A_{t+1}/δ_t, which can be written, using (3),

[5] Over a short period, this is approximately $\frac{A_t}{\delta_t}$ (Barro (2006)). For the sake of simplicity, we write the equations in terms of the asset-to-debt ratio rather than the leverage ratio, which is just $1/(\Lambda_t - 1)$.

in the following form:

$$\frac{A_{t+1}}{\delta_t} = \Lambda_t e^{\gamma + u_{t+1}} (1-b) \tag{7}$$

Given the value of the SDF in each of the three relevant states, and the payoff on the debt in each state, we can compute the expected return as:

$$\ln ER^B \approx (1-p\pi)\ln R^F + p\pi \left[\Lambda_t e^{\gamma+\frac{1}{2}\sigma^2} \cdot E(1-b \mid b > b^*) - 1\right] \tag{8}$$

This says that the (log of) the expected return offered by this security is a weighted average of two components, the face return (i.e. assuming no default), R^F, and the return in the default scenario, the latter being the bondholder's claim on the normal growth in output, $\Lambda_t e^{\gamma+\frac{1}{2}\sigma^2}$, scaled down by the proportionate fall in output that brings about the default. Note that the impact of the asset-to-debt ratio, Λ_t, is complicated. In the first place, a higher value of Λ_t (lower leverage) makes the return greater in the event of default, and also raises the critical size of the contraction needed to trigger a default, b^*, and hence reduces the default probability, π. At the same time, the greater security associated with lower leverage (higher Λ_t) lowers the equilibrium nominal return, other things being equal.[6]

The return on default-free government debt, which is the riskless rate in this model, is simply:[7]

$$\ln R = \rho + \theta\gamma - \frac{1}{2}\theta^2\sigma^2 + p\left[1 - E(1-b)^{-\theta}\right] \tag{9}$$

[6] Explicitly, the equation for the face return is:

$$\begin{aligned} \ln R^F &= \rho + \theta\gamma - \frac{1}{2}\theta^2\sigma^2 \\ &\quad + p\left\{1 - (1-\pi)E\left[(1-b)^{-\theta} \mid b \le b^*\right] - \pi\Lambda_t e^{-\rho+(1-\theta)\gamma+\frac{1}{2}(1-\theta)^2\sigma^2} E\left[(1-b)^{1-\theta} \mid b > b^*\right]\right\} \end{aligned}$$

[7] as in Barro (2006), in the special case when the default probability (denoted q in his model) is zero.

This says that the riskfree rate is the sum of the rate of time preference, ρ, and the marginal value of output (and consumption), $\theta\gamma$, less components relating to risk in the normal and abnormal states. The first is the familiar convexity adjustment, while the second is the expected value of incremental consumption in the disaster state. Note that, since $0 < b < 1$, the term in square brackets is negative as long as $\theta > 1$, which we take to be the case. In other words, we assume that in the crisis state, the intertemporal substitution (insurance) effect dominates the income effect, as in Barro (2006).

5 The Credit Spread and Equity Risk Premium

In our model, the credit spread is equivalent to the difference between the face return and the risk-free rate i.e. the gap between the rates promised by corporate and government riskless debt. Comparing (8) and (9), we get:

$$\begin{aligned}\ln R^F - \ln R &= \\ & p(1-\pi)\left\{E(1-b)^{-\theta} - E\left[(1-b)^{-\theta} \mid b \leq b^*\right]\right\} \\ & +\pi p\left\{E(1-b)^{-\theta} - \Lambda_t e^{-\rho+(1-\theta)\gamma+\frac{1}{2}(1-\theta)^2\sigma^2} \cdot E\left[(1-b)^{1-\theta} \mid b > b^*\right]\right\} \end{aligned} \tag{10}$$

which can be viewed as the sum of the required compensation for the expected loss in default and the premium associated with this type of systematic risk. As Elton et al (2001) emphasise, the latter component will be present as long as investors are risk-averse and, in fact, in the light of the evidence they and others have produced, actually accounts for much of the spread.

To understand (10), note that when the probability of a crisis, p, is zero, there is no gap between the returns on Government and private sector debt, since default can only occur in crises. Another limiting case is the "superprime borrower" for whom b^* is near 100% i.e. the borrower who would only be forced into default if income fell almost to zero. In this case, the conditional and unconditional expectations in the first curly bracket are almost the same thing, so that this component is very small. Likewise, the second term involves $p\pi$, which will be extremely small, (the probability of a crisis multiplied by the probability of default, which is also tiny in this case). In sum, the credit spread will be small, as intuition would lead us to expect, for a good quality borrower.

From (10), it is clear that leverage will increase the credit spread, other things being equal. To see that this is the case, note that the final term in square brackets will be substantially greater than unity if $\theta > 1$, because the mean of the distribution of $1 - b$ in default scenarios will be small. Moreover, this effect will be reinforced because higher leverage (lower Λ_t) makes the firm more vulnerable to a downturn. In other words, it reduces b^* and raises $(1 - b^*)$, making $E(1 - b^*)^{1-\theta}$ even smaller, and further widening the credit spread.

As far as equity is concerned, leverage means that the payoff to shareholders is simply the residue of output after the bondholders have received a sum no greater than the face value of the debt plus the accrued interest. In the event of default, the return on equity is, of course, zero. Hence the return on the levered equity is just:

$$R^L = \frac{A_{t+1} - \beta_t R^B}{P_t - \beta_t} \approx \frac{A_{t+1} - \beta_t R^B}{A_t - \beta_t} \tag{11}$$

which is related to the unlevered return by the familiar weighted average cost of capital formula:

$$R^E = \frac{\Lambda_t - 1}{\Lambda_t} R^L + \frac{1}{\Lambda_t} R^B \tag{12}$$

where R^E is the return on the underlying asset (i.e. on the unlevered equity), given by:

$$\ln R^E = \rho + \theta\gamma - \frac{1}{2}\theta^2\sigma^2 + \theta\sigma^2 - p\left[E(1-b)^{1-\theta} - E(1-b)\right] \qquad (13)$$

and $R^L$and R^B are the returns on the levered equity and on the firm's bonds respectively. The equity risk premium can then be derived from (11) and (9). Although our main concern in this paper is with the pricing of debt, we also generate results for equities as an additional check on the plausibility of our results, given that the data are more plentiful and more accurate than they are for bond markets, a point emphasised by Huang and Huang (2003).

We now turn to the question of how far it is possible to explain the stylised facts within the framework of the model set out here, using parameter values taken from Barro (2006) and the more extensive data collected in Barro and Ursua (2008).

6 Calibration

The key element in calibrating the model is the annual crisis probability, p, which was estimated as 1.7% in Barro (2006) and 3.7% in Barro and Ursua (2008), and the frequency distribution of the output contraction, b. In view of the problematic nature of historic data, and the fact that the value of p is critically dependent on the definition of a crisis, we calibrate for a number of different values in the range indicated by the facts for the last century. It must be emphasised however that, while the equity risk premium puzzle as originally posed by Mehra and Prescott (1985) was originally based on the experience of the twentieth century, the credit risk spread has largely been considered in the context of datasets which were shorter and hence possibly less representative. In particular, in comparing our calibration results with current levels of the credit spread, we need to bear in mind that the market may be anticipating a very different crisis frequency in the future than in the past.

In view of its importance, however, we should make clear the channels through which it impacts on the credit spread. On the one hand, the higher is the probability of a crisis, p, the wider must be the credit spread, for the obvious reason that, in a more uncertain world, risk-free assets must be more and risky assets less valuable, other things being equal. On the other hand, a greater value for the riskless security and lower R reduces the cost of debt service and thereby raises b^*, the critical size of output contraction needed to trigger a default. This latter mechanism, taken on its own, reduces the conditional probability of default, π. The credit spread is determined by the unconditional probability of default, which is the product of p and π. Higher p raises this probability directly, but indirectly (via a fall in π) reduces it. The net effect is therefore complex, but as will be seen from our results, invariably positive in practice.

In Tables 1 and 2, we take as given the estimates of the rate of time preference, ρ, the trend growth rate, γ, and the standard deviation of normal shocks, σ, and show how the results are affected by changes in the crisis probability for different values of the leverage ratio, and θ, the elasticity of intertemporal substitution. Note that, as far as the last parameter is concerned, we only take the values 3.0, 3.5 and 4.0, all well within the range usually considered reasonable.

It can be seen that, for both parameter sets, the model generates plausible values of the riskless rate (in the zero to 3% range), for the return on leveraged equity (the 7% to 11% range) and for the equity risk premium (5% to 10%).

As far as the credit spread is concerned, note that the model generates values that are as high as those actually observed, and even greater in some cases. For example, taking the

3.7% probability from Barro and Ursua (2008), we get spreads of 36, 73 or 147 points for the three values of the risk aversion parameter, $\theta = 3.0$, 3.5 and 4.0 respectively, with a leverage ratio of only 50%. At higher levels of leverage, the credit spread widens dramatically, to 99, 184 and 337 points at 75%, for example.

Note that the reason why this model yields such a large credit spread is to be found in the correlation it generates between the discount factor and the loss in default, so that corporate debt inflicts heavy losses on investors in precisely those states where consumption is most valuable. By contrast, Government debt is riskless and therefore benefits from the "flight to quality", so that its equilibrium return is reduced by the possibility of a severe contraction in output.

At the same time, our results confirm the consensus view (e.g. Huang and Huang (2003), Elton et al (2001), Bhamra et al (2007)) that the credit spread overwhelmingly reflects the default risk premium rather than the expected loss. In fact, the cost of default makes a negligible contribution for almost all values of the parameters. One possible explanation for this result is that the recovery rates we generate look high relative to observed levels. There are two reasons for this. First, we make no allowance here for the substantial costs lenders have in reality to pay for recovering their loans. Secondly, as Huang and Huang (2003) explicitly recognise in their continuous time model, in practice lenders are usually unable or unwilling to call in loans as soon as the value of the entreprise falls to the face value of the debt (in fact, Huang and Huang (2003) explicitly assume foreclosure at the 60% boundary). In the context of our model, however, lenders have no reason to delay foreclosure, so they capture a higher proportion of the assets. This in turn makes the expected loss in default small.

7 Conclusions

We have shown that a generalised version of the Barro (2006) crisis model to allow for defaultable corporate debt can account for the broad facts about the credit spread for a range of plausible values of the parameters and for two alternative estimates of the frequency distributions of the large output contractions experienced in the last century. However, like most other authors, we find that very little of the credit spread appears to be explained by expected default loss; it appears to be almost all risk premium. It is quite possible that this result is due to sampling error of the peso-problem type. In other words, disasters - which by definition are rare events - may either be more likely than is apparent from the datasets on which we base our parameterisations, or may simply be believed by the market to be more likely. Future research will need to be directed to this issue.

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TABLE 1: CALIBRATIONS WITH BARRO (2006) PARAMETERS (ρ = 3% γ = 2.5% σ = 2%)

range found in historic data […,...] **Highlighted data in normal (bold italic) font match histroric data for single target range (all four target ranges)**

		p = 0.015			p = 0.017			p = 0.020			p = 0.025		
			ϴ			ϴ			ϴ			ϴ	
	leverage	3	3.5	4	3	3.5	4	3	3.5	4	3	3.5	4
risk-free													
	0.5	5.32%	3.91%	1.06%	4.66%	2.89%	-0.49%	3.66%	1.37%	-2.81%	1.99%	-1.16%	-6.69%
[-1%,3%]	0.75	5.32%	3.91%	***1.06%***	4.66%	***2.89%***	-0.49%	3.66%	***1.37%***	-2.81%	***1.99%***	-1.16%	-6.69%
	1	5.32%	3.91%	1.06%	4.66%	***2.89%***	-0.49%	3.66%	***1.37%***	-2.81%	***1.99%***	-1.16%	-6.69%
	1.5	5.32%	3.91%	1.06%	4.66%	***2.89%***	-0.49%	3.66%	***1.37%***	-2.81%	***1.99%***	-1.16%	-6.69%
leveraged-equity		3	3.5	4	3	3.5	4	3	3.5	4	3	3.5	4
return	0.5	9.13%	9.86%	10.42%	8.95%	9.62%	10.09%	8.68%	9.25%	9.80%	8.22%	8.63%	9.02%
[7%,11%]	0.75	9.41%	10.23%	***10.87%***	9.26%	***10.02%***	10.60%	9.04%	***9.72%***	10.18%	***8.66%***	9.20%	9.78%
	1	9.57%	10.41%	11.10%	9.44%	***10.23%***	10.85%	9.24%	***9.97%***	10.46%	***8.92%***	9.51%	10.12%
	1.5	9.81%	10.68%	11.35%	9.70%	***10.53%***	11.12%	9.54%	***10.29%***	10.77%	***9.26%***	9.88%	10.17%
leveraged -risk premiu		3	3.5	4	3	3.5	4	3	3.5	4	3	3.5	4
	0.5	3.81%	5.96%	9.36%	4.29%	6.72%	10.58%	5.02%	7.87%	12.62%	6.23%	9.79%	15.71%
[5%,10%]	0.75	4.09%	6.32%	***9.81%***	4.61%	***7.13%***	11.08%	5.38%	***8.34%***	12.99%	***6.67%***	10.36%	16.46%
	1	4.25%	6.51%	10.04%	4.78%	***7.34%***	11.34%	5.58%	***8.60%***	13.28%	***6.93%***	10.68%	16.80%
	1.5	4.49%	6.78%	10.29%	5.05%	***7.63%***	11.61%	5.88%	***8.91%***	13.58%	***7.27%***	11.04%	16.86%
default-level		3	3.5	4	3	3.5	4	3	3.5	4	3	3.5	4
output fall b*	0.5	64.00%	64.00%	64.00%	64.00%	64.00%	64.00%	64.00%	64.00%	68.00%	64.00%	64.00%	68.00%
	0.75	52.00%	52.00%	52.00%	52.00%	52.00%	52.00%	52.00%	52.00%	52.00%	52.00%	52.00%	59.00%
	1	45.00%	45.00%	49.00%	45.00%	45.00%	49.00%	45.00%	49.00%	49.00%	49.00%	49.00%	52.00%
	1.5	36.00%	36.00%	36.00%	36.00%	36.00%	36.00%	36.00%	36.00%	36.00%	36.00%	36.00%	36.00%
credit spread		3	3.5	4	3	3.5	4	3	3.5	4	3	3.5	4
	0.5	0.09%	0.17%	0.32%	0.10%	0.19%	0.36%	0.12%	0.22%	0.00%	0.14%	0.28%	0.00%
[0.5%, 4.7%]	0.75	0.56%	0.99%	***1.74%***	0.64%	***1.12%***	1.98%	0.75%	***1.32%***	2.32%	***0.93%***	1.65%	2.50%
	1	0.93%	1.59%	2.69%	1.05%	***1.80%***	3.05%	1.24%	***2.09%***	3.59%	***1.53%***	2.62%	4.19%
	1.5	1.37%	2.27%	3.79%	1.55%	***2.57%***	4.30%	1.82%	***3.03%***	5.06%	***2.28%***	3.79%	6.32%
expected default loss		3	3.5	4	3	3.5	4	3	3.5	4	3	3.5	4
	0.5	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	0.75	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.03%	0.02%	0.02%	0.03%	0.02%	0.02%
	1	0.04%	0.04%	0.04%	0.05%	0.04%	0.04%	0.05%	0.05%	0.04%	0.06%	0.06%	0.04%

1.5	0.08%	0.08%	0.08%	0.09%	0.09%	0.08%	0.10%	0.10%	0.09%	0.12%	0.12%	0.10%
default premium	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**
0.5	0.08%	0.17%	0.32%	0.10%	0.19%	0.36%	0.11%	0.22%	0.00%	0.14%	0.28%	0.00%
0.75	0.54%	0.97%	1.73%	0.61%	1.10%	1.96%	0.72%	1.30%	2.31%	0.91%	1.63%	2.48%
1	0.89%	1.55%	2.65%	1.00%	1.76%	3.01%	1.18%	2.04%	3.54%	1.47%	2.56%	4.14%
1.5	1.29%	2.19%	3.72%	1.46%	2.49%	4.22%	1.72%	2.93%	4.96%	2.15%	3.67%	6.22%
face-return	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**
0.5	5.41%	4.07%	1.38%	4.75%	3.08%	-0.13%	3.77%	1.59%	-2.81%	2.13%	-0.88%	-6.69%
0.75	5.88%	4.90%	2.80%	5.29%	4.01%	1.49%	4.40%	2.69%	-0.49%	2.93%	0.49%	-4.19%
1	6.25%	5.49%	3.75%	5.71%	4.69%	2.56%	4.89%	3.47%	0.77%	3.52%	1.46%	-2.50%
1.5	6.69%	6.18%	4.85%	6.20%	5.47%	3.81%	5.48%	4.40%	2.24%	4.27%	2.63%	-0.36%
expected bond return	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**
0.5	5.41%	4.07%	1.38%	4.75%	3.08%	-0.13%	3.77%	1.59%	-2.81%	2.13%	-0.88%	-6.69%
0.75	5.86%	4.88%	2.79%	5.27%	3.99%	1.47%	4.38%	2.67%	-0.51%	2.90%	0.46%	-4.20%
1	6.21%	5.45%	3.71%	5.66%	4.65%	2.52%	4.84%	3.42%	0.73%	3.46%	1.40%	-2.54%
1.5	6.61%	6.10%	4.78%	6.11%	5.38%	3.73%	5.37%	4.30%	2.15%	4.14%	2.51%	-0.47%
recovery rate	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**
0.5	91.07%	92.24%	94.69%	91.64%	93.13%	96.12%	92.51%	94.49%	100.00%	93.99%	96.86%	100.00%
0.75	81.98%	82.75%	84.43%	82.44%	83.45%	85.53%	83.14%	84.52%	87.23%	84.33%	86.38%	84.42%
1	79.89%	80.47%	79.76%	80.30%	81.08%	80.69%	80.93%	79.98%	82.12%	79.94%	81.56%	76.31%
1.5	68.27%	68.60%	69.46%	68.58%	69.06%	70.16%	69.05%	69.76%	71.23%	69.85%	70.97%	73.10%
conditional default	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**
probability 0.5	1.67%	1.67%	1.67%	1.67%	1.67%	1.67%	1.67%	1.67%	0.00%	1.67%	1.67%	0.00%
0.75	8.33%	8.33%	8.33%	8.33%	8.33%	8.33%	8.33%	8.33%	8.33%	8.33%	8.33%	5.00%
1	15.00%	15.00%	13.33%	15.00%	15.00%	13.33%	15.00%	13.33%	13.33%	13.33%	13.33%	8.33%
1.5	15.00%	15.00%	13.33%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%

TABLE 2: CALIBRATIONS WITH BARRO and URSUA (2008) PARAMETERS (ρ = 3% γ = 2.5% σ = 2%)

range found in historic data […,...]   **Highlighted data in normal (bold italic) font match histroric data for single target range (all four target ranges)**

		p = 0.027			p = 0.037			p = 0.047		
		θ			θ			θ		
	leverage	**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**
risk-free										
	0.5	4.87%	3.17%	-0.32%	2.85%	***0.08%***	-5.14%	***0.82%***	-3.01%	-9.95%
[-1%,3%]	**0.75**	4.87%	3.17%	-0.32%	***2.85%***	***0.08%***	-5.14%	***0.82%***	-3.01%	-9.95%
	1	4.87%	3.17%	-0.32%	***2.85%***	***0.08%***	-5.14%	***0.82%***	-3.01%	-9.95%
	1.5	4.87%	3.17%	-0.32%	2.85%	0.08%	-5.14%	***0.82%***	-3.01%	-9.95%
leveraged-equity return		**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**
	0.5	8.68%	9.37%	9.86%	8.04%	***8.50%***	8.73%	***7.38%***	7.63%	7.59%
[7%,11%]	**0.75**	8.97%	9.70%	10.28%	***8.39%***	***8.97%***	9.37%	***7.83%***	8.21%	8.40%
	1	9.17%	9.93%	10.51%	***8.65%***	***9.24%***	9.63%	***8.12%***	8.58%	8.80%
	1.5	9.42%	10.21%	10.78%	8.97%	9.59%	9.92%	***8.51%***	8.97%	9.19%
leveraged -risk premium		**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**
	0.5	3.81%	6.20%	10.18%	5.20%	***8.42%***	13.86%	***6.55%***	10.64%	17.54%
[5%,10%]	**0.75**	4.10%	6.53%	10.61%	***5.55%***	***8.89%***	14.51%	***7.01%***	11.22%	18.35%
	1	4.30%	6.76%	10.83%	***5.80%***	***9.16%***	14.76%	***7.30%***	11.59%	18.75%
	1.5	4.55%	7.04%	11.11%	6.12%	9.51%	15.05%	***7.69%***	11.97%	19.14%
default-level output fall b*		**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**
	0.5	58.70%	66.20%	66.20%	66.20%	66.20%	66.20%	66.20%	66.20%	66.20%
	0.75	54.50%	54.50%	57.20%	54.50%	57.20%	58.70%	57.20%	57.20%	58.70%
	1	48.00%	48.00%	48.00%	48.00%	48.00%	50.30%	48.00%	50.30%	52.50%
	1.5	36.70%	38.10%	38.10%	38.10%	38.10%	38.10%	38.10%	38.10%	41.90%
credit spread		**3**	**3.5**	**4**	**3**	**3.5**	**4**	**3**	**3.5**	**4**
	0.5	0.32%	0.53%	1.07%	0.36%	***0.73%***	1.47%	***0.46%***	0.93%	1.87%
[0.5%, 4.7%]	**0.75**	0.73%	1.37%	2.54%	***0.99%***	***1.84%***	3.37%	***1.23%***	2.33%	4.28%
	1	1.03%	1.89%	3.48%	***1.41%***	***2.59%***	4.73%	***1.80%***	3.26%	5.92%
	1.5	1.45%	2.55%	4.56%	1.98%	3.50%	6.25%	***2.51%***	4.45%	7.85%

expected loss in default	3	3.5	4	3	3.5	4	3	3.5	4
0.5	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.01%	0.01%	0.00%
0.75	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%	0.03%	0.02%	0.02%
1	0.03%	0.03%	0.03%	0.04%	0.04%	0.04%	0.05%	0.05%	0.04%
1.5	0.07%	0.07%	0.06%	0.09%	0.08%	0.07%	0.10%	0.10%	0.08%

default premium	3	3.5	4	3	3.5	4	3	3.5	4
0.5	0.32%	0.53%	1.07%	0.36%	0.73%	1.47%	0.46%	0.93%	1.86%
0.75	0.71%	1.35%	2.53%	0.97%	1.82%	3.35%	1.21%	2.31%	4.27%
1	1.00%	1.86%	3.45%	1.37%	2.55%	4.70%	1.74%	3.21%	5.88%
1.5	1.38%	2.49%	4.50%	1.89%	3.42%	6.18%	2.41%	4.35%	7.76%

face-return	3	3.5	4	3	3.5	4	3	3.5	4
0.5	5.19%	3.70%	0.75%	3.21%	0.81%	-3.67%	1.29%	-2.08%	-8.08%
0.75	5.59%	4.54%	2.22%	3.84%	1.92%	-1.76%	2.06%	-0.67%	-5.67%
1	5.90%	5.06%	3.16%	4.26%	2.67%	-0.41%	2.62%	0.25%	-4.03%
1.5	6.31%	5.72%	4.24%	4.82%	3.58%	1.12%	3.34%	1.44%	-2.11%

expected bond return	3	3.5	4	3	3.5	4	3	3.5	4
0.5	5.18%	3.70%	0.75%	3.20%	0.81%	-3.67%	1.28%	-2.08%	-8.09%
0.75	5.57%	4.52%	2.21%	3.82%	1.90%	-1.78%	2.03%	-0.70%	-5.69%
1	5.86%	5.03%	3.13%	4.21%	2.63%	-0.44%	2.57%	0.20%	-4.07%
1.5	6.25%	5.66%	4.18%	4.74%	3.50%	1.04%	3.23%	1.34%	-2.19%

recovery rate	3	3.5	4	3	3.5	4	3	3.5	4
0.5	89.55%	76.37%	78.61%	76.74%	78.56%	82.21%	78.19%	80.88%	86.17%
0.75	78.80%	79.59%	77.32%	80.13%	77.56%	74.58%	77.45%	79.58%	77.67%
1	75.78%	76.39%	77.79%	76.97%	78.16%	77.83%	78.20%	77.32%	77.73%
1.5	74.97%	73.80%	74.85%	74.43%	75.33%	77.16%	75.50%	76.92%	74.29%

conditional default probability	3	3.5	4	3	3.5	4	3	3.5	4
0.5	2.03%	0.68%	0.68%	0.68%	0.68%	0.68%	0.68%	0.68%	0.68%
0.75	3.38%	3.38%	2.70%	3.38%	2.70%	2.03%	2.70%	2.70%	2.03%
1	5.41%	5.41%	5.41%	5.41%	5.41%	4.73%	5.41%	4.73%	4.05%
1.5	10.14%	9.46%	9.46%	9.46%	9.46%	9.46%	9.46%	9.46%	7.43%