TACTICAL OPTIMIZATION, HOW TO FORECAST RISKS ?

Jean-François BOULIER*, Pierre SEQUIER** and Grégory TAILLARD***

* SINOPIA Asset Management
Chief Investment Officer
66, rue de la Chaussée d'Antin, 75009 Paris, France
e-mail: jfboulier@sinopia.fr

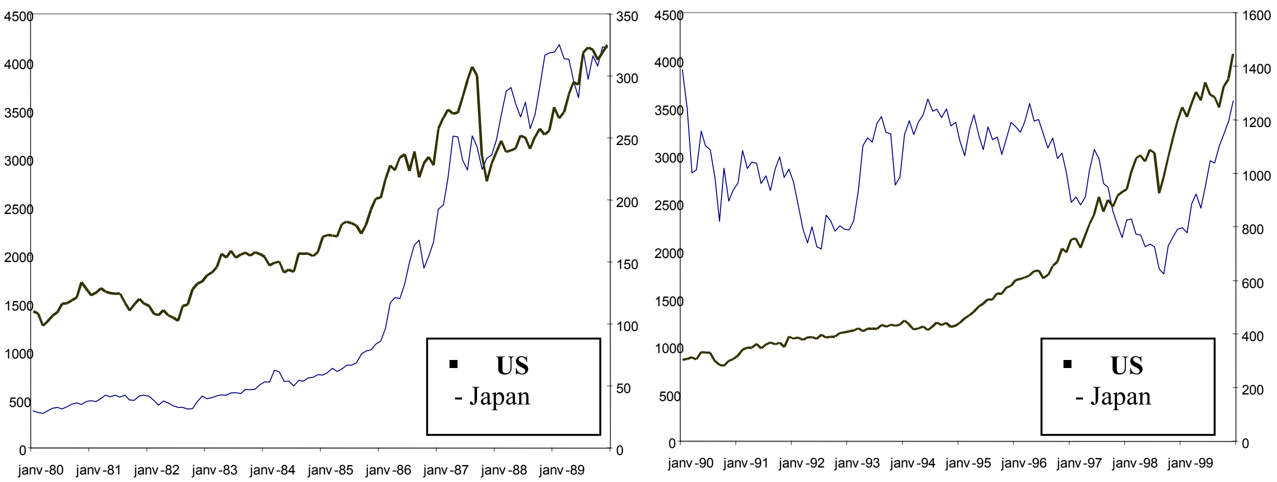
** SINOPIA Asset Management
Head of Research
66, rue de la Chaussée d'Antin, 75009 Paris, France
e-mail: psequier@sinopia.fr

*** SINOPIA Asset Management
Researcher
66, rue de la Chaussée d'Antin, 75009 Paris, France
e-mail: gtaillard@sinopia.fr

Abstract

This paper deals with one of the most classical but critical issue of the asset management activity: how to efficiently allocate the assets in a portfolio according to their expected returns? A part of the answer comes from the capacity of forecasting the volatilities and correlation of the assets. As the results of an optimization program are very sensitive to these parameters, the estimation process must be led carefully. This article shows that historical covariance matrices actually contain much noise and must then be retreated with statistical tools in order to have a more robust risk model. Then, the optimization step is performed and gives a more intuitive and reliable allocation for the manager.

A European investor willing to diversify her equity exposure to the global developed markets at the beginning of the 90's would have primarily invested the international part of her portfolio in US and Japanese stocks. A much more difficult question to address is : in what proportion? Optimization can provide an answer if expected returns and expected risks are available. Among those parameters, it is well known that expected returns have a dramatic impact on optimal portfolio allocations. It is therefore natural for investors to devote a lot of time and energy to forecast returns. Typically, no investment professional would take the last 10 year realized performance as a forecast for an asset return. Surprisingly, when it comes to the risks, historical volatilities and correlations are very often used as an estimate of the future variance-covariance matrix. Figure 1 presenting US and Japanese stock market performances in the eighties and the nineties illustrates how misleading past correlations can be as a forecast. An optimization performed on the basis of historical estimates at the beginning of 1990 would have rely on a significantly positive US-Japan correlation whereas the ex-post number was on average much lower. There is no doubt that history can provide some guidance in the selection of the risk parameters but although statistical theory says that historical covariance matrices are the best estimator of an independent identically distributed process, assessing the future volatilities of the markets is not an estimation problem and should rather be considered as a forecasting issue.



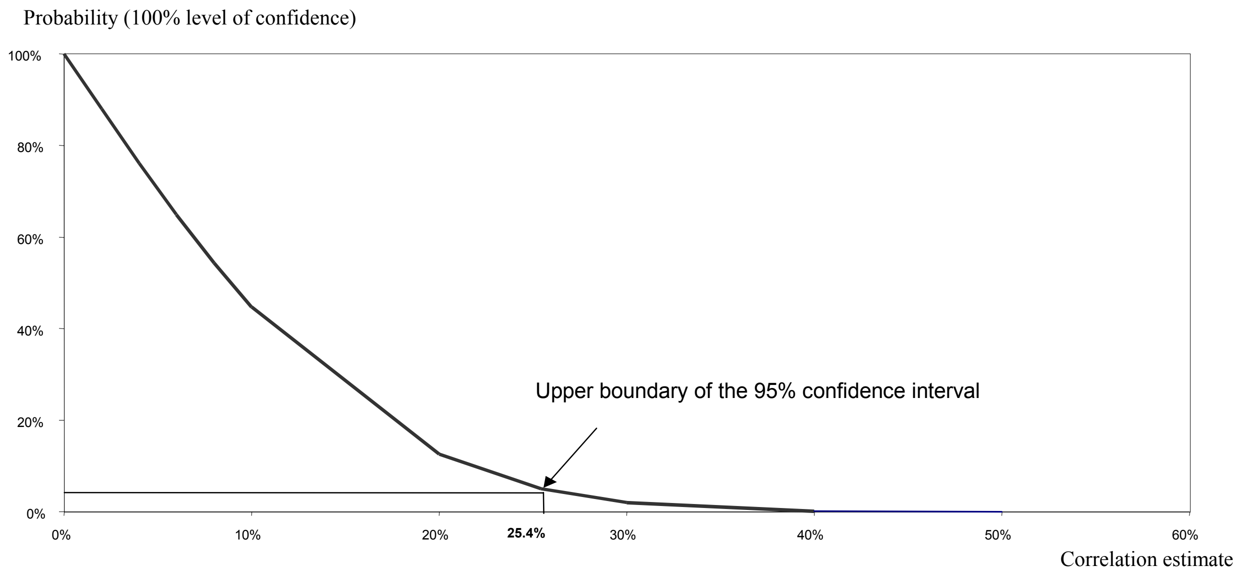
Japanese and US equities performances in the 80's and 90's
(Figure 1)

A lot of articles on portfolio optimization have been published since Harry Markowitz seminal paper some fifty years ago. Indeed, optimization is the cornerstone of the financial theory, from which equilibrium relationship like CAPM have been derived. Yet, when practitioners try to use optimizers they experience, after some excitement, a number of disappointments as soon as their assumptions depart somewhat from the standard equilibrium, in particular as far as expected returns are concerned. Michaud (1998) even describes these tools as "error optimizers", because once an expected return of one single equity market, maybe by chance, diverges from others expected returns, the optimizer tends to overemphasize this discrepancy in the outcoming allocation. A number of remedies have been proposed, among others, additional constraints on the deviation from a benchmark such as ranges for the asset weighs in the portfolio. More refined ways, such as sophisticated models for covariance matrix (like Garch specifications), penalizations of the «volatility» of the volatility, or some sort of Bayesian approach to the input (Jorion, 1986) have been tried. At the end of the day, a lot of efforts have been spent to find robust ways to make the optimization work, most of them in the direction of more sophistication and thus more parameters to estimate from history. The difficulty to forecast risk parameters as illustrated by the above example of US and Japanese stocks has incited us to try in the opposite direction : less sophistication but more robustness leading to more stable parameter estimates.

A model aimed at forecasting risk

As mentioned before, the key point for a risk model built in order to be used in an optimization procedure is to be forward looking. To build optimal portfolios, a very precise analysis of past risks is not very useful while being able to anticipate the main sources of uncertainty in the future is really crucial. The model presented in this section has been designed with this objective in mind, focusing on the modeling of stable sources of risks and correlations rather than more subtle but less reliable relationships between the various asset returns.

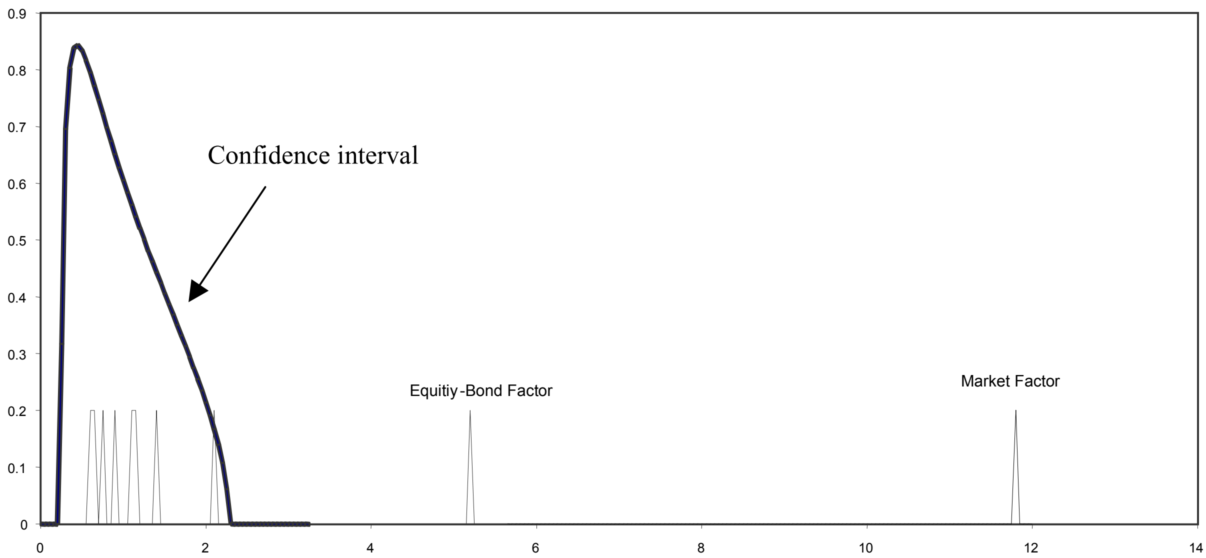
Once investors have recognized that they face a risky environment, they try to figure out the major sources of uncertainties in order to «structure» their thoughts. Take for example what regulators did in order to control market risks in banks. Before allowing the most advanced banks to use their «home cooked» internal model, the Basles paper of 1993 proposed a standard model, that could be applied to every market participant. To clarify things, it started with the simple idea that there is a general market risk and apart from that, a variety of specific risks. This might as well be applied to stock market positions or to bond portfolios.



Level of Confidence for rejecting a zero correlation given a correlation estimate (Figure 2)

Now, let’s go back to our portfolio risks with this natural idea and try to evaluate it. In statistical terms, the question is: can we reject the hypothesis that portfolios are subject to a global market risk and to specific risks uncorrelated to the former? At this stage, statistical theory can provide a test or even better, a confidence interval, for the parameters. Figure 2 gives the half width of a confidence internal, around a zero correlation, for a typical sample of historical data (says monthly return over 5 years). As an example, at a 95 % probability, you cannot claim that a 25.4 % correlation is different from 0. This means that one has to be very cautious about the low negative correlation that might suggest there is a hedging potential between two assets, when in fact, there is only an estimation error.

There is an even more sophisticated approach to this parameter uncertainty situation that can help us in the multi-dimension problem that we face. Random matrix theory (Bouchaud, 1999) provides tools to make the difference between any given correlation matrix and the matrix of a pure random process. These tests judge how the eigen values of a given correlation matrix are significantly different from one which would correspond to a pure noise process. Because of the noise in the sample and estimation process, one must be cautious to keep the eigen values that are high enough to be of some statistical significance. The actual interval for the eigenvalues of a random correlation are compared to those of the historical correlation matrix of the 21 most developed stock markets and bond markets computed over 5 years. Figure 3 shows this comparison. In the end, only two factors seem to be structural, the rest can be considered as noises that are mutually independent.



Eigen values of 1990-2000 covariance matrix of bonds and equities returns in 21 counties
(Figure 3)

For global asset allocation purposes, as a result, the structure of the risks can be summarized by two general risk factors, a general bond risk and pure equity risk and uncorrelated specific factors. How did we come up with analysis ?
The two eigen vectors of the preceding matrix corresponding to the two significant eigen values shown above are respectively an equally weighted portfolio of the 42 assets included in the universe, the market portfolio and a combination of positive weights on equity markets and negative ones on bond markets. A very simple transformation of these two factors leads to two factors that are even easier to interpret: a global bond factor described by a straight forward bond portfolio and a "pure equity" factor, which would correspond to a global equity portfolio whose sensitivity to interest rates has been immunized.

From an economic point of view the meaning of the two factors is straightforward : global markets are integrated, bond markets more than equities markets, and the remaining return of major asset classes can be considered as specific risk of various sorts. This transparent interpretation of the two common risk factors is a very important feature of the model. As much as the robustness of the statistical results presented above, the common economic sense it underlines, provides confidence in the model stability. In a sense, this conclusion support some simplified APT approach of global investments with only two factors prevailing: interest rates and "pure equity" factors.

Clearly, we cannot claim that no other common factors exist. But they do not lead to a permanent and significant risk structure in the random matrices analysis that could be relied on, in an optimization process. Precisely, one source of disappointment in the use of optimizers comes from the fact that optimization procedures look for extreme correlations. An easy way to see this is the limit case of a correlation equal to one. Two assets almost perfectly correlated with different risk-adjusted expected returns offer a quasi arbitrage opportunity. An optimizer will heavily (infinitely) loaded the portfolio on this opportunity leading to corner solutions that can result in disasters if the initial correlation was spuriously high.

It could also be argued, that the covariance matrix of asset return is an inadequate measure of the risks of the investments. We know for sure, that during financial crises, return distributions depart widely from normality. Therefore, covariance matrices provide little guidance to the risk management during these periods of extreme volatilities. Nevertheless, such crisis in developed financial markets remain infrequent and quite short and thus one can use with relative safety normality distribution when it comes to the allocation of the portfolio.

Further more, many authors have shown (Longin and Solnik 1999) that correlations during crises tend to increase. The above risk model is very consistent with this feature. Indeed, if for a short period there is a spike in market factor variances while specific variances remain constant, correlation estimates between market returns will increase as a consequence of the global factor crisis.

As the covariation between global stock and bond markets can be captured with two factors, a global bond factor and a global equity factor, one can decompose the excess returns above the local cash rate on any given bond or equity market as follows :

$$r_i^b = \beta_i^b r_B + sr_i^b \tag{1}$$

where

r_i^b excess return of bond market *i*

r_B excess return of key bond portfolio

β_i^b sensitivity of bond market *i* to key bond portfolio

sr_i^b specific return of bond market *i*

$$r_j^e = \beta_j^e r_E + sr_j^e \tag{2}$$

r_j^e excess return of equity market *j*

r_E excess return of key equity portfolio

β_j^e sensitivity of equity market *j* to key equity portfolio

sr_j^e specific return of equity market *j*.

To take into account the correlation between the key portfolios B and E, the return r_E is itself decomposed further into :

$$r_E = \lambda r_B + r_{PE} \tag{3}$$

Where r_{PE} is the «pure equity» return, defined as the part of r_E that is not correlated to the global bond factor. In this setting λ stands for the sensitivity of the global equity factor to the global bond factor.

Key portfolios are investable portfolios that are closely linked to the factors mentioned earlier[1]. Their weights in the country markets is therefore related to the financial importance of the various markets. If an historical period was representative enough of bond or equity markets behavior, these weights could be obtained from a principal component analysis. In practice, other weights are also possible because many portfolios are very close to the former in terms of tracking errors. Among others, market capitalization or GDP weights could also be used. In any case, the two key portfolios end up being global portfolios respectively invested in bond or equity markets. In this setting, any specific return of sb_i and se_j are independent random variables characterized by their specific variances. One recognizes, here, the classical distinction between general market risk and specific risk applied to the global financial markets. Obviously, betas and specific risks can be obtained from a regression, once again, if the sample period is representative enough[2]. Nevertheless, we would rather suggest to estimate the specific risks by taking into account the size on the economy, the liquidity of the capital and equity markets, and also general financial environment, like economic or monetary integration in the near term.

Optimization in the new risk structure.

Let's start with classical unconstrained optimization, before turning to the tactical allocation process, which requires a more detailed framework. Let's assume that the forecasting steps of the process are completed : all the parameters of the risk model described above have been estimated and expected return on the assets have been computed. These expected returns are respectively denoted $\bar{r}_i^b$ for the bonds and $\bar{r}_j^e$ for the equities. The decomposition of the asset returns formalized in the equations (1), (2) and (3) gives naturally the same expression for the expected returns:

$$\bar{r}_i^b = \beta_i^b \bar{r}_B + \overline{sr}_i^b$$
$$\bar{r}_j^e = \beta_j^e (\lambda \bar{r}_B + \bar{r}_{PE}) + \overline{sr}_j^e$$

In a very general framework, the mean variance optimization program leads to the following solution :

[1] Key portfolios can also be seen as the projection of unobservable and uninvestable factors on the universe of investable assets

[2] Parameters derived from the regression of the assets return on those of the key portfolios have to be adjusted to take into account the fact that the key portfolios represent approximations of the real factors.

$$\mathbf{X} = \frac{1}{a}\mathbf{\Sigma}^{-1}\bar{\mathbf{r}}$$

where $\mathbf{X}$ is the allocation vector, $\mathbf{\Sigma}$ the covariance matrix $\bar{\mathbf{r}}$ the expected return vector and a is the risk aversion of the investor.

If one just applies this formula with a pure historical covariance matrix, one faces two major difficulties. First, the allocation is very sensitive to the estimation errors on the correlation coefficients. Secondly, the relationship between the assets expected returns and the portfolio composition is very unclear. Using the forward looking factor model presented in the former section is much more satisfactory. This choice actually makes the overall optimization process much simpler and straightforward. Indeed, the covariance matrix has a particular form and the allocations derived in this framework are much more intuitive as the optimal mix of exposures to specific risks and systematic factors.

The exposure to the specific risk of the asset *i* is given by[3] :

$$Xs_i = \frac{\overline{sr}_i}{a\sigma s_i^2} \tag{4}$$

where σs_i^2 is the specific variance.

As they are also uncorrelated to each other, the respective exposures to the systematic factors B and PE have the same expression :

$$X_B = \frac{\bar{r}_B}{a\sigma_B^2} \text{ and } X_{PE} = \frac{\bar{r}_{PE}}{a\sigma_{PE}^2} \tag{5}$$

where σ_B^2 (resp. σ_{PE}^2) is the variance of the bond factor (resp. the pure equity factor).

Rearranging from (4) and (5) with the decomposition of assets returns in key portfolio plus specific components, the optimal asset weights can be derived as follows:

$$X_j^e = Xs_j^e + w_j^e\left(X_{PE} - \sum_j \beta_j^e Xs_j^e\right) \tag{6}$$

$$X_i^b = Xs_i^b + w_i^b\left(X_B - \lambda X_{PE} - \sum_i \beta_i^b Xs_i^b\right) \tag{7}$$

where Xs_j^e and Xs_i^b come from (4)

and w_j^e (resp. w_i^b) is the weight of the equity market *j* (resp. the bond market *i*) in the pure equity key portfolio (resp. the bond key portfolio).

[3] Superscripts for bonds and equities have been dropped for the sake of clarity.

The resulting optimal weights in the assets can be analyzed in more details. These equations show that the allocation in the various assets is a combination of the relative specific attractiveness, through Xs_i^b or Xs_j^e and of the forecasted returns of key portfolios B and PE.

From a theoretical point of view, the above results can be considered as a generalization of the optimization presented by Black and Treynor (1973) in the case of domestic equity portfolios. These authors consider a risk model for domestic equities where general market (the index) is the only systematic factor and the return of each equity is a combination of a market component plus a purely specific return.

An simplified example :

In order to illustrate this approach to optimization, we consider now a simple example of global asset allocation among US, European and Japanese bond and stock markets assuming that currency risks are fully hedged to the Euro.

In this setting, only the returns in the excess of local short term rate, that we shall call premia in the remainder of this paper are of interest. Indeed, it is well known that after currency hedging to the Euro, the return of any foreign asset is equal to the short term in Euro plus the return in local currency minus the short term rate in this foreign currency, that is to say the excess return. One could debate whether it is necessary or not to fully hedge the foreign holdings. No doubt that this is a simplification. Given the fact that currency exposures do not provide any constant premium but instead increase quite a bit the overall volatility of the portfolio, we decide here to hedge totally.

Let's start with an analysis of the risk structure of this investment universe limited to six asset classes, yet the major ones. Table 1 gives the risk characteristics of the universe[4] with a decomposition in global risk and specific risk for the six asset classes (represented by Salomon Brothers indices for bonds and MSCI indices for stocks). As one can see from Table 1, Japan tend to show more specific risks than the two other markets which are quite close in term of beta and specific risks.

[4] The figures have been rounded for the sake of simplicity

Asset	Specific Risk	Global Risk	Beta		Factor Composition
Bonds					
U.S.	2.57%	4.80%	1.10		40%
Japan	3.10%	4.28%	0.80		20%
Euro Zone	2.45%	4.43%	1.00		40%
Global Bond		**4.00%**	**1.00**		**100%**
Equities					
U.S.	6.26%	14.68%	1.00		50%
Japan	12.52%	16.42%	0.80		10%
Euro Zone	7.17%	15.68%	1.05		40%
Global Equity		**14.00%**	**1.00**		**100%**

Structural characteristics of the risk model
(Table 1)

In order to complete the risk model estimation, the equity factor has been regressed on the bond factor. The λ coefficient is found to be equal to 1.40 and the volatility of the pure equity factor is 12.83%. Although volatility of the equities is poorly explained by fixed income markets, the equity factor is more sensitive to the interest rates movements than the bond factor.

Table 3 gives the allocation of a global portfolio for an investor who has a risk aversion of 5, in the context of expected premiums shown in the first column.

Asset	Expected premium	Specific premium	Risk exposures		Allocation
Bonds					
U.S.	1.05%	-0.03%	-8.5%		2.4%
Japan	0.80%	0.02%	3.3%		8.8%
Euro Zone	1.00%	0.02%	6.7%		17.6%
Key Bond Portfolio	**0.98%**		**27.3%**		**28.8%**
Equities					
U.S.	7.00%	0.03%	1.5%		35.5%
Japan	7.50%	1.92%	24.5%		31.3%
Euro Zone	6.80%	-0.52%	-20.2%		7.0%
Key Equity Portfolio	**6.97%**		**68.0%**		**73.9%**

Global Allocation without benchmark
(Table 3)

Let's describe in more details the way the optimal allocation is obtained. First, the expected returns of the 2 key portfolios are computed as a weighted average of the assets expected return. The specific premiums are then derived using the decomposition given by the equations (1) and (2).

Let's take the example of the Japanese bond market.
The expected return of the key bond portfolio is :

$0.4 \times 1.05\% + 0.2 \times 0.80\% + 0.4 \times 1.00\% = 0.98\%$.
And thus the Japan bonds market specific premium is equal to :
$0.80\% - 0.8 \times 0.98\% = 0.02\%$.

From equation (4), we then deduct the optimal exposure to the specific risk of the Japan bonds market : $0.02\%/(5 \times 3.10\%^2) = 3.3\%$.

The exposure to the key equity portfolio (resp. key equity bond portfolio) reported in Table 3 represents in fact the term in equation (6) (resp. equation (7)) which is between the parenthesis, it means the contribution of the systematic factors in the optimal allocation. In this particular case, we have:

$$\sum_j \beta_j^e Xs_j^e = \sum_i \beta_i^b Xs_i^b = 0 . \quad (6)$$

So, the exposure to the key equity portfolio is just the same as the exposure to the pure equity factor which is, according to the equation (5), equal to the pure equity expected premium ($6.97\% - 1.4 \times 0.98\% = 5.60\%$) divided by its variance and the risk aversion of the investor :
$5.60\% /(5 \times 12.83\%^2) = 68.0\%$.

Since the key equity portfolio gives also an exposure to the bond factor of $1.40 \times 68.0\% = 95.2\%$, the later has to be deduced of the desired level of exposure, which is $0.98\% /(5 \times 4.00\%^2) = 122.5\%$, in order to obtain the amount to be invested in the key bond portfolio ($122.5\% - 95.2\% = 27.3\%$).

Finally, the portfolio allocation in the Japan bonds market, reported in the last column of Table 3, is the sum of its contribution to the global bond exposure ($20\% \times 27.3\%$ where 20% is the weight of Japan in the key bond portfolio) and its optimal specific one (3.3%).

Real life Tactical Allocation

In practice, the overall allocation process starts with a strategic view that is translated into a benchmark plus an active risk budget for the asset manager. In this section, the global benchmark is composed as follows: 50% Salomon Brothers World Government Bond Index (fully hedged to the Euro) and 50% MSCI World (also fully hedged to the Euro.)
The risk budget constraint imposes that active risk should be kept below 3% of tracking error (the volatility of the benchmark being roughly 10%).

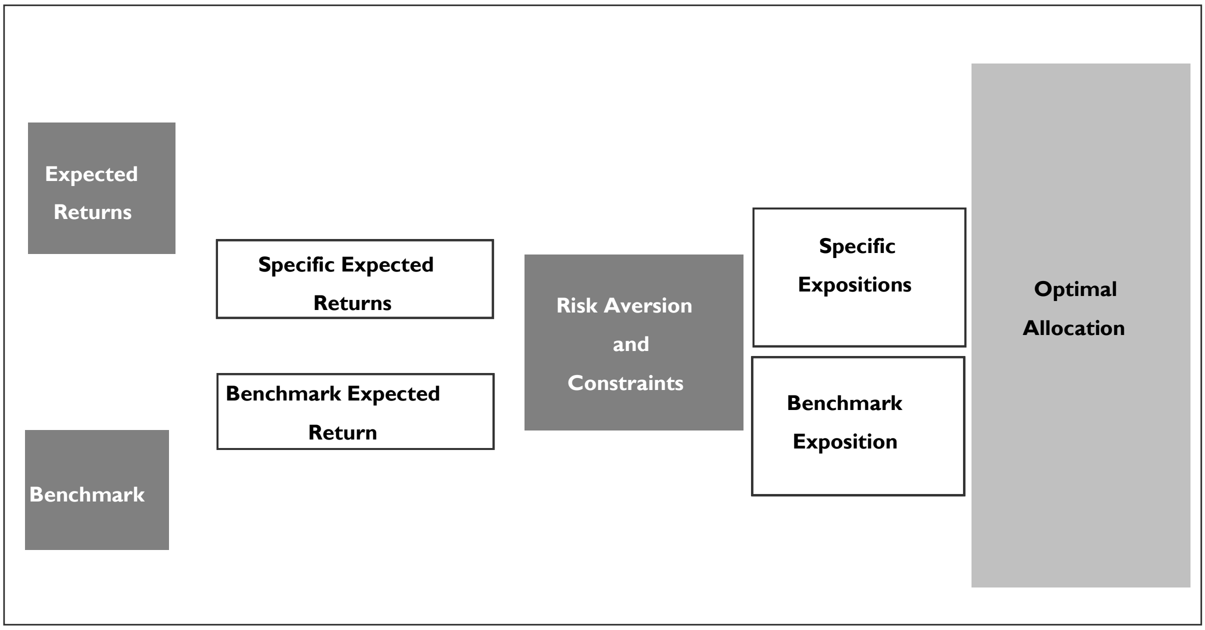
Table 4 presents the set of expected premiums derived from Sinopia's Tactical Asset Allocation models at the beginning of January 2000 for a 12 month horizon. As one can see they correspond to bearish views on stocks but quite positive on bonds. These expected premiums deviate from their average to a large extent offering interesting tactical opportunities as for example the global equity market expected premium shows.

Assets		Expected premium
BONDS	Australia	2.81%
	Canada	2.53%
	Denmark	4.27%
	U.S.	3.54%
	Japan	-3.17%
	Norway	1.82%
	New-Zeland	4.73%
	U.K.	1.54%
	Sweden	4.47%
	Switzerland	1.70%
	Euro Zone	4.28%
GLOBAL BONDS		3.03%

Assets		Expected premium
EQUITIES	U.S.	-27.02%
	Japan	1.86%
	Germany	-35.09%
	France	-20.09%
	U.K.	-13.02%
	Italy	-13.14%
	Switzerland	-5.43%
	Belgium	-1.92%
	Netherlands	-22.78%
	Spain	-2.27%
	Denmark	-1.66%
	Sweden	-67.12%
	Canada	-16.13%
	Australia	-11.01%
	New-Zeland	13.83%
	Norway	-37.78%
	Finland	-42.12%
	Austria	12.62%
	Singapore	-35.39%
	Hong-Kong	-52.55%
	Portugal	9.97%
	Ireland	-10.29%
GLOBAL EQUITIES		-19.11%

Asset classes 1 year expected returns (January 2000)
(Table 4)

These expected premiums together with the risk forecasts explained previously are used to perform the tactical optimization along the process described in Figure 4.



Tactical optimization sketch
(Figure 4)

For this "back testing" example, the resulting portfolio is strongly underweighting stocks (-20%) and long in bonds (+30%) because premiums of the later are higher than usual. Ex ante tracking error is 2.9% and expected premium of - 0.11%.

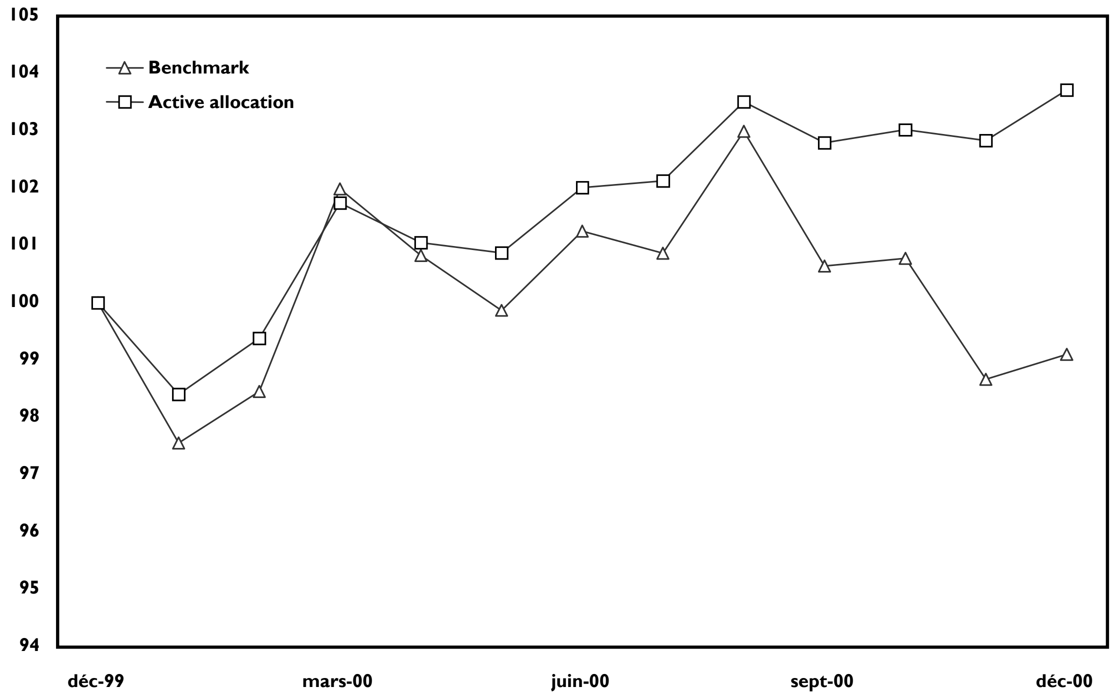
	Assets	Benchmark	Tactical Allocation
BONDS	Australia	0.3%	0.0%
	Canada	1.4%	0.0%
	Denmark	0.6%	6.3%
	U.S.	13.5%	21.5%
	Japan	12.9%	3.9%
	Norway	0.0%	0.0%
	New-Zeland	0.0%	2.5%
	U.K.	2.9%	0.0%
	Sweden	0.5%	7.1%
	Switzerland	0.2%	0.0%
	Euro Zone	17.7%	36.9%
GLOBAL BONDS		**50.0%**	**78.12%**

Average TE desired	**3.00%**
Ex-ante TE	**2.92%**
Ex-Post TE	**3.07%**

	Assets	Benchmark	Tactical Allocation
EQUITIES	U.S.	25.2%	8.7%
	Japan	5.9%	5.5%
	Germany	2.0%	0.0%
	France	2.6%	1.3%
	U.K.	4.8%	3.3%
	Italy	1.0%	1.2%
	Switzerland	1.4%	1.8%
	Belgium	0.2%	1.1%
	Netherlands	1.3%	0.2%
	Spain	0.7%	1.8%
	Denmark	0.2%	1.2%
	Sweden	0.8%	0.0%
	Canada	1.4%	0.7%
	Australia	0.7%	0.7%
	New-Zeland	0.0%	1.5%
	Norway	0.1%	0.0%
	Finland	0.6%	0.0%
	Austria	0.0%	1.4%
	Singapore	0.3%	0.0%
	Hong-Kong	0.6%	0.0%
	Portugal	0.1%	1.3%
	Ireland	0.1%	0.4%
GLOBAL EQUITY		**50.0%**	**32.19%**

Allocation as of January 2000
(Table 5)

The active bets corresponding to the above allocation proved to be quite successful, as can be seen in Figure 5, when year 2000 performances of the benchmark and of the active allocation are compared. The outperformance being 5% for a level of ex-post tracking error of 3.1%, the information ratio is then around 1.4.



Performances during year 2000
(Figure 5)

Conclusion

Dealing with investment is dealing with an uncertain future. Therefore, a disciplined approach to investment requires a view on expected returns but also on expected risks which are often not considered.

In this article , we show a way to build a forecasted covariance matrix consistent with expected returns on asset classes. We argue that no more than two global factors remain statistically and financially relevant when global asset allocation is concerned as the unstability of the further factors makes their practical use , at best, ineffective...

This robust approach has been illustrated in a simplified example and a true life situation. Other authors have shown that an information ratio of 1 (fairly large in the active management community) cannot be checked ex-post with less than 5 years, at a confidence level of 1%. We can bet with confidence that more than 5 years of performance using tactical optimization are necessary to check if it provides better performances, measured by information ratios for example.

In the meantime, we claim that this approach is more realistic, more transparent and much more intuitive. Reasonable investors should not refrain from using optimizers. They should instead select an optimizer that help them allocate between investment opportunities in a consistent way with their active views… looking towards the future. Beware of the rear mirror optimizers

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