The CAPM and the Risk Appetite Index: theoretical differences and empirical similarities*

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Abstract

This paper provides a theoretical and an empirical analysis of the Risk Appetite Index (RAI), a measure of investors' risk aversion proposed by Kumar and Persaud (2001, 2002). Our theoretical analysis shows that the RAI distinguishes between risk and risk aversion only under restrictive assumptions on the distribution of returns and the nature of the shocks affecting assets' riskiness. Although these assumptions are theoretically restrictive, we need to verify empirically to what extent they alter the behavior of the RAI. We do this by comparing the RAI with a measure of risk aversion "in the short-term", which we derive from the estimation of a CAPM – a model that does not require those restrictive assumptions. Using data of international asset prices in the last six years, we find that estimates are surprisingly similar. We explain this result by proving that under a certain condition the RAI may be regarded as a robust estimator of the risk aversion parameter in a CAPM. This condition requires the ratio between the variance of assets' returns and the variance of assets' riskiness to be approximately constant – a condition that is met in our sample.

JEL classification: G11, G12
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1 Introduction

The behavior of international asset prices in last two years has been often explained on the basis of changes in risk aversion. For instance, according to the IMF, the decline in equity prices recorded between March 2002 and mid-March 2003 was mostly due to an increase in investors' risk aversion connected with the geopolitical tensions that ended up with the war in Iraq (IMF (2003a)). By the same token, both the IMF and the BIS explained the subsequent recovery in equity prices with a decline in risk aversion (see IMF (2003b) and BIS (2004)). These explanations have been supported empirically with a variety of indicators, usually created by private financial analysts.

The positive attitude of market practitioners towards measuring risk aversion contrasts sharply with the general skepticism that prevails in the academic research. For instance, in his classical book on the economics of uncertainty, Laffont (1993) has an entire chapter on 'Measuring Risk Aversion and Risk', without a single reference to empirical works![1] In his seminal contribution to the theory of risk-bearing, Arrow (1970) has inferred the value of risk aversion parameters on the basis of the properties of Von Neumann and Morgenstern utility functions.[2] Then, in theoretical models economists have typically used specific utility functions (generally with a constant absolute or relative risk aversion), mainly because of mathematical convenience. This skepticism derives not only from the unobservability of individuals' preferences, but also from the observational equivalence between risk and risk aversion: in fact, an increase (decrease) in either of them causes asset prices to decline (rise) and risk premia to increase (decrease).

In the financial community, instead, market practitioners have developed a number of indicators based on the behavior of asset prices, that they interpret as measures of investors' risk aversion. Most, if not all, of these indicators, however, do not necessarily reflect changes in risk aversion.

In this literature, one can distinguish between two main classes of indicators.[3] One class is based on weighted averages of risk premia on a wide range of assets, including, e.g., technology stocks, industrial countries' junk bonds and emerging countries' currencies.[4] The basic hypothesis is that if

[1]In fact, Laffont concludes: "It is of course difficult to obtain sufficient information about an agent's preferences, to know whether his absolute risk aversion increases or decreases (since this requires information about the third derivative of his utility function)." (Laffont (1993, p. 24), cited from Hartog et al. (2002)).

[2]Starting from the boundedness property of utility functions, Arrow (1970) concludes that the relative risk aversion should be approximately 1 – a condition implying that preferences are represented by a logarithm function, as first suggested by Bernoulli (1738).

[3]For a brief survey of the indicators used by private banks see IMF (2003, Box 3.1).

[4]An example is the well known "Liquidity, Credit, and Volatility Index" released by the J.P. Morgan Chase in October 2002, and recently revised at the IMF by Dungey et al. (2003).

the cross section of assets is sufficiently large, then the underlying overall risk may be regarded as constant across time. In this case, changes in the weighted average of risk premia may be attributed to changes in risk aversion. This hypothesis, however, is highly questionable, especially in periods of financial turmoil, when international global integration is likely to spill over the effects of idiosyncratic country shocks.

Another class of measures is based on options prices. Options provide a spectrum of observations for each expiry date (one observation for each quoted strike price). This multiplicity of prices allows to construct probability density functions (PDFs) representing forward-looking forecasts of the distribution of prices of the underlying asset. The PDFs estimated from options prices are risk-neutral and do not provide accurate forecasts. Then, one can attribute the difference between predicted and actual values to the true risk aversion of the representative investor. Hence, a measure of risk aversion can be obtained by comparing risk-neutral PDFs with historical distributions of prices.[5] While this procedure may be correct on average, in specific periods of time investors may expect risk to change with respect to the average risk incorporated in historical data. This will affect the PDFs, thereby distorting the measure of risk aversion.[6]

Thus, previous indicators may fail to distinguish between risk and risk aversion. Recently, Kumar and Persaud (2001, 2002) have made an attempt to break the observational equivalence between risk and risk aversion by exploiting a special feature of asset pricing models. According to these authors, pricing models are such that changes in risk aversion modify the rank of assets' expected returns relative to the rank of assets' riskiness, whilst changes in assets' riskiness do not affect the relative ranks. Then, they build an indicator of investors' risk aversion, called Risk Appetite Index (RAI), based on the rank correlation between the expected excess return and the riskiness of a cross section of assets.

In this work, we examine the RAI both theoretically and empirically. In the theoretical part, we refine a previous analysis of Misina (2003), who gathers the conditions under which the RAI can distinguish between risk and risk aversion into two propositions (Section 2). Next, building on Kumar and Persaud (2002) and Misina (2003), we examine the RAI in the context of the Capital Asset Pricing Model (CAPM) (Section 3).[7] We focus on the

[5] See Bliss and Panigirtzoglou (2004) and Grande and Pericoli (2004).

[6] The academic literature has also followed other lines of reaserch. Some measures of risk aversion are obtained from survey data on household asset holdings, as in the tradition of Friend and Blume (1975) (Guiso and Paiella (2003) is a recent example using survey data). Measures drawing on experimental data also have a long history (see, e.g., Gordon et al. (1975) or the recent paper by Bossaerts and Plott (2002)). Finally, another important stream of research is based on the estimation of the Capital Asset Pricing Model, which we survey in Section 4.

[7] Pericoli and Sbracia (2004) show that these conditions are not fulfilled, even in the *ad hoc* asset pricing model proposed by Kumar and Persaud (2001).

CAPM because this model is the prototype of modern asset pricing theory. Its main prediction, that equilibrium expected returns are proportional to their covariance with the aggregate risk, is shared with virtually any other pricing model that has been taken to the data.[8] We show that the RAI distinguishes between changes in assets' riskiness and changes in investors' risk aversion only under restrictive assumptions on the distribution of returns and the nature of the shocks affecting assets' riskiness. Specifically, given a sufficiently large number of assets, we need to assume that returns are independent and that the shocks affecting assets' riskiness are idiosyncratic.

Although these assumptions are theoretically restrictive, we need to verify empirically to what extent they alter the behavior of the RAI. We can do this by comparing the RAI with a measure of risk aversion "in the short-term", which we derive from the estimation of a CAPM – a model that does not require either independent returns or specific assumptions on the nature of the shocks. Using data on international stock prices during the last six years, we show that the two estimates are surprisingly similar. Therefore, by focusing on the statistical properties of the RAI, we prove that under a certain condition this indicator may be regarded as a robust estimator of the risk aversion parameter in a CAPM. This condition, which requires the ratio between the variance of assets' returns and the variance of assets' riskiness to be approximately constant, is met in our sample (Section 4). Section 5 concludes.

2 Theoretical foundations

The first element needed in order to build the RAI is the expected excess return (excess return hereafter) on each asset i, that we denote with R_i^{ex}, $i = 1, ..., n$. The excess return R_i^{ex} is the difference between the expected return on the risky asset i, $E(R_i)$, and the return on the risk free asset, R_f:[9]

$$R_i^{ex} = E(R_i) - R_f \ . \tag{1}$$

The expected return on asset i, in turn, can be viewed as the difference between its expected price plus the expected dividend, denoted with L_i, and its current price P_i:

$$E(R_i) = L_i - P_i \ , \tag{2}$$

[8] Generally, it is the meaning of aggregate risk that differs across models: in the standard CAPM it is the return of the market portfolio; in Lucas (1978) and Breeden (1979) it is aggregate consumption.

[9] Hereafter E denotes the expectation operator, Var the variance operator, Cov the covariance operator and $Corr$ the correlation operator. All the operators refer to unconditional moments.

where both L_i and P_i are expressed in terms of logarithms.[10] In asset pricing models, given a change in a parameter the adjustment occurs through current prices that, in turn, make expected and excess returns change. Thus, the adjustment in excess returns occur through changes – *with the opposite sign* – in current asset prices.

The second element in the RAI is the riskiness of each asset i, that we denote with λ_i. In the following sections, the parameter λ_i will be defined precisely according to the considered asset pricing model. We will see that such definition may turn out to be critical in order to assess the properties of the pricing model and, in turn, the appropriateness of the RAI as a measure of market's risk aversion.

As pointed out by Misina (2003), the RAI stems from an important property. We say that a change in a parameter of the asset pricing model that affects assets' excess returns yields a *rank effect* if the following condition holds:

$$\text{if } \lambda_i > \lambda_j \;\Rightarrow\; \text{either } \Delta R_i^{ex} > \Delta R_j^{ex} \;\forall i \neq j \;\text{ or } \Delta R_i^{ex} < \Delta R_j^{ex} \;\forall i \neq j. \quad (3)$$

Property (3) states that the "rank effect" obtains when a change in a parameter of the pricing model determines changes in each asset's excess return that are monotone (either increasing or decreasing) in the riskiness of each asset.[11] Definition (3) leaves indeterminate both the sign of the change in expected returns (which can be positive or negative) and the sign of the monotonicity relationship between the change in excess return and the riskiness of each asset (increasing or decreasing). These indeterminacies give origin to four possible cases. We argue below that in all possible cases, *property (3) affects the rank correlation between excess returns and risks in a cross section of assets.*

In general, there are several measures of rank correlation between two variables (see Stuart and Ord (1991, Chapter 26) for a brief overview). Following Kumar and Persaud (2001, 2002) in this paper we use Spearman's measure of rank correlation (denoted with ρ^s) which takes values in the interval $[-1, 1]$; specifically, $\rho^s = 1$ ($\rho^s = -1$) when the rank of the values of one variable is the same as (inverse of) the rank of the values of the other variable.

In order to understand why the rank effect affects the rank correlation, suppose that, before the rank effect shows up, such correlation is less than

[10] We have adopted the standard definition of expected returns (see, e.g., Cochrane (2001, Chapter 1)). Kumar and Persaud (2001), instead, define L_i as the *long-run price* of asset i.

[11] The rank effect as defined by property (3) generalizes the one stated by Misina (2003), who considers only changes in excess returns that are *increasing* in assets' riskiness. Our definition is more relevant for the analysis of the RAI, because we can show that *also decreasing* changes in excess returns modify the rank correlation between assets' excess returns and risks.

1. This assumption means that there are at least two assets, say assets i and j, such that $\lambda_i > \lambda_j$ and $R_i^{ex} < R_j^{ex}$. Hence, consider a change in a parameter that causes a rank effect and, for now, assume that this effect gives rise to an increase in excess returns (i.e. ΔR_i^{ex} and ΔR_j^{ex} are both positive). Suppose also that property (3) holds because we observe increasing changes in excess returns; namely, $\Delta R_i^{ex} > \Delta R_j^{ex} > 0$. Then, if the increase in the excess return of asset i is sufficiently larger than the one of asset j, it can reverse the relationship between excess returns into $R_i^{ex} > R_j^{ex}$, thereby strengthening the rank correlation between excess returns and risks. Analogously, suppose that the rank effect gives rise to a decrease in excess returns (i.e. ΔR_i^{ex} and ΔR_j^{ex} are both negative) and that property (3) holds with $\Delta R_j^{ex} < \Delta R_i^{ex} < 0$; then, if the decrease in R_j^{ex} is sufficiently larger (in absolute terms) than the decrease in R_i^{ex}, the relationship between the excess returns on the two assets can be reversed into $R_i^{ex} > R_j^{ex}$, and this will strengthen the rank correlation between excess returns and risks.

By the same token, assume that the rank correlation between excess returns and risks is larger than -1, so that there exist at least two assets, say assets i and j, such that $\lambda_i > \lambda_j$ and $R_i^{ex} > R_j^{ex}$. Assume also that the change in the parameter determines a rank effect with decreasing and positive excess returns (i.e., $\lambda_i > \lambda_j$ and $\Delta R_i^{ex} < \Delta R_j^{ex}$ with $\Delta R_i^{ex}, \Delta R_j^{ex} > 0$). Then, the resulting rank correlation may weaken, as the change in excess returns can turn the relationship between R_i^{ex} and R_j^{ex} into $R_i^{ex} < R_j^{ex}$. The same result obtains if $\Delta R_i^{ex} < \Delta R_j^{ex}$ and ΔR_i^{ex} and ΔR_j^{ex} are both negative.

Thus, the rank effect – as defined by condition (3) – tends to modify (either strengthen or weaken) the rank correlation between excess returns and risks.

Now suppose that in an asset pricing model property (3) is fulfilled *only* by changes in the risk aversion parameter, whilst changes in risk do not fulfill it. Then, we could exploit this property in order to discriminate between changes in risk and changes in risk aversion. Specifically, changes in asset prices that turn out to modify the rank correlation between excess returns and risks will be due to changes in risk aversion; moreover, if changes in asset prices do not modify the rank correlation between excess returns and risks, then they can be attributed to changes in risks.

Following Misina (2003), the conditions under which the RAI can be used to discriminate between changes in risk and changes in risk aversion can be conveniently gathered into two propositions. The first proposition specifies that changes in risk aversion have a rank effect; namely:

Proposition 1 *A change in investors' risk aversion has a rank effect on excess returns across different assets.*

Of course, this proposition cannot be verified empirically, since risk aversion is an unobservable parameter. Moreover, we cannot use its consequences on the rank correlation between excess returns and risks to detect changes in risk aversion because, in principle, a rank effect can show up for reasons other than changes in risk aversion. Therefore, we can introduce a second proposition that addresses both issues. Specifically, we can assume that *only* changes in risk aversion have a rank effect, or, equivalently, if we assume that risk and risk aversion are the sole parameters of the pricing model that can change over time, then the second proposition states that changes in risk do not have a rank effect:

Proposition 2 *A change in the riskiness of assets does* ***not*** *have a rank effect on excess returns across different assets.*

Thus, when both propositions hold, the rank effect can be used to break the observational equivalence between risk and risk aversion. Specifically, we can use the rank correlation between excess returns and risks to detect changes in investors' risk aversion.

In the following section we will examine whether Propositions 1 and 2 can be proved in the context of a standard asset pricing models such as the CAPM.

3 The Risk Appetite Index and the CAPM

In Kumar and Persaud (2002), the authors motivate the RAI by considering changes in risk and in risk aversion in the CAPM. However, they focus only on the induced changes in the excess return on the *market portfolio* (i.e. on the optimal portfolio of the representative investor) and do not calculate explicitly the effect of the parameter changes on the excess return on *each risky asset*. The latter calculations are critical to verify whether Propositions 1 and 2 hold and, therefore, to establish whether the RAI can be a measure of changes in risk aversion.

Kumar and Persaud (2002) start their analysis with a well-known relationship between the excess return and the variance of the market portfolio. For the sake of simplicity, suppose that there are only risky assets in the market. If investors prefer frontier portfolios – defined as the portfolios with the minimum variance in the class of the portfolios with the same expected rate of return – then the following relationship holds (see Huang and Litzenberger (1988, Chapter 3)):

$$\sigma_m^2 = a\left[E\left(R_m\right)\right]^2 + bE\left(R_m\right) + c\ , \tag{4}$$

where R_m is the stochastic return on the market portfolio, σ_m^2 denotes its variance, and a, b and c are constants which depend on the expected returns on each risky asset and the variance-covariance matrix of asset returns. Equation (4) defines the *portfolio frontier* – i.e. the locus of all frontier portfolios – which is a parabola in the σ_m^2-$E(R_m)$ space (the Risk-Return space).

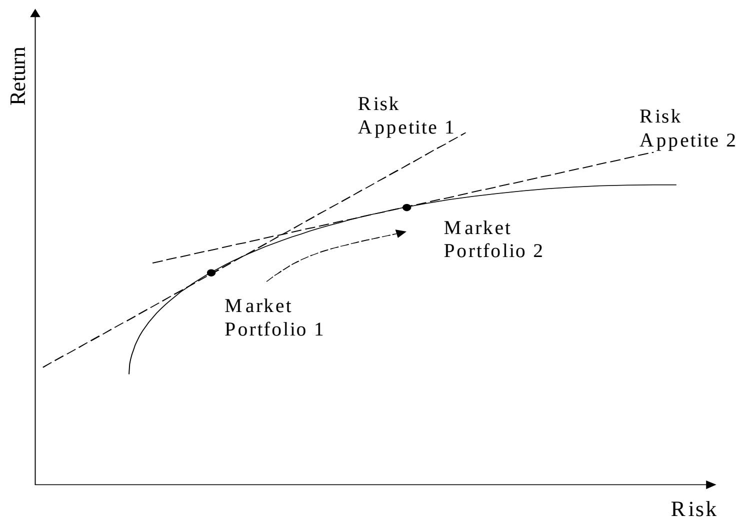

Figure 1: Effect of a change in the risk aversion

The slope of the curve (4) is the investors' risk aversion (see Kumar and Persaud (2002) or Cochrane (2001, Chapter 5)). Hence, changes in risk aversion determine a shift of the optimal portfolio that modifies both the expected return and the variance of the market portfolio, as illustrated in Figure 1.

Kumar and Persaud (2002) also consider an alternative scenario in which a simultaneous change in the riskiness of all assets occurs. However, they focus a very specific change: namely, one that gives rise only to a change in a single parameter of equation (4), that is the parameter c of the parabola. If c changes, say it goes from c to $c' > c$, it modifies only the riskiness of the market portfolio, without changing its expected return. Figure 2 illustrates this effect.

By comparing the different consequences of these two scenarios, the authors conclude that changes in risk aversion modify the rank correlation between expected returns and risks, while a simultaneous increase in the riskiness of all assets does not affect it. This claim motivates the RAI as a measure of market's risk aversion.

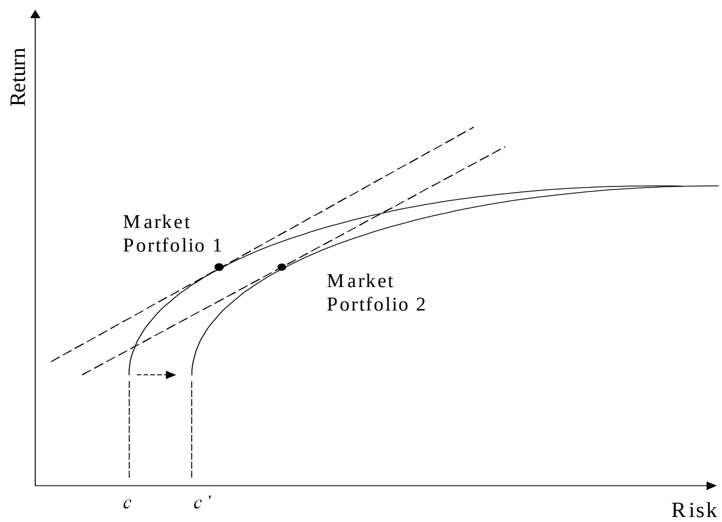

Figure 2: Effect of a simultaneous change in the riskiness of all assets

However, by focusing only on the implications on the market portfolio of parameter changes, Kumar and Persaud neglect the implications on the excess returns of each risky assets, which are essential for the validity of the RAI. For instance, in the case represented in Figure 1, a change in the return of the market portfolio does not necessarily imply that assets' excess returns change monotonically in their riskiness, as Proposition 1 requires. Similarly, the case represented in Figure 2 does not exclude that the change in the assets' excess returns yields a rank effect, as Proposition 2 establishes.

Thus, even within the very specific change in assets' riskiness considered by Kumar and Persaud (2002), the validity of the RAI remains questionable. Therefore, in the following section we examine analytically the effect of changes in risk and risk aversion and we show that the authors' conclusions are, in general, incorrect.

3.1 The CAPM with exponential utility and normal returns

Following Cochrane (2001, Chapter 9) and Misina (2003), in this section we verify whether Propositions 1 and 2 hold in a standard CAPM with multivariate normal returns and identical investors with Constant Absolute Risk Aversion (CARA) preferences. Appendix A.1 will further show that, if we focus on an appropriate measure of risk aversion, then a more general CAPM with heterogeneous agents with risk averse preferences would provide the same conclusions as those obtained with the simple CAPM analyzed in this section.

Let us consider a single consumer, interpreted as a representative agent of a large number of identical consumers, with preferences represented by the CARA utility:

$$u(C) = -e^{-\gamma C} \ , \tag{5}$$

where γ is the Arrow-Pratt coefficient of absolute risk aversion.

This representative investor has initial wealth W, which can be splitted between a risk free asset paying R_f and a set of n risky assets paying a stochastic return $R = (R_1, ..., R_n)$. Let $a = (a_1, ..., a_n)$ denote the amount of wealth invested in each asset i with $a_i \in \mathbb{R}$, $\forall i = 1, ..., n$. The budget constraint then implies:

$$W = a_f + \sum_{i=1}^{n} a_i \ , \tag{6}$$

while consumption will be given by:[12]

$$C = a_f R_f + \sum_{i=1}^{n} a_i R_i \ . \tag{7}$$

We also assume that asset returns are multivariate normally distributed with mean $E(R) = (E(R_1), ..., E(R_n))$ and variance-covariance matrix Σ:

$$R \sim N(E(R), \Sigma) \ . \tag{8}$$

The hypothesis (8) implies that consumption, which is an affine transformation of multivariate normal returns, will be (univariate) normally distributed:

$$C \sim N(\mu_C, \sigma_C^2) \ ,$$

with $\mu_C = a_f R_f + a' E(R)$ and $\sigma_C^2 = a' \Sigma a$. Hence, using a property of normal distributions we can write:

$$E[u(C)] = E\left(-e^{-\gamma C}\right) = -e^{-\gamma \mu_c + \frac{\gamma^2}{2} \sigma_C^2} \ .$$

We can now solve the investors' maximization problem:

$$\max_{a_f, a'} \left\{ -\exp\left[-\gamma \left(a_f R_f + a' E(R) \right) + \frac{\gamma^2}{2} a' \Sigma a \right] \right\} \ , \tag{9}$$

[12] We are implicitly assuming a two-period framework where agents invest in the first period, and, in the second period, returns are distributed and consumption occurs.

subject to the budget constraint (6). Using the constraint (6) we obtain $a_f = W - a'\underline{1}$. In addition, since the $\exp(.)$ is a monotonic function, we can rewrite the problem (9) as:

$$\max_{a'} \left\{ \gamma \left(R_f \left(W - a'\underline{1} \right) + a'E(R) \right) - \frac{\gamma^2}{2} a'\Sigma a \right\} \ .$$

First order conditions imply:

$$\gamma \left(-R_f \underline{1} + E(R) \right) - \gamma^2 \Sigma \overline{a} = 0 \ .$$

Therefore: $E(R) - \underline{1}R_f = \gamma\Sigma\overline{a}$, where $\overline{a}$ is the vector whose elements $\overline{a}_i$ are the optimal amounts of wealth invested in each risky asset i, and where $\underline{1}$ is a vector of ones. It follows that the solution of the problem (9) is:

$$\overline{a} = \Sigma^{-1} \frac{E(R) - \underline{1}R_f}{\gamma} \ . \qquad (10)$$

Of course, the budget constraint (6) implies that the optimal amount of wealth invested in the risk free asset is: $\overline{a}_f = W - \underline{1}'\overline{a}$. It is important to note that: (i) the concavity of the utility function (5) implies that the solution (10) is unique; (ii) each parameter $\overline{a}_i$ is a solution of the problem for *given* parameter values R_f, γ and Σ, and for *given* expected returns $E(R)$.

Thus, the total return on investors portfolio is $\overline{a}_f R_f + \overline{a}'R$, where the latter addendum is the return on the risky portfolio, which we denote by R_m. Of course, as investors have the same preferences, they will also hold the same risky portfolio; in other words, R_m is the return on the risky market portfolio. Note also that the assumption of CARA preferences implies that the amount invested in each risky asset is independent from wealth. Hence, if investors were heterogeneous in their level of wealth, they would buy the same amounts of risky assets and different amounts of the risk free asset, the latter depending on their level of wealth.

In order to obtain the standard formulation of the CAPM, note that: $Cov(R, R_m) = Cov(R, \overline{a}'R) = \Sigma\overline{a}$. Denote with R^{ex} the vector of the excess returns on each asset, i.e. $R^{ex} = E(R) - \underline{1}R_f$; analogously, denote with R_m^{ex} the excess return to the market portfolio, i.e. $R_m^{ex} = E(R_m) - R_f$. Then, rearranging expression (10), we obtain:

$$R^{ex} = \gamma \cdot Cov(R, R_m) \ . \qquad (11)$$

3.2 The rank effect

We can now verify whether Propositions 1 and 2 hold; i.e. whether changes in the risk aversion parameter yield a rank effect (Proposition 1), whilst changes in risk do not cause a rank effect (Proposition 2).

We have noted above that the optimal coefficients $\overline{a}$ defined by (10), from which equation (11) originates, represent the unique solution of the investors' problem (9) for given parameters R_f, γ and Σ, and for given expected returns. Now suppose that investors hold the optimal portfolio $(\overline{a}_f, \overline{a}')$ with rate of return $\overline{a}_f R_f + \overline{a}' R$. We can ask what happens when one parameter changes. In equation (10), optimal quantities are obtained for given prices. Then, in this model only one between prices and quantities can change. Of course, it is reasonable to assume that, for a given optimal allocation $(\overline{a}_f, \overline{a})$, the adjustment after a change in a parameter will occur through prices – i.e. through the excess returns R_i^{ex}. Recall, also, that for each asset i an increase (decrease) in R_i^{ex} occurs through a decrease (increase) in the asset price P_i. In other words, after a parameter change quantities remain fixed, equal to $\overline{a}$, and prices, i.e. excess returns, adjust.

A preliminary step to verify whether Propositions 1 and 2 hold concerns the definition of the riskiness of each asset i. Misina (2003) defines it as

$$\lambda_i = Cov\left(R_i, R_m\right) \ . \tag{12}$$

It is immediately clear that, with definition (12), the riskiness of some assets may be negative. Even if one is ready to accept this incongruity, we can see that, in general, the RAI does not appear to provide a reliable measure of the risk aversion.

Given (12), we can rewrite equation (11) as:

$$R_i^{ex} = \gamma \lambda_i \ .$$

Hence, consider a change in the risk aversion parameter γ:

$$\frac{\partial R_i^{ex}}{\partial \gamma} = \lambda_i \ .$$

Thus, Proposition 1 is established in this model.

Now consider a change in the riskiness λ_i:

$$\frac{\partial R_i^{ex}}{\partial \lambda_i} = \gamma$$

It would seem that Proposition 2 is established, because $\partial R_i^{ex} / \partial \lambda_i$ are constant for any i; then, a simultaneous increase in the riskiness of all assets does not seem to yield a rank effect. However, as Misina (2003) points out, the result that the derivatives of each asset's excess returns with respect to its riskiness are constant does not necessarily establish Proposition 2: one has to consider explicitly which parameter has caused the increase in assets' riskiness.

In order to prove that Proposition 2 does not hold, we just need to provide a counterexample, which we borrow from Misina (2003).

Example. *Assume that there are only two assets, denoted with i and j, with variances σ_i^2 and σ_j^2 and covariance σ_{ij}. The CAPM is:*

$$\begin{aligned} R_i^{ex} &= \gamma \left(\overline{a}_i \sigma_i^2 + \overline{a}_j \sigma_{ij} \right) \\ R_i^{ex} &= \gamma \left(\overline{a}_j \sigma_j^2 + \overline{a}_i \sigma_{ij} \right) \ . \end{aligned}$$

Suppose that the riskiness of asset i changes because the covariance of asset i with asset j increases; then:

$$\frac{\partial R_i^{ex}}{\partial \sigma_{ij}} = \gamma \overline{a}_j \ .$$

Clearly, the change in covariance will also affect the riskiness of asset j; therefore:

$$\frac{\partial R_j^{ex}}{\partial \sigma_{ij}} = \gamma \overline{a}_i \ .$$

Now if $\overline{a}_i \neq \overline{a}_j$, then $\frac{\partial R_i^{ex}}{\partial \sigma_{ij}} \neq \frac{\partial R_j^{ex}}{\partial \sigma_{ij}}$, which gives rise to a rank effect.

This example highlights a general problem (see Misina (2003, page 12)): changes in the riskiness of one asset will affect expected returns also on other assets and will, in turn, be affected by changes in the riskiness of other assets. The previous example shows that this type of dependence may give origin to a rank effect, unless all assets are equally weighted ($a_i = a_j \ \forall (i, j)$)

An obvious way to preclude the possibility of these patterns is to assume that asset returns are independent. In this case, the CAPM becomes:

$$R_i^{ex} = \gamma \overline{a}_i \sigma_i^2 \ ,$$

where $\sigma_i^2 = Var(R_i)$. In this model, $\lambda_i = \overline{a}_i \sigma_i^2$. It is immediately clear that a change in risk aversion yields a rank effect:

$$\frac{\partial R_i^{ex}}{\partial \gamma} = \lambda_i \ .$$

Consider, however, a change in σ_i^2:

$$\frac{\partial R_i^{ex}}{\partial \sigma_i^2} = \gamma \overline{a}_i \ ,$$

which may yield a rank effect; for instance, if there are only two assets, say assets i and j, then a simultaneous change in their riskiness yields a rank effect as long as $\overline{a}_i \neq \overline{a}_j$. Thus, even with independent returns, the RAI cannot discriminate between a change in risk aversion and a simultaneous change in the riskiness of all assets, unless $a_i = a_j \ \forall (i, j)$.

Another possibility is to consider independent returns and a idiosyncratic shock. We can show that the rank correlation may be affected also in this case. However, if the cross section of assets is sufficiently large (as it should be in the CAPM), it is reasonable to presume that the change in correlation is small. In fact, consider a idiosyncratic shock to a single asset, say asset i, and suppose that assets i and j are such that $\lambda_i > \lambda_j$ while $R_i^{ex} < R_j^{ex}$. If $\overline{a} > 0$, an increase in σ_i^2 causes an increase in λ_i that, therefore, does not change the inequality $\lambda_i > \lambda_j$; in addition, it causes an increase in R_i^{ex} that can reverse the relationship with R_j^{ex} into $R_i^{ex} > R_j^{ex}$, thereby increasing the rank correlation. Now, since asset i is the only asset for which we observe some change (in both its riskiness and its return), we may expect that, if the cross section of assets is large, the change in the rank correlation is rather small.

Finally, note that an alternative way of preceding is to change the definition of riskiness of asset i. However, if $\lambda_i \neq Cov\left(R_i^{ex}, R_m^{ex}\right)$, then equation (11) shows that Proposition 1 may not be established. E.g., suppose one puts $\lambda_i = \sigma_i^2$; then

$$R_i^{ex} = \gamma \left(\overline{a}_i \lambda_i + \sum_{j \neq i} \overline{a}_j \sigma_{ij} \right) .$$

Therefore, $\partial R_i^{ex} / \partial \gamma$ is no longer proportional to λ_i, denying Proposition 1.

3.3 A summary

The previous analysis has focused on the properties of the RAI in the context of the standard CAPM. This analysis shows that one can prove that the RAI distinguishes between changes in risk and changes in risk aversion only under very restrictive assumptions.

First, drawing on Misina (2003) we have shown that Proposition 1 and 2 do not hold if asset returns covariate and the assets in the market portfolio are not equally weighted. Second, to account for the case of unequally weighted assets we have considered the assumptions of independent returns. In this case, we have proven that the RAI cannot distinguish between a change in risk aversion and a simultaneous change in the riskiness of all assets, unlike the claim of Kumar and Persaud (2001, 2002). Next, with independent returns and unequally weighted assets, if the shock to asset riskiness is idiosyncratic we need the further assumption that the cross section of assets is sufficiently large.

Thus, given that the assumption of equally weighted assets is too restrictive, proving Propositions 1 and 2 in the context of a CAPM requires independent returns, idiosyncratic shocks to assets' riskiness, and a large cross section of assets.

4 An application

In this section we use a modified version of the classical methodology build by Fama and MacBeth (1973) to estimate the risk aversion parameter from a standard CAPM. Departing somewhat by the traditional practice in which risk aversion is estimated on a cross section of assets referred to long time periods (generally, from 5 to 10 years), here we run our estimates on a cross section of monthly data.[13] Thus, we obtain a short-term measure of risk aversion, that we compare with the RAI. Our results will show that, in our sample, these estimates and the RAI are almost equivalent.

4.1 Methodology

Our objective is to estimate the model:

$$R_{i,t}^{ex} = \gamma_t \lambda_{i,t} \tag{13}$$

on a cross section of assets i, $i = 1, ..., n$, at time t. Hence, we want to obtain an estimate of the risk aversion parameter γ_t, for each time t.[14] The assets included in our analysis are the stock market indices of ten sectors of the United States, the euro area and Japan, while time periods are calendar months.[15] To estimate equation (13), we need the excess returns $R_{i,t}^{ex}$ and the regressors $\lambda_{i,t}$ – which represent the covariance of asset i with the market portfolio at time t.

In order to determine the excess returns $R_{i,t}^{ex}$ – that, we recall, are equal to $E(R_{i,t}) - R_f$ – we have to find the expected returns $E(R_{i,t})$. The standard practice followed by the literature is to use rational expectations and assume that:

$$R_{i,t} - E(R_{i,t}) = \varepsilon_{i,t}$$

where $\varepsilon_{i,t}$ is a white noise. Therefore, model (13) becomes:

$$R_{i,t} - R_f = \gamma_t \lambda_{i,t} + \varepsilon_{i,t} \; . \tag{14}$$

In order to obtain the regressor $\lambda_{i,t}$ we use the first step of the "two-pass" procedure of Fama and MacBeth (1973). Hence, for each asset i we

[13] For instance, Black, Jensen and Scholes (1972) consider non-overlapping 5-year periods, Fama and MacBeth (1973) use overlapping periods from 5 to 8 years, Sharpe (1965) uses a single 10-year period.

[14] The parameter γ_t is the *Arrow-Pratt coefficient of absolute risk aversion* in the context of the theoretical model (11); it can be interpreted as an *aggregate relative risk aversion of the economy* in the context of the more general model (20) (see Appendix A.1).

[15] We use end-of-period value-weighted monthly stock indices of 10 sectors (level 3 of the Financial Times' classification) of general indices of the three main areas (United States, the euro area and Japan). Indices are in US dollars. The source is *Thomson Financial Datastream.*

run the rolling regression:

$$R^{ex}_{i,k} = \alpha_i + \beta_{i,t} R^{ex}_{m,k} + \eta_{i,k} \quad ,$$

on a time-series of data with k that goes from $t - H + 1$ to t, where $R^{ex}_{i,k}$ is the excess return on asset i at time k, α_i is an asset-specific constant, $\beta_{i,t}$ is the asset "beta" (namely, it is the covariance between the return on asset i and the return on market portfolio divided by the variance of the market portfolio), $R^{ex}_{m,k}$ is the excess return on the market portfolio, $\eta_{i,k}$ is the residual.[16] We run this regression with a moving window of 36 months (i.e. $H = 36$) and obtain a point estimate of the parameter $\widehat{\beta}_{i,t}$ for each asset i and time period t.[17] Note that the product between the variance of R_m in the H months before period t and $\widehat{\beta}_{i,t}$ provides an estimate of $\lambda_{i,t}$, denoted with $\widehat{\lambda}_{i,t}$.

We can now estimate equation (14) on cross-sectional data. Plugging directly an estimate of $\lambda_{i,t}$ into equation (14) would cause an errors-in-variable problem. Therefore, following the literature, we reduce the cross-sectional variability by including in the regression two other explanatory variables, denoted with z_1 and z_2.[18] These variables are: the logarithm of market capitalization (Schwert (1983)) and the "systematic" skewness (Kraus and Litzenberger (1978)).[19] Hence, for each time period t we estimate:

$$R^{ex}_{i,t} = k_t + \gamma_t \widehat{\lambda}_{i,t} + b_{1,t} z_{1,i,t} + b_{2,t} z_{2,i,t} + \varepsilon_{i,t} \ ,$$

where γ_t is our short-term measure of risk aversion, k_t is a constant, and

[16] For the market portfolio, we use the World Stock Market index, computed by Thomson Financial Datastream. Note that our application only includes stock prices. On this point, the Roll critique (Roll (1977)) pointed that the model's validity may depend on the assets included in the portfolio: the CAPM, in fact, should include all assets, tradable and non-tradable, tangible or intangible, that adds to the world wealth. However, Stambaugh (1982) constructed a number of market portfolios, which included also government bonds, corporate bonds, Treasury bills, real estate and consumer durables, and finds that even when stocks represent only 10 per cent of the market portfolio, inferences about the model are the same as those obtained with a stocks-only index.

[17] The proper specification of the CAPM requires that asset weights in the market portfolio do not change over time. In the World Stock Market index, that we have adopted as a proxy for the market portfolio, weights do change. However, given the large number of assets contained in that index, this is usually considered a good working approximation (see Ferson et al. (1987) for further discussions on this issue).

[18] Recall that the errors-in-variable problem would cause the estimated risk aversion to be smaller than the true one. Two alternative ways to address this problem are: clustering the assets according to their estimated betas (Black, Jensen and Scholes (1972) and Fama and MacBeth (1973)) or using maximum likelihood estimates to avoid the need of separate steps (Gibbons (1982)).

[19] The "sistematic" skewness is introduced in order to account for the possible effect of higher order moments of the utility function of the representative investor. Following Kraus and Litzenberger (1978), we compute it as: $E[(R_i - E(R_i))(R_m - E(R_m))^2]/E[(R_m - E(R_m))^3]$.

$\varepsilon_{i,t}$ is the residual.[20] To address heteroskedasticity, we use the Newey-West estimator (with the Bartlett window) and we cancel the observations outside the interval $\pm 2\sigma$.

Finally, in order to obtain the RAI, for each time t we compute the rank correlation between risks and returns using the estimates of $\lambda_{i,t}$ and the returns $R_{i,t}$. Following Kumar and Persaud (2001, 2002), among the possible measures of rank correlation we choose Spearman's. Therefore, the RAI is: $\rho^s\left(R_{i,t}, \widehat{\lambda}_{i,t}\right)$.

4.2 Results

The results of our estimates are illustrated in Figure 3. In order to get smoother series, for both indicators we present a (centered) moving average of five terms.

Both indicators show two phases of sharp increase in risk aversion: the former goes form the beginning of 1998 to the first half of 1999, the second starts at the end of the year 2000 and vanishes during the year 2003. In 2003, in particular, risk aversion either stabilized (according to the CAPM indicator) or decreased (according to the RAI).

Although these results seem quite reasonable, we should note that our short-term indicator often takes negative values, a well-know problem at the monthly frequencies (see IMF (2003, Box 3.1)). In addition, even when the estimates are positive, they are still rather small relative to the values that one could obtain at lower frequencies. On the one hand, this outcome could be due to an unsatisfactory solution of the errors-in-variable problem that we have discussed above. On the other hand, it could be due to more structural problems that occur at these frequencies (for instance, negative returns are dominant in some sub-periods).

[20] Recall that according to the Sharpe-Lintner version of the CAPM, k_t should be zero. The Black version, instead, allows for $k_t \neq 0$; in this case, $k_t + R_{f,t}$ is the return on the zero-covariance portfolio.

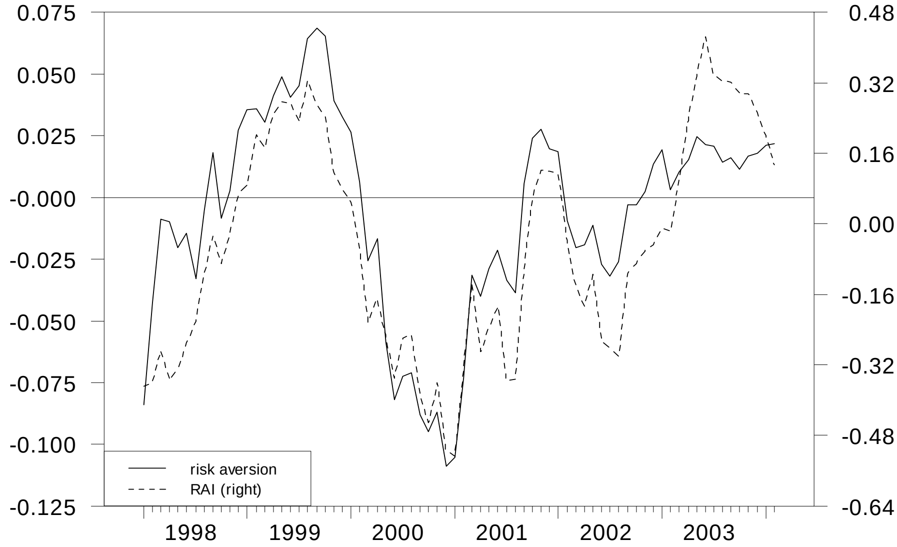

Figure 3: RAI and short-term risk aversion from a CAPM

In addition, the behavior of the risk aversion illustrated in Figure 3 does not appear to be robust and, therefore, results needs to be taken with some caution. In particular, results are not robust to the assets included in the regression (while including bond prices does not modify the results, adding stock indices of emerging countries often alters the pattern of risk aversion), the choice of the frequency (monthly, quarterly and annual returns provide different results), the method chosen to address the errors-in-variable problem.

4.3 A comparison

The most striking feature of Figure 3 is the similarity of the behavior of the two indicators. In particular, the correlation between the RAI and our short-term measure of risk aversion obtained by the CAPM is 0.8. This result is robust to essentially any choice made in our application.[21] Thus, despite the theoretical differences discussed in Section 4, the two indicators provide essentially the same results. We can explain this apparent conundrum by focusing on the statistical properties of both indicators.

[21] Specifically, we have performed a sensitivity analysis that included the following changes: the assets included in the analysis (we have included bond prices of industrial countries and stock prices of emerging countries); the explanatory variables z_i selected to address the errors-in-variable problem (we experimented a model without z_1 and/or z_2); the size of the moving window in the time-series regression (besides $H = 36$, we have also tried $H = 24$ and $H = 60$).

The OLS estimate of γ_t in model (14) are:

$$\widehat{\gamma}_t = \frac{Cov(R_t, \widehat{\lambda}_t)}{Var(\widehat{\lambda}_t)} \ ,$$

that we can rewrite as:

$$\widehat{\gamma}_t = \rho(R_t, \widehat{\lambda}_t) \cdot c_t \ , \tag{15}$$

where $\rho(R_t, \widehat{\lambda}_t)$ denotes the linear correlation between R_t and $\widehat{\lambda}_t$ for the cross section of assets at time t, and:

$$c_t = \left[\frac{Var(R_t)}{Var(\widehat{\lambda}_t)} \right]^{1/2} \ .$$

Now recall that the Spearman's rank correlation ρ^s may be regarded as a robust statistic for the linear correlation ρ. For instance, if $\rho = 1$ ($\rho = -1$) then $\rho^s = 1$ ($\rho^s = -1$). In addition, any change in the rank that increases (decreases) ρ^s also increases (decreases) ρ (see, e.g., Huber (1981, Chapter 8) and Stuart and Ord (1991, Chapter 26)). Thus, if the ratio c_t, that we call variance ratio, is sufficiently stable with respect to $\rho(R_t, \widehat{\lambda}_t)$, then $\widehat{\gamma}_t$ is proportional to $\rho(R_t, \widehat{\lambda}_t)$. In that case, being $\rho^s(R_t, \widehat{\lambda}_t) \simeq \rho(R_t, \widehat{\lambda}_t)$, the indicators $\widehat{\gamma}_t$ and $\rho^s(R_t, \widehat{\lambda}_t)$ would be almost equivalent.

The ratio c_t is indeed stable in our sample. Figure 4, in fact, shows the variance of c_t and the variance of $\rho^s(R_t, \widehat{\lambda}_t)$ (that is approximately equal to that of $\rho(R_t, \widehat{\lambda}_t)$). When the former is sufficiently small with respect to the latter, this means that the behavior of $\widehat{\gamma}_t$ is mainly driven by the correlation coefficient (see equation (15)). Figure 4 shows that the variance of c_t is about one thousand times smaller than the variance of $\rho^s(R_t, \widehat{\lambda}_t)$. Therefore, c_t is sufficiently stable and this explains the equivalence of the results obtained with the two indicators.

Thus, although the theoretical analysis performed in Sections 3 and 4 suggested that one needs very restrictive assumptions to prove that the RAI is an indicator of risk aversion, the preliminary evidence illustrated in Figure 3 shows that the RAI could be a good proxy of a standard CAPM-based risk aversion indicator. Being a robust measure of the linear correlation between $R_{i,t}$ and $\lambda_{i,t}$, as long as the variance ratio c_t is stable the RAI is, de facto, a proxy of the risk aversion parameter of the CAPM. Further studies, however, are needed along two main directions. First, one should verify whether c_t is approximately constant also in other applications. Second, a Montecarlo simulation should try to specify under what conditions one can benefit from the robust estimates provided by ρ^s.

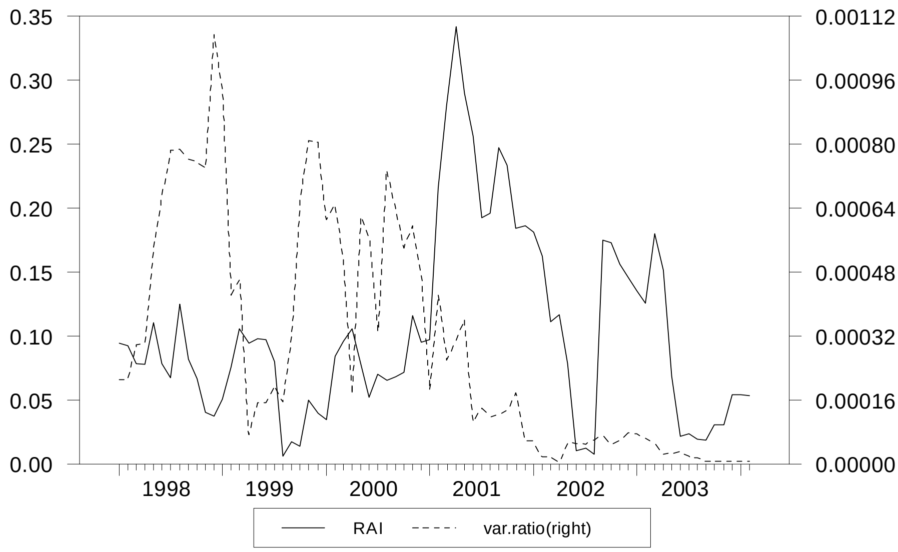

Figure 4: Variances of the RAI and of the variance ratio

5 Conclusion

We provide a theoretical and an empirical analysis of the Risk Appetite Index, recently proposed by Kumar and Persaud (2001, 2002). This indicator aims at measuring the degree of risk aversion of the representative investor.

In our theoretical analysis, we prove that the RAI cannot correctly identify risk aversion neither in the *ad hoc* pricing model proposed by Kumar and Persaud (2001), nor, in general, in the standard CAPM. In particular, in the context of the CAPM, we show that the RAI measures investors' risk aversion only under very restrictive assumptions. Specifically, if the assets in the market portfolio are not equally weighted, the RAI requires independent asset returns, idiosyncratic shocks to assets' riskiness, and a sufficiently large cross section of assets.

Despite these assumptions are theoretically very restrictive, it is necessary to evaluate empirically whether they significantly affect the behavior of the RAI. This can be done by comparing the RAI with a measure of risk aversion "in the short-term", which we derive from the estimation of a CAPM – a model that does not require either independent returns or specific assumptions on the nature of the shocks. We consider an application to the behavior of international asset prices since 1998 as a case-study. The comparison shows that results are surprisingly similar. Therefore, by focusing on its statistical properties, we prove that under a certain condition the RAI may be regarded as a robust estimator of the risk aversion parameter in a CAPM. This condition requires that the ratio between the variance of assets' returns and the variance of assets' riskiness is approximately constant – a condition that is met in our sample. However, further studies are needed to verify whether that condition also holds in other applications, and to identify the shocks that may induce to prefer a robust estimator like the RAI.

A Appendix

A.1 Heterogenous agents with risk averse preferences

Section 4 was based on the restrictive assumptions that agents are identical, and that their preferences are given by CARA utility functions. Following Huang and Litzenberger (1988), here we discuss a more general setting where agents are heterogeneous in both preferences and wealth. In this more general model, our conclusions on the RAI will remain essentially unchanged once that we focus on an appropriately specified *global risk aversion* parameter. This is because this setting leads to version of the CAPM which has the same functional form as equation (11).

We assume that preferences are represented by increasing and concave utility functions. With respect to the previous section, we will maintain the assumption that asset returns are multivariate normally distributed. In addition, we will redefine the problem in terms of shares of wealth rather that in value terms. Namely, denote by $a_{i,h}$ the *amount* of wealth invested in asset i by investor h; then, the *share* of investor h's wealth, W_h, invested in such asset is:

$$w_{i,h} = \frac{a_{i,h}}{W_h}$$

The total wealth of the N investors in the economy is $W_m = \sum_{h=1}^{N} W_h$. In equilibrium, the total wealth W_m is equal to the total value of the assets. We denote with $w_{i,m}$ the portfolio weight of asset i in the market portfolio, namely:

$$w_{i,m} = \sum_{h=1}^{N} w_{i,h} \frac{W_h}{W_m}$$

Using the budget constraint (6), we can rewrite the consumption (7) of the investor h with wealth W_h as:

$$C_h = \left(W_h - \sum_{i=1}^{n} w_{i,h} W_h \right) R_f + \sum_{i=1}^{n} w_{i,h} W_h R_i = W_h \left[R_f + \sum_{i=1}^{n} w_{i,h} \left(R_i - R_f \right) \right] \tag{16}$$

The maximization problem of such investor, whose preferences are represented by the increasing and concave function $u_h \in \mathbb{C}^2$,becomes:

$$\max_{a'} \left\{ E \left[u_h \left(W_h R_f + \sum_{i=1}^{n} w_{i,h} W_h \left(R_i - R_f \right) \right) \right] \right\} . \tag{17}$$

Let us assume that a solution to (17) exists. Since u_h is concave, the first order (necessary) conditions are also sufficient and are:

$$E\left[u_h'\left(W_h R_f + \sum_{i=1}^{n} \overline{w}_{i,h} W_h \left(R_i - R_f\right)\right) \cdot \left(R_i - R_f\right)\right] = 0 \quad \forall i = 1, ..., n, \tag{18}$$

where the coefficients $\overline{w}_{i,h}$ are the optimal shares of wealth invested in asset i by individual h. The optimal consumption of such investor then is:

$$\overline{C}_h = W_h R_f + \sum_{i=1}^{n} \overline{w}_{i,h} W_h \left(R_i - R_f\right) \ .$$

Using the definition of covariance, we can rewrite condition (18) as:

$$E\left[u_h'\left(\overline{C}_h\right)\right] \cdot E\left(R_i - R_f\right) + Cov\left[u_h'\left(\overline{C}_h\right), R_i\right] = 0.$$

In addition, using Stein's lemma we find:

$$Cov\left[u_h'\left(\overline{C}_h\right), R_i\right] = E\left[u_h''\left(\overline{C}_h\right)\right] \cdot Cov\left(\overline{C}_h, R_i\right)$$

which we can substitute back into the previous expression and, recalling that $E\left(R_i - R_f\right) = R_i^{ex}$, we have:

$$E\left[u_h'\left(\overline{C}_h\right)\right] \cdot R_i^{ex} = -E\left[u_h''\left(\overline{C}_h\right)\right] \cdot Cov\left(\overline{C}_h, R_i\right) \ . \tag{19}$$

We can now define the *global absolute risk aversion* of the investor h as:

$$\Gamma_h = -\frac{E\left[u_h''\left(\overline{C}_h\right)\right]}{E\left[u_h'\left(\overline{C}_h\right)\right]} \ .$$

Dividing both terms of (19) by $E\left[u_h''\left(\overline{C}_h\right)\right]$, summing across the N investors and rearranging we obtain:

$$R_i^{ex} = \left(\sum_{h=1}^{N} \Gamma_h^{-1}\right)^{-1} \cdot \sum_{h=1}^{N} Cov\left(\overline{C}_h, R_i\right).$$

Note that the first term in brackets of the right hand side is the harmonic mean of the investors' global absolute risk aversions. Moreover we can write:

$$\begin{aligned}
\sum_{h=1}^{N} Cov\left(\overline{C}_h, R_i\right) &= Cov\left(\sum_{h=1}^{N} \overline{C}_h, R_i\right) \\
&= Cov\left\{\left[\sum_{h=1}^{N} W_h R_f + \sum_{h=1}^{N}\sum_{i=1}^{n} \overline{w}_{i,h} W_h \left(R_i - R_f\right)\right], R_i\right\} \\
&= Cov\left(\sum_{i=1}^{n}\sum_{h=1}^{N} \overline{w}_{i,h} W_h R_i, R_i\right) \\
&= Cov\left(W_m \sum_{i=1}^{n} \overline{w}_{i,m} R_i, R_i\right) \\
&= W_m Cov\left(R_m, R_i\right).
\end{aligned}$$

Substituting into the previous expression and putting

$$\Gamma = W_m \left(\sum_{h=1}^{N} \Gamma_h^{-1}\right)^{-1} ,$$

which represent a sort of *aggregate relative risk aversion of the economy*, we get:

$$R_i^{ex} = \Gamma \cdot Cov\left(R_i, R_m\right) , \tag{20}$$

that has exactly the same functional form as (11).

Equation (11) shows that our conclusions on the risk appetite index do not depend on the specific hypothesis made for the investors' preferences: as long as asset returns are multivariate normally distributed (a hypothesis that cannot be rejected for monthly and quarterly data) they hold for any non satiated and risk averse preferences.

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