Quantitative Forecast Model for the Application of the Black-Litterman Approach

Franziska Becker* und Marc Gürtler**

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The estimation of expected security returns is one of the major tasks for the practical implementation of the Markowitz optimization. Against this background, in 1992 Black and Litterman developed an approach based on (theoretical established) expected equilibrium returns which also accounts for subjective investors' views. In contrast to historical estimated returns, which lead to extreme asset weights within the Markowitz optimization, the Black-Litterman model generally results in balanced portfolio weights. However, the existence of investors' views is crucial for the Black-Litterman model and with absent views no active portfolio management is possible. Moreover problems with the implementation of the model arise, as analysts' forecasts are typically not available in the way they are needed for the Black-Litterman-approach. In this context we present how (publicly available) analysts' dividend forecasts can be used to determine an a-priori-estimation of the expected returns and how they can be integrated into the Black-Litterman model. For this purpose confidences of the investors' views are determined from the number of analysts' forecasts as well as from a Monte-Carlo simulation. After introducing our two methods of view generation, we examine the effects of the Black-Litterman approach on portfolio weights in an empirical study. Finally, the performance of the Black-Litterman model is compared to alternative portfolio allocation strategies in an out-of-sample study that has not been presented in literature before to the best of our knowledge.

- All authors will attend the conference.
- Franziska Becker will present the paper.

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1 Introduction

Portfolio theory focuses on the question, how an investor can at the best invest money in assets that are available on the market. In this realm, basic work has been done by Harry Markowitz (1952, 1959). He suggests that, concerning the portfolio composition of N assets, risk-averse investors should comply with expected value μ_P and variance σ_P^2 of the total portfolio return. Assuming constant absolute risk aversion, the following preference function is being maximized:[1]

$$\phi = \mu_P - \frac{\lambda}{2}\sigma_P^2 = X'\mu - \frac{\lambda}{2}X'\Sigma X \qquad \to \max_X ! \tag{1}$$

Here, X is the N-vector of asset weights: $X = (x_1, \ldots, x_N)'$, Σ describes the $N \times N$ variance-covariance matrix of asset returns, μ describes the expected return vector and λ the risk aversion parameter.

One problem of portfolio theory is the estimation of the required parameters: expected values, variances and covariances of individual asset returns. The input parameters are being estimated and the optimization procedure assumes that they are the true values of the return moments. However, future returns are random variables and their true values are different from their expected values. In many cases, extreme short sale positions result from the optimization algorithms or, if portfolio weights are bounded between zero and one, a lot of assets will not be incorporated into the optimal portfolio. If the parameters have been correctly estimated, the resulting portfolio weights obviously lead to the highest preference level. However, if the parameters deviate from the forecasts, the only slightly diversified portfolio could achieve a poor preference level, if the respective chosen assets develop suboptimal. Furthermore, the optimal weight vector

$$X = \frac{1}{\lambda}\Sigma^{-1}\mu \tag{2}$$

that has been determined from maximization problem (1) is very sensitive to the incoming parameters. Marginal changes in the expected returns can already result in great differences in

[1] A utility function with constant absolute risk aversion and normally distributed returns involves this certainty equivalent, which can be maximized instead of the expected utility. See Anderson/Bancroft (1952), p. 37; Freund (1956).

the portfolio weights.[2] Regarding changes in variances and covariances, the sensitivity of weights is not as strong as with changes in the expected returns.[3]

The procedure that has been developed by Black and Litterman (1992) combines equilibrium expected returns with subjective views or prior beliefs of investors and has widely been accepted in practice.[4] If views are only proposed for some assets, only these weights vary from the associated market weights. As a matter of fact, predictions (views) first have to be made in order to deviate from the intuitive market weights. If own predictions cannot be made, the Black-Litterman procedure does not provide the possibility of applying active portfolio management. Even if individual predictions can be made for asset returns, these are based on single opinions – either an own prediction or an analyst's forecast. In this contribution, we describe two possibilities of how views can be quantified for the Black-Litterman procedure with the help of valuation models and a multitude of analysts' forecasts. Furthermore, we apply an to our knowledge unprecedented out-of-sample performance analysis of the Black-Litterman procedure.

Herold (2003) describes an approach, in which the Black-Litterman procedure can be employed with qualitative analysts' forecasts. In the framework of active portfolio management, the optimal portfolio is chosen in a way to reach a given tracking error. However, the composition of the portfolio is based on one analyst's forecasts and not on a number of analysts' forecasts. Jones (2007) generates the views on the basis of a factor model. The view confidences are determined from historical return variances and covariances. In our contribution, we do not use the historical variance-covariance matrix for the calculation of the confidence probabilities but the confidence in views is directly deduced from analysts' forecasts.

To begin with, in chapter 2 the basics of the Black-Litterman procedure are explicated. Afterwards, our both methods for generating views for the Black-Litterman procedure are described in chapter 3. For this purpose, in section 3.1 we resort to the number of analysts' forecasts, and in section 3.2 a Monte-Carlo simulation for the creation of prior beliefs is described. In the empirical examination in chapter 4, input and output parameters of the methods described in chapter 3 are analyzed with the help of real capital market data. Finally, an out-

[2] A sensitivity study can be found for instance in Kempf/Memmel (2002).
[3] See Chopra/Ziemba (1993).
[4] Cf. Herold (2004), p. 285. Goldman Sachs employed for instance the Black-Litterman model. See He/Litterman (1999).

of-sample performance of the methods is determined in section 4.3. Chapter 5 summarizes our findings.

2 The Black-Litterman Procedure

2.1 Basic Idea

In their model, Black and Litterman (BL) (1992) combine equilibrium expected returns with investor views, in order to calculate a new vector of expected returns μ_{BL} which is then integrated into the Markowitz optimization. The purpose of optimization with these new input parameters is to gain relatively balanced portfolios without the implementation of long-only constraints or other restrictions. The equilibrium expected returns are derived from the market portfolio via a reverse optimization.[5]

First of all, it is assumed that the return vector r of N regarded assets is normally distributed with $N \times 1$ expected return vector μ and $N \times N$ variance-covariance matrix Σ:

$$r \sim N(\mu, \Sigma). \tag{3}$$

The variance-covariance matrix is supposed to be known[6] and is estimated traditionally with the unbiased historical estimator. However, the vector of expected returns is a random vector that follows a normal distribution with known parameters Π, τ and Σ:

$$\mu \sim N(\Pi, \tau\Sigma). \tag{4}$$

Π is the $N \times 1$ expected return vector of the market portfolio and serves as a neutral reference point. The calculation of Π will be described in section 2.2. The variance-covariance matrix of the expected return vector μ is chosen as a multiple of the variance-covariance matrix of returns r with scaling factor $\tau > 0$. This factor is not predetermined in the model but since Black and Litterman assume that the uncertainty about the expected return is smaller than the uncertainty (in this case the variance-covariance matrix) of returns, they suggest to choose a relatively small τ.[7]

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[5] Strictly speaking a proxy for the market portfolio is chosen, as the market portfolio could not be reproduced. See Roll (1977).

[6] Cf. Black/Litterman (1992), p. 43.

[7] Cf. Black/Litterman (1992), p. 34.

2.2 The Neutral Reference Point

Black and Litterman (1992) use the market portfolio as a starting point for the expected returns.[8] Thereby, the expected returns of the market portfolio are calculated via a reverse optimization. By assuming an investor i with preference function (1) and solving equation (2) for the expected return vector μ,

$$\mu = \lambda_i \cdot \Sigma \cdot X. \tag{5}$$

is obtained. Thus, the vector of expected asset returns μ is a multiple of the product of variance-covariance matrix Σ and the asset weight vector. The optimal asset weights x_j are given through the proportion of the market capitalization of the single security to the overall market capitalization for a given market portfolio of N assets:

$$x_j = \frac{\eta_j \cdot P_j}{\sum_{i=1}^{N} \eta_i \cdot P_i}. \tag{6}$$

Here, η_j denotes the number of shares of security j on the capital market and P_j the current price of security j. Finally, the only missing component for the calculation of μ by using equation (5) is the risk aversion λ. Taking the capital asset pricing model (CAPM) as a basis, the risk aversion parameter is determined as the market price of risk.[9] If parameter λ_i is undetermined, the relative expected returns can still be identified with the help of (5) and (6). If the calculated share returns μ are used in a Markowitz Optimization, as expected, weights of the market portfolio are obtained for any λ_i.

2.3 Investors' Views

As an additional opinion, investors can express k views or prior beliefs about the returns in the following form:

$$Q = P'\mu + \varepsilon, \text{mit } \varepsilon \sim N(0, \Omega). \tag{7}$$

Q is a $k \times 1$ vector of k forecasted return expectations and P is a known $N \times k$ view matrix. The $k \times 1$ vector ε follows a normal distribution with expected value zero and a variance-covariance matrix Ω (view confidence matrix). Following Black/Litterman, Ω is a diagonal

[8] For the selection of an adequate reference return, Black and Litterman also discuss historical expected returns of the individual securities, equal expected returns within the asset classes as well as risk adjusted expected returns. However, they arrive at the conclusion that market returns in contrast to the other strategies generate the most balanced and intuitive portfolios.

[9] Cf. Pitts (1998), pp. 123. Drobetz (2003) sets the risk aversion parameter to be one.

matrix,[10] thus, the views are independent.[11] This assumption should be regarded in a rather critical way, since views in one industry sector are surely not independent of each other. The entries on the diagonal identify the investor's confidence concerning the respective view: the bigger the entry, the less certain is the investor concerning his forecast. The beliefs can be indicated absolute as well as relative and there is no need for a view for every asset. The setting up of equation (7) will be exemplified via the following example.

Example 2.1:

(a) The investor is sure that the expected return of share 1 will amount to 20 %.

(b) With a probability of 70 %, the investor believes that the difference of the expected returns between an equally weighted portfolio of share 1 and share 3 and an equally weighted portfolio of share 4, 5 and 6 will amount to 5 % to 7 %.

For an investment horizon of 6 stocks in the form of equation (7), these views are set up in the following way:

$$\begin{pmatrix} 0,2 \\ 0,06 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0,5 & 0 & 0,5 & -0,33 & -0,33 & -0,33 \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}, \tag{8}$$

$$\text{with } \varepsilon \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 9,317 \cdot 10^{-5} \end{pmatrix}\right).$$

The first entry of vector Q contains the expected return of share 1 with 20 %. View (a) is specified in absolute terms. For this purpose, the first entry (for the first stock) of the first view row in matrix P' contains the value one. Since the investor is sure of this view, variance ω_{11} of error term ε_1 is zero. Forecast (b) is a relative view. The average of the return interval is the second entry of vector Q and the second row of matrix P' is related to view (b). Since the relative comparison is related to portfolios between stocks 1, 3 and stocks 4, 5 and 6, the re-

[10] Cf. Black/Litterman (1992), p. 35.

[11] If the off-diagonal entries are zero, it can be deduced that the views are uncorrelated. By assuming a normal distribution the uncorrelated entries correspond to independent entries. See for this purpose Fahrmeir et al. (1997), p. 349.

spective column contains 1/2 for stocks 1 and 3 and -1/3 for stocks 4, 5 and 6.[12] The view confidence is expressed through a probability that has to be converted to a variance. The more certain an investor is concerning his belief, the smaller the variance is at the respective position in matrix Ω. Exemplarily, entry ω_{22} is derived in the appendix. The second column of matrix P exclusively contains zeroes, since neither absolute nor relative forecasts have been made for stock 2.

In order to derive the distribution of Q, we need the distribution of the expected returns μ that have been assumed normally distributed in (4): $\mu \sim N(\Pi, \tau\Sigma)$. Overall, for the distribution of the investor's prior expected returns we get:

$$Q \sim N\left(P'\Pi, P'\tau\Sigma P + \Omega\right). \tag{9}$$

2.4 Combination of Both Perspectives

With the help of the investors' views, not sole information about the distribution of returns $E(r) = \Pi$ and $Cov(r, r') = \Sigma$ is used for the portfolio optimization but the conditional distribution of the returns, with the investors' views given. The expected return vector and variance-covariance matrix of the conditional distribution result in:[13]

$$E(r \mid Q) \equiv \mu_{BL} = \Pi + \tau\Sigma P\left(P'\Sigma P\tau + \Omega\right)^{-1}\left(Q - P'\Pi\right), \tag{10}$$

$$Var(r \mid Q) = \Sigma + \tau\Sigma - \tau^2\Sigma P\left(P'\Sigma P\tau + \Omega\right)^{-1} P'\Sigma. \tag{11}$$

The expected return vector of the conditional distribution is a matrix-weighted average of expected values of the individual distributions. In the literature, an alternative notation is often used:

$$E(r \mid Q) = \left[(\tau\Sigma)^{-1} + P\Omega^{-1}P'\right]^{-1} \cdot \left[(\tau\Sigma)^{-1}\Pi + P\Omega^{-1}Q\right], \tag{12}$$

which can be transformed into (10) without further ado.[14]

If the uncertainty about own beliefs converges to infinity, the investor only trusts in the equilibrium expected returns:

[12] Similarly, relative views could be generated with value-weighted portfolio weights. Cf. Drobetz (2003), pp. 220.
[13] Cf. Herold (2004), p. 289, Memmel (2004), p. 119.
[14] Cf. Pitts (1998), p. 144, Black/Litterman (1992), p. 42; Lindley/Smith (1972).

$$\lim_{\Omega \to \infty} E(r \mid Q) = \Pi . \tag{13}$$

Similarly, only market expectations are trusted, if the uncertainty about expected returns converges to zero:

$$\lim_{\tau \to 0} E(r \mid Q) = \Pi . \tag{14}$$

However, if the investor is certain about his forecasts, i.e. $\Omega = 0$, the following conditional expected value is obtained:

$$E(r \mid Q) = \Pi + \tau\Sigma P\left(P'\Sigma P\tau\right)^{-1}\left(Q - P'\Pi\right). \tag{15}$$

Since relative forecasts could be made between stocks, expected BL-returns do not directly result from investor's prior returns Q. However, if absolute views with confidence probability one are made for single stocks i, the respective expected BL-return $\mu_{BL,i}$ is equal to the view return Q_i.

Using the conditional expected value of the Black-Litterman model for the Markowitz-optimization, more balanced portfolios result. If there is only an absolute forecast for one asset, still all further expected returns of the conditional distribution are changed by the covariance matrix of returns Σ. But when substituting the expected return vector of the conditional distribution into a portfolio optimization, unchanged portfolio weights for all shares for which no forecasts have been made are obtained. However, an adapted weight for the asset for which a view has been given is calculated. This is, of course, only true for the case of non-binding restrictions. However, if the sum of weights is restricted to be one, the relative weights for assets without views remain unchanged.[15]

2.5 Problems With the Application of the Black-Litterman Model

As has been described before, for the Black-Litterman model, own views about return expectations of the investment horizon are required. Institutional investors are often only familiar with single market segments, as, for example, the segment "Stocks Europe", but do not have expertise in the field of "Stocks Asia". However, if these investors want to invest in segments which they cannot make forecasts about, using the Black-Litterman model, they would "only" realize the market portfolio in the particular field. Private investors, who possibly do not have any information about stocks, would likewise only realize the market portfolio, which turns

[15] Cf. Herold (2004), pp. 289. Furthermore Herold applies a case study in which the traditional mean-variance optimization of Markowitz is compared to the Black-Litterman model, see Herold (2004), pp. 370.

the Black-Litterman model to be dispensable. Additionally, analysts do not typically make predictions as in example 2.1, thus this possibility of generating forecasts for deviations of the market portfolio weights is not given in reality, either.

In order to make use of analysts' expertise for the formulation of views and to have the possibility of diverging from the market portfolio in less conversant segments, in the following, a model will be described, which generates beliefs for the Black-Litterman model with the help of analysts' dividend forecasts.

3 The Use of Analysts' Forecasts

3.1 Determining Views From the Number of Analysts' Forecasts

Historical expected equity returns have proved to be poor estimators for future returns.[16] Due to the necessity of finding better estimators for future returns, the literature for the determination of implied equity returns on the basis of valuation models has made a rapid development during the last years. Meanwhile, there are many empirical examinations of valuation models (e.g., dividend discount model, discounted cash flow model, residual income model, Ohlson/Jüttner-Nauroth (2005) model), which are based on analyst forecasts concerning several parameters and the models achieve good results.[17] In the following, views based on the dividend discount model according to Williams (1938) and Gordon (1959, 1966) will be derived. Views can be deduced in almost the same manner on the basis of other valuation models.

In the dividend discount model, the expected stock return $\mu_i^{(t)}$ is calculated at a given point in time t on the basis of the market value of equity $EK_i^{(t)}$ of a company i as the internal interest rate for the time series of expected dividend payments. Predominantly, a two-phase model is used. In the first phase of duration T, detailed estimations of the dividends $D_i^{(t)}$ by a company i are available. For the remaining time, a constant growth g_i of dividends is assumed:

$$EK_i^{(t)} = \sum_{\tau=1}^{T} \frac{D_i^{(t+\tau)}}{(1+\mu_i^{(t)})^{\tau}} + \frac{D_i^{(t+T)} \cdot (1+g_i)}{(\mu_i^{(t)} - g_i) \cdot (1+\mu_i^{(t)})^{T}} . \tag{16}$$

We can access the

[16] See Elton (1999), Jorion (1986), Kempf and Memmel (2002).

[17] See for instance Botosan/Plumlee (2005), Courteau et al. (2001), Francis et al. (2000), Gebhardt et al. (2001), Gode/Mohanram (2003), Easton (2004), Daske et al. (2006).

- mean D
- median M
- standard deviation σ_D
- highest estimation of D D_{hi}
- lowest estimation of D D_{lo}
- and the number of analysts‘ forecasts for D NE

of expected dividends for the next three years, for every stock and every month. The data are from providers of financial data (in this article Thomson Financial Datastream). Now the question arises, how the confidence interval and the confidence probability of the views can be determined using the data of analysts' forecasts at hand.

In validity tests for the evaluation of calculated expected returns from the above mentioned valuation models, a regression of expected returns with different risk factors, such as Beta, debt-equity ratio, market capitalization or information risk is often carried out.[18] Thereby, a certain relation of a risk factor and the expected returns is assumed and it is analyzed, whether this relation can be proved. Concerning the "information risk", it is assumed that a larger amount of information available of a company reduces its cost of capital. As a measure for the information risk, Botosan/Plumlee (2005), for example, take the distance between lowest and highest forecast of a stock as a basis. They emphasize that this distance reflects the uncertainty of the forecast. Brennan et al. (1993) point out that prices of companies with larger analyst coverage react more quickly to market information. Gebhardt et al. (2001) implement these examination results by also taking the number of analysts' forecasts as a measure for the information risk – the more analysts' forecasts at hand, the lower the cost of equity capital should be. In this article, we also assume that the uncertainty of analysts' forecasts is larger for fewer analysts' forecasts and vice versa.

In the following, the number of analysts' forecasts will be linked to the view confidence. In order to generate the return interval $\mu_{i,lo}^{(t)}$ and $\mu_{i,hi}^{(t)}$ of analysts' views, such as in example 2.1 5 % and 7 %, we initially calculate two expected returns per point in time t and stock i with the help of the highest and lowest analysts‘ dividend expectation for this stock and time:

[18] See Botosan/Plumlee (2005), Gebhardt et al. (2001).

$$EK_i^{(t)} = \sum_{\tau=1}^{3} \frac{D_{i,lo}^{(t+\tau)}}{(1+\mu_{i,lo}^{(t)})^{\tau}} + \frac{D_{i,lo}^{(t+3)} \cdot (1+g_{i,lo})}{(\mu_{i,lo}^{(t)} - g_{i,lo}) \cdot (1+\mu_{i,lo}^{(t)})^3}, \quad (17)$$

$$EK_i^{(t)} = \sum_{\tau=1}^{3} \frac{D_{i,hi}^{(t+\tau)}}{(1+\mu_{i,hi}^{(t)})^{\tau}} + \frac{D_{i,hi}^{(t+3)} \cdot (1+g_{i,hi})}{(\mu_{i,hi}^{(t)} - g_{i,hi}) \cdot (1+\mu_{i,hi}^{(t)})^3}. \quad (18)$$

The entries for the view confidence matrix Ω will be calculated with the help of the number of analysts‘ forecasts. For each stock, point in time and prediction variable $D_i^{(1)}, D_i^{(2)}$, and $D_i^{(3)}$, there is a number of analysts who have made forecasts. Thus, for the VW stock in October, there are, for example, mean dividend forecasts $D_{VW,Okt}^{(1)}, D_{VW,Okt}^{(2)}, D_{VW,Okt}^{(3)}$ for the next, the one after next and the then following year and for each of these forecasts, we have the number of analysts giving these forecasts at hand: $NE_{VW,Okt}^{(1)}, NE_{VW,Okt}^{(2)}, NE_{VW,Okt}^{(3)}$. Abarbanell/Bernard (2000) and Courteau et al. (2001) measure a strong influence of the terminal value $\left(D_i^{(t+T)} \cdot (1+g_i)\right) / \left((\mu_i^{(t)} - g_i) \cdot (1+\mu_i^{(t)})^T\right)$ on the estimation of expected return. Due to the great importance of the last term in (16), the number of analysts’ forecasts for the last (third) year is taken as a basis for the confidence probability. As a starting point, the maximal number of analysts’ forecasts that are made over the whole period of time in question for every single stock, is marked with a confidence probability of 100 %.[19] Thus, if the maximum number of forecasts made for stock 10 is 40 and 40 analysts make forecasts for the third year of stock 10 at one point in time, the confidence probability for the particular interval is 100 %. If no forecast was made for the third year of one asset at one point in time, the confidence probability would amount to 0 %. The confidence probability is then linearly interpolated between 0 and 100 % based on the number of analysts’ forecasts for the third forthcoming year for every stock at every point in time. Thus, the forecast according to (7) looks like this:

$$\begin{pmatrix} \frac{\mu_{1,lo}^{(t)} + \mu_{1,hi}^{(t)}}{2} \\ \frac{\mu_{2,lo}^{(t)} + \mu_{2,hi}^{(t)}}{2} \\ \vdots \\ \frac{\mu_{N,lo}^{(t)} + \mu_{N,hi}^{(t)}}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{pmatrix}, \text{mit } \varepsilon \sim N\left(\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} \omega_{11} & 0 & \cdots & 0 \\ 0 & \omega_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_{NN} \end{pmatrix} \right). \quad (19)$$

In this case, the view matrix P is an identity matrix, as an absolute forecast is available for every stock. Equation (10) is then simplified as follows:

[19] The maximum number is determined for each stock separately.

$$E\left(r \mid Q\right) = \Pi + \tau\Sigma\left(\Sigma\tau + \Omega\right)^{-1}\left(Q - \Pi\right). \tag{20}$$

If analysts' forecasts are missing for some stocks in the portfolio, these can be ignored in the view matrix without further ado. At last, we get market weights as optimal weights for shares without views.

3.2 Determining Views by a Monte-Carlo Simulation

Moreover, the view confidence matrix Ω could also be determined by directly converting the standard deviations of the analysts' dividend forecasts into standard deviations for investors' expected returns. To this end, a Monte-Carlo-simulation is carried out in order to compute the standard deviation of expected returns with the following equation:

$$EK_i^{(t)} = \sum_{\tau=1}^{3} \frac{D_{i,s}^{(t+\tau)}}{(1+\mu_{i,s}^{(t)})^{\tau}} + \frac{D_{i,s}^{(t+3)} \cdot (1+g_{i,s})}{(\mu_{i,s}^{(t)} - g_{i,s}) \cdot (1+\mu_{i,s}^{(t)})^{3}}. \tag{21}$$

For a stock i at time t and every parameter which is required for the calculation of the expected return, s=1, …, S random variables are picked, with S being the number of simulation runs. We draw a

- normally distributed $D_{i,s}^{(t+1)}$ with mean value $D_i^{(t+1)}$ and standard deviation $\sigma_{D,i}^{(t+1)}$ (provided by Thomson Financial): $D_{i,s}^{(t+1)} \sim N\left(D_i^{(t+1)}, \sigma_{D,i}^{(t+1)}\right)$
- $D_{i,s}^{(t+2)} \sim N\left(D_i^{(t+2)}, \sigma_{D,i}^{(t+2)}\right)$ and $D_{i,s}^{(t+3)} \sim N\left(D_i^{(t+3)}, \sigma_{D,i}^{(t+3)}\right)$
- and a uniformly distributed growth rate with $g_{i,s} \in [g_{i,s,lo}, g_{i,s,hi}]$.

On the basis of one respective simulated data set s: $D_{i,s}^{(t+1)}, D_{i,s}^{(t+2)}, D_{i,s}^{(t+3)}, g_{i,s}$, an expected return $\mu_{i,s}^{(t)}$ is numerically computed with (21). Thus, we get S expected returns, from which the variance of expected returns of stock i at time t is determined. Finally, the variances are inserted on the diagonal of matrix Ω, and the Black-Litterman model can be applied.

3.3 Further Models for Estimation of View Confidences

3.3.1 Historical Variance-Covariance Matrix

A further possibility of determining the view confidence matrix Ω is to use the historical variance-covariance matrix of the specified views. Here it is assumed that relations of stocks in the past are also valid for the future. Thus, we detach ourselves from “forecasting“ analysts’ estimations and the view confidence matrix Ω is:

$$\Omega = \mathrm{diag}\left(P'\Sigma P\right). \tag{22}$$

If we have a look at a certain view portfolio, that is, a certain row k from the transposed view matrix (for example from (8)) and multiply it by variance-covariance matrix Σ and then by column vector P_k, we get the portfolio variance of the view portfolio

$$\omega_{kk} = P_k'\Sigma P_k. \tag{23}$$

Since, in the case of our approach, the view matrix is an identity matrix, the variance-covariance matrix is simplified to:

$$\Omega = \mathrm{diag}\left(\Sigma\right). \tag{24}$$

3.3.2 Proposal of He and Litterman (1999)

A different implementation is proposed by He and Litterman (1999). The elements of the view confidence matrix of view k ω_{kk} are defined in such a way that they equal the historical variances of view k multiplied by τ:

$$\omega_{kk} / \tau = \mathrm{diag}\left(P_k'\Sigma P_k\right). \tag{25}$$

This definition has the advantage that parameter τ does not need to be specified in the equation for calculating the expected returns according to Black-Litterman (12). Substituting $\Omega = \mathrm{diag}\left(P'\Sigma P\right)\cdot\tau$ into (10), we get:

$$\begin{aligned} E\left(r \mid Q\right) &= \Pi + \tau\Sigma P\left(P'\Sigma P\tau + \mathrm{diag}\left(P'\Sigma P\right)\cdot\tau\right)^{-1}\left(Q - P'\Pi\right) \\ &= \Pi + \tau\Sigma P\left(\tau\cdot\left(P'\Sigma P + \mathrm{diag}\left(P'\Sigma P\right)\right)\right)^{-1}\left(Q - P'\Pi\right) \\ &= \Pi + \Sigma P\left(P'\Sigma P + \mathrm{diag}\left(P'\Sigma P\right)\right)^{-1}\left(Q - P'\Pi\right). \end{aligned} \tag{26}$$

However, by this we have made an implicit assumption regarding the relation between matrix Ω and parameter τ. The division of variance of view k ω_{kk} and τ equals the historical variance-covariance matrix of the view-portfolio P_k. Thus, the entries in matrix Ω will be much smaller than in the above described case, since parameter τ is less than one. Hence, investors’ views are given more importance.

4 Empirical Examination

4.1 Examination of the Method on the Basis of Number of Analysts' Forecasts

In the following, the Black-Litterman model is applied with real capital market data. For this purpose, monthly data from 12/01/1993 to 01/01/2008 of all stocks of HDAX and DAX100, respectively, are available.[20] Data is extracted from the Thomson Financial Datastream database. Since DAX100 has been replaced by HDAX not before 03/24/2003, it will form the basis of our empirical examination in the beginning. DAX100 has been composed of 30 DAX shares and 70 MDAX shares. However, HDAX is composed of 30 DAX shares, 50 restructured MDAX shares and 30 TecDAX-shares. Insofar, we are considering 100 shares until 03/24/2003 and 110 shares afterwards. Only stocks that are included in the index (either DAX100 before April 2003 or HDAX from April 2003 on) at a specified point in time are considered in the optimization at this time. Furthermore, we examine, whether all data are available for the estimation of input parameters for the Markowitz optimization– for example, the share returns of the last 36 points in time for the estimation of the historically expected return. If, however, not all data that are required for the empirical examination are available, the respective share is not used in the optimization for this point in time. This procedure allows for an optimization that, at a certain point in time, contains a stock, which is no longer in the index after this point in time – thus, in this empirical examination a survivorship bias is nonexistent. The number of shares which are optimized over time fluctuates between 34 and 66.

The period of portfolio optimization starts on 01/01/1997. The preceding data are, for example, used for the calculation of the variance-covariance matrix or the historical mean value of realized returns.[21] The variance-covariance matrix is calculated on the basis of the Single-Index-Model. Although a historical variance-covariance matrix on the basis of the last 36 monthly returns could be calculated, the number of shares in the optimization exceeds 36.

[20] The data are from Thomson Datastream.

[21] The variance-covariance matrix is calculated on the basis of the last 36 points in time (three years) in every point in time. For the computation of the historical means 36 months are used too.

Thus, the resulting variance-covariance matrix would not be invertible and the optimization could not be applied.[22]

In Figure 4.1, the equilibrium expected returns calculated according to (5) and the historical expected returns of 12/01/2007 are presented.[23] At this point in time, 64 shares enter into the optimization. Negative historical returns appear on the basis of the last 36 months for four stocks. On the contrary, most shares feature much higher historical returns than equilibrium expected returns. For the determination of portfolio weights with (2), the historical mean vector of returns μ_{hist} and the vector of equilibrium returns Π are substituted for the vector of expected returns μ. For vector Π, the composition of the "market portfolio" obviously results immediately. For the weight calculation, in the first instance we assume that short sales are permitted for all stocks.[24]

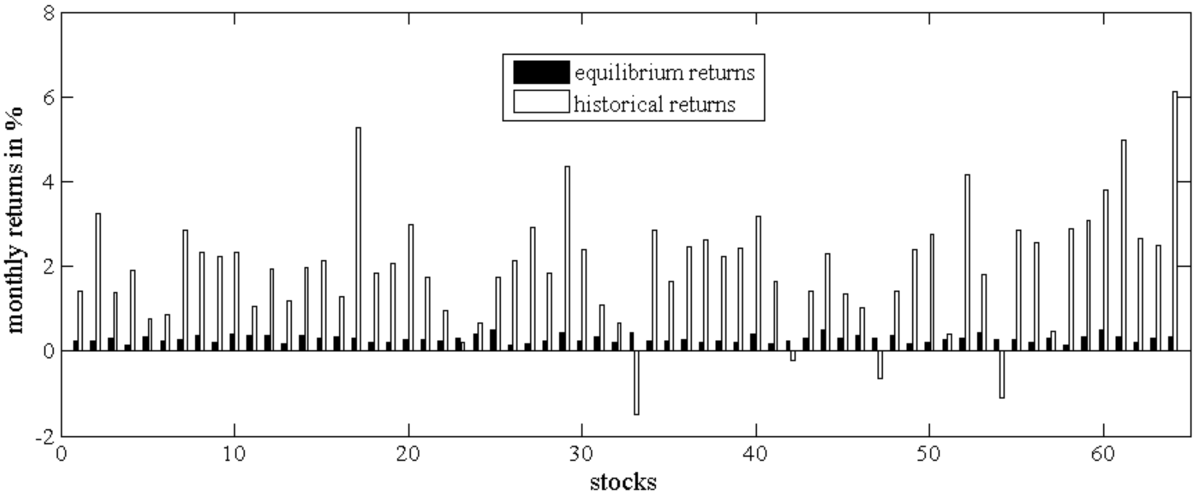

Figure 4.1: Equilibrium Returns and Historical Returns

The optimal portfolio weights, which result from the historical expected return vector, are illustrated in Figure 4.2. The first chart shows the market weights and the second chart depicts the extreme weights which result from historical returns. They fluctuate between -4.3 and 4.6,

[22] Furthermore the estimation error for the estimation of the variance-covariance matrix within the single index model is lower up to a time period of five years compared to the historical variance-covariance matrix. See Briner/Connor (2008), p. 12.

[23] The risk aversion parameter is set to two for the figures as the mean of all considered risk aversion parameters. Parameter τ is set to 0.03, which relates to a return history of 35 months: $1/35 \approx 0.03$ within the framework of a Bayesian derivation of the Black-Litterman model. For a sensitivity analysis of the parameters see Drobetz (2003). Moreover, all following figures correspond to the date 12/01/2007, thus the last point in time with an optimization.

[24] The sum of portfolio weights differs from one. The difference of one minus sum of asset weights is invested in the riskless asset. See Kempf/Memmel (2003), p. 520, Drobetz (2003), p. 231.

while, according to the construction, the market weights are all positive, between zero and one and add up to one. An Investor would most likely not realize these extreme weights.

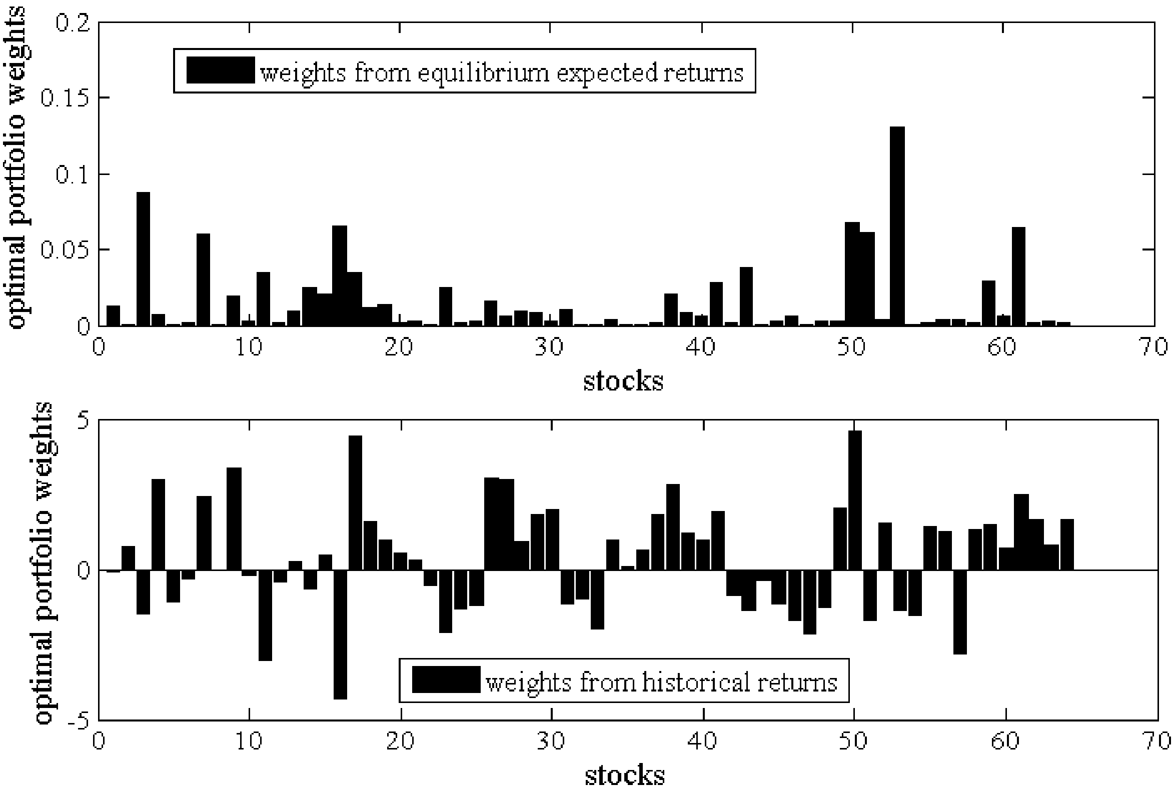

Figure 4.2: Optimal Portfolio Weights Resulting from Equilibrium and Historical Returns

Figure 4.3 illustrates the returns from analysts' forecasts for the Black-Litterman model. The black bars represent the expected returns μ_{lo} from the dividend discount model for the following out-of-sample performance examination, that are calculated from the lowest analysts' dividend forecasts D_{lo} with the lowest growth rate that is given for every individual share. The white bars represent the expected returns μ_{hi}, that are calculated from the highest analysts' dividend forecasts D_{hi} with the respective highest growth rate. The growth rates are individually estimated for every share. In the literature, different growth rates are applied for the calculation of terminal values. By way of example, the inflation rate or growth rate of the gross national income serve as estimations for the growth rate of dividends.[25]

[25] See for instance Wallmeier (2005), p. 141, Stotz (2004), p. 191.

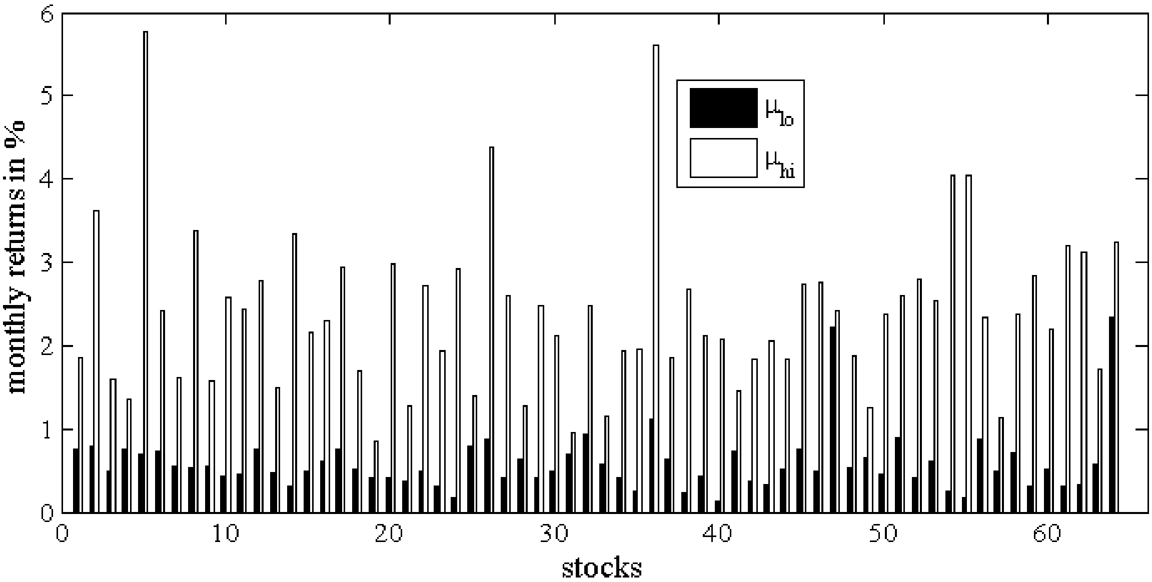

Figure 4.3: Lowest and Highest Expected Return of the Dividend Discount Model

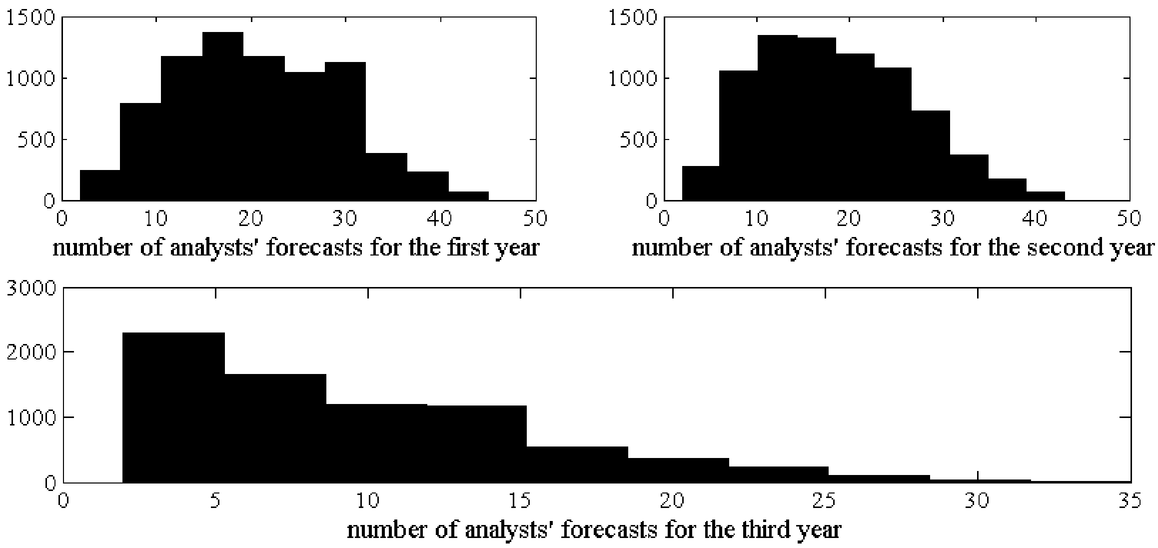

Figure 4.4: Histogram of the Number of Analyst Forecasts

Figure 4.4 shows histograms of the number of analysts' forecasts for estimations $D^{(1)}, D^{(2)}$ and $D^{(3)}$. The maximum number of analysts' forecasts for the whole period and all shares amounts to 45, the minimum number to two.[26] 34 to 66 shares over a period of 132 months are available for the histogram; overall 7569 values. The histograms of estimations for the first and second forthcoming year are similar. The distribution of number of estimations

[26] If there is only one analyst forecast, the stock is not contained in the optimization, as there is no standard deviation of the estimations.

per share is thus approximately the same. In comparison to the first two charts, the third chart reveals that much less forecasts are made for the third forthcoming year. The further away the estimation period, the less estimations are made by analysts.

Figure 4.5 presents an overview of the input parameters for the calculation of the expected Black-Litterman returns μ_{BL} according to (10) for the first 30 shares on 12/01/2007. For purposes of clarity, only the values of the first 30 of 64 shares are depicted below. The here assumed relation of a higher number of analysts' forecasts involving a higher confidence furthermore implies that a higher number of analysts' forecasts also results in a lower standard deviation. The first chart shows the return difference $\mu_{hi} - \mu_{lo}$, which enters the calculation of view standard deviations according to (30). The higher the return difference, the higher the view standard deviation is. The second chart shows the number of analysts' dividend forecasts for the third forthcoming year, that is utilized for the calculation of the view confidence in our approach. The highest number of analysts' forecasts can be found for share 16 with 25 fore-

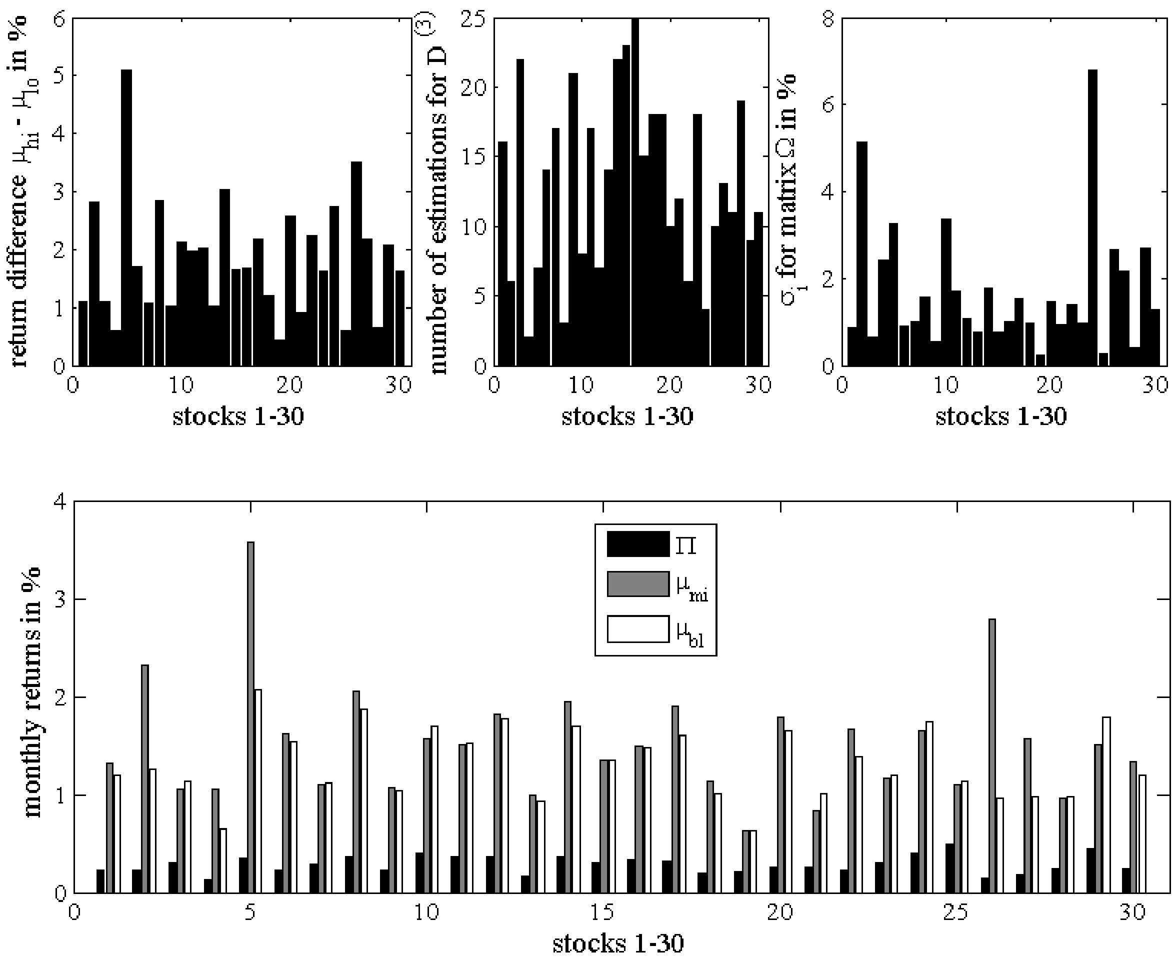

Figure 4.5: Overview of Returns in the Black-Litterman Model

casts, the lowest number has been made for share 4 with 2 forecasts. The third chart in figure 4.5 depicts the standard deviations of the view confidence matrix Ω. Share 19 has the lowest standard deviation, which is a result of the relatively high number of analysts' forecasts (18 forecasts) and the low return difference. On the contrary, share 24 has the highest view standard deviation. There, the number of analysts' forecasts is relatively small (4 forecasts) and the return difference $\mu_{hi} - \mu_{lo}$ is relatively high. The fourth chart in figure 4.5 shows the equilibrium returns Π, the mean return from the dividend forecasts $\mu_{mi} = (\mu_{hi} + \mu_{lo})/2$ for vector Q in (7) and the expected BL-return μ_{BL}, which is calculated from the previous returns according to (10). There is evidence that the expected BL-returns by trend are closer to the expected returns from the analysts' dividend forecasts. Share 2 features a relatively high standard deviation, which is also reflected in the higher deviation between the expected return from the dividend discount model and the BL-return. However, the level of standard deviation does not necessarily indicate the deviation. For instance, the BL-return for share 24 almost equals the dividend discount model-return, even though the standard deviation is higher than for share 2, where the return difference is clearly higher. The reason is the dependency among shares that enter the calculation of the BL-return in form of variance-covariance matrix of returns Σ.

The optimal portfolio weights according to Black-Litterman are then calculated for the first 30 shares from (2) with the BL-returns μ_{BL}. In Figure 4.6, the BL-weights are represented by white bars and are opposed to the market weights (black bars) and the weights that are obtained if the "pure" implied dividend returns μ_{mi} (grey bars) are inserted. The weights of the Black-Litterman model are much more balanced than the pure dividend weights. Even if the expected Black-Litterman-returns deviate from the dividend returns to a minor degree (cf. Figure 4.5), this is not the case for the weights. The weights based upon μ_{BL} predominantly lie between the market weights and the "dividend weights" on the basis of analysts' forecasts. For the 30 shares the share weights are balanced – for example, the weights for shares 2, 9 and 15 are close to the market weights, the ones for shares 1, 6 and 25 tend towards the dividend weights. All in all, we can say that the weights are by far not as extreme as the optimal weights obtained with historical return forecasts (cf. figure 4.2). The BL-weights fluctuate between a minimum weight of -0.76 for share 21 and a maximum weight of 0.74 for share 47 on 12/01/2007.

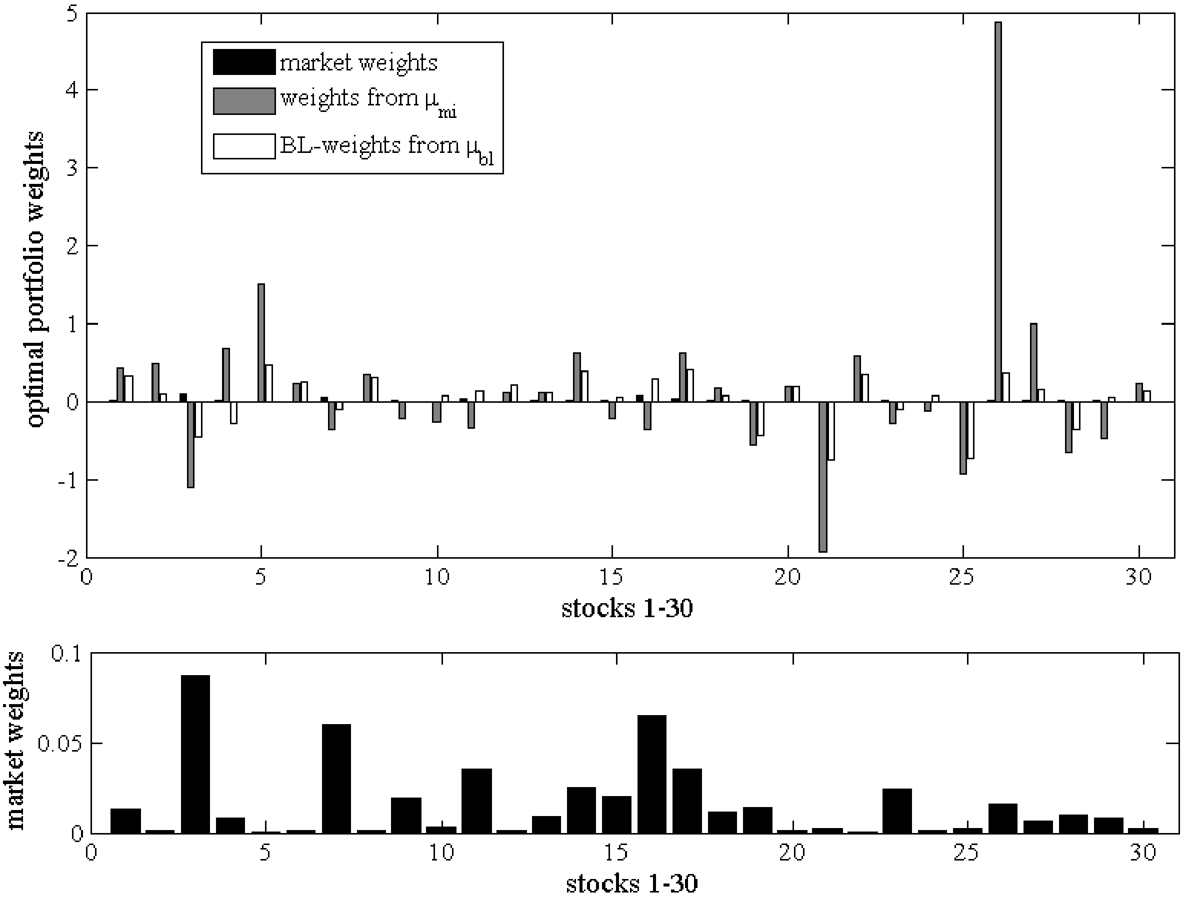

Figure 4.6: Optimal Portfolio Weights

4.2 Examination of Further Methods for Generating the View Confidence Matrix Ω

Figure 4.7 shows the standard deviations calculated according to the different approaches for the determination of the view confidence matrix Ω. The black bars of the first chart represent the standard deviations on the basis of the number of analysts' forecasts and the white bars display the standard deviations calculated with the Monte-Carlo simulation. The latter ones are clearly smaller. Views with lower standard deviations are assigned a higher loading in the calculation of the expected BL-return, thereby the resulting weights deviate stronger from the market weights. The optimal portfolio weights from the Monte-Carlo simulation, which are depicted with black bars in the second chart of Figure 4.7, are always larger compared to the weights of our approach (number of analysts' forecasts) when comparing their absolute values.

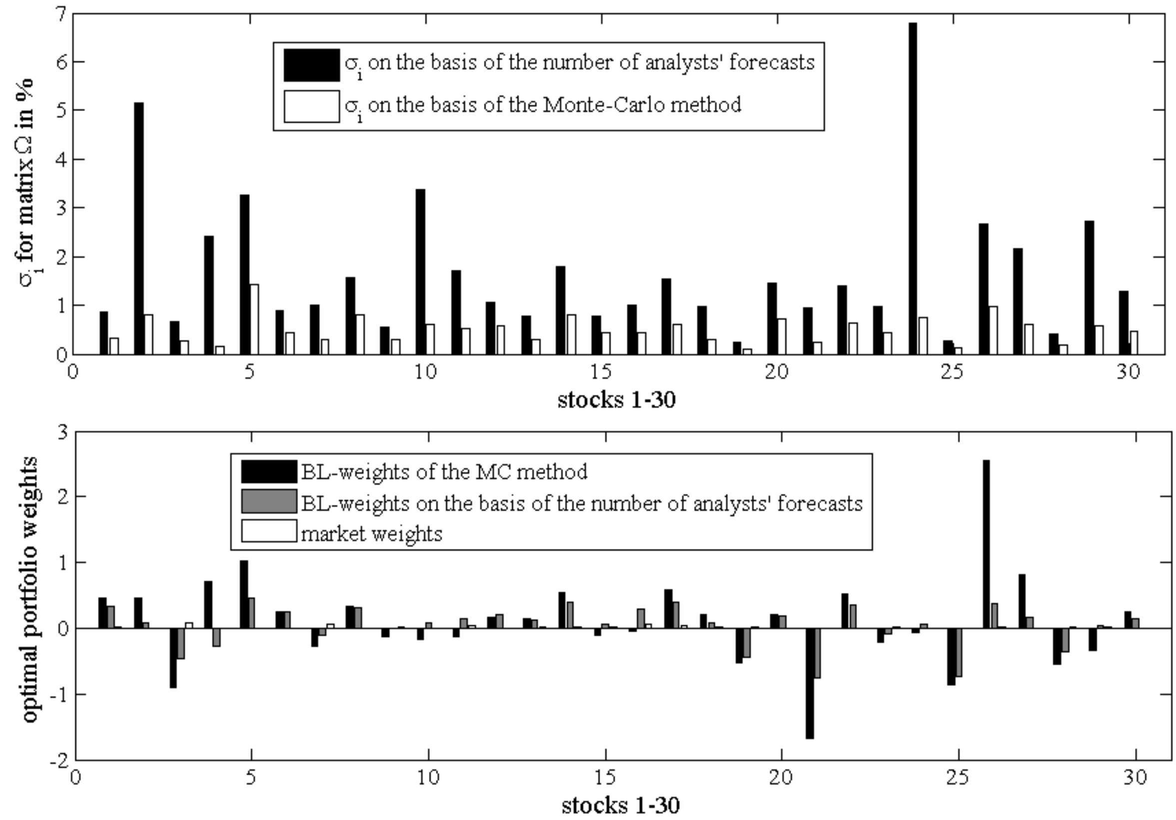

Figure 4.7: Standard Deviations of Ω with Different Approaches and Resulting Optimal Portfolio Weights

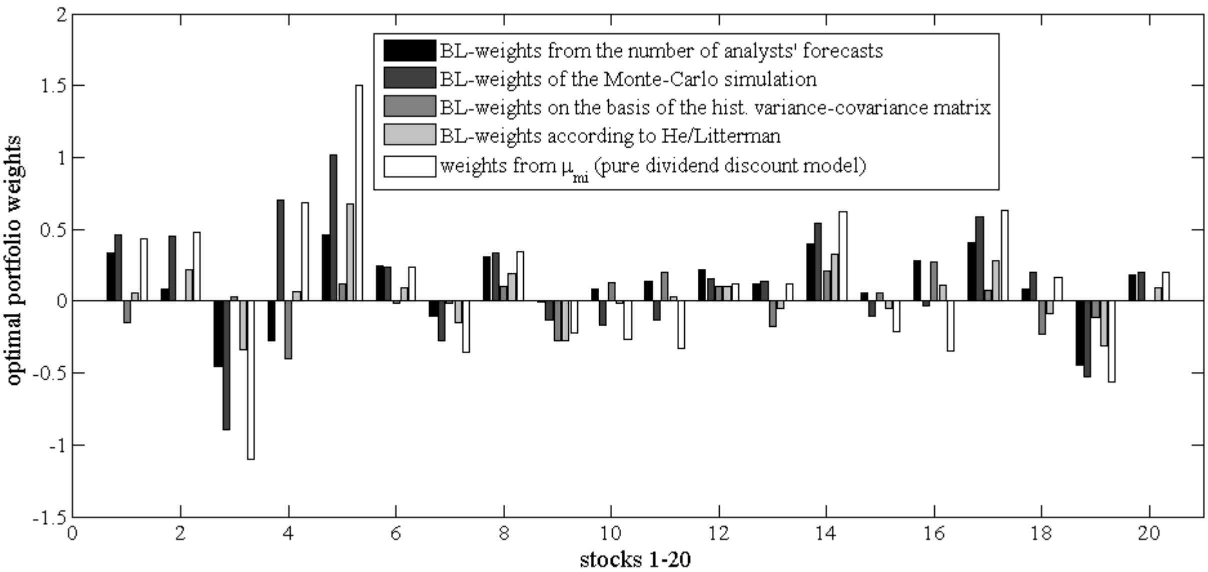

Figure 4.8: Overview of Portfolio Weights Resulting From the Different BL-Approaches

Furthermore, in Figure 4.8 the resulting portfolio weights of all introduced BL-approaches for the first 20 of 64 shares on 12/01/2007 are presented. For the purpose of a clearly represented illustration only 20 in place of 30 shares are displayed. In absolute values, the weights of the pure dividend discount model (white bars) are the highest except for few values. Moreover, the weights of the He-Litterman approach are closer to the weights of the dividend discount model compared to the weights of the historical BL-approach, as expected.

The performance of the four methods for the calculation of BL-returns (number of analysts' forecasts, Monte-Carlo simulation, historical variance-covariance matrix, He-Litterman) is compared to several benchmark strategies in the following paragraph.

4.3 Performance of the Selected Approaches

In a rolling optimization procedure, monthly out-of-sample returns and the resulting performances are calculated from 02/01/1997 to 01/01/2008.[27] We assume that at a specific time t, only the past and current data of times $t-s,\ldots t$, with $s>0$ are known. Based on this information, the expected returns for time t and the optimal portfolios and benchmarkportfolios for the following nine strategies are determined subsequently:

1. expected returns of the dividend discount model μ_{mi},
2. BL-returns on the basis of the number of analysts' forecasts $\mu_{BL,AS}$,
3. BL-returns on the basis of the historical variance-covariance matrix,
4. BL-returns according to He/Litterman (1999),
5. BL-returns from the Monte-Carlo simulation $\mu_{BL,MC}$,
6. historical expected returns μ_{hist},
7. portfolio composed according to a Bayesian estimator,
8. equilibrium expected returns μ_{markt} - market portfolio,
9. equally weighted portfolio $x_i = 1/N$.

The rolling optimization procedure is displayed in Figure 4.9. The expected BL-returns are calculated at each point in time t with current information and analysts' forecasts of time t, as mentioned before. Subsequently, the weight vectors are determined with the different expected returns and the resulting portfolio compositions are kept fixed for one month from t to t+1. At time t+1, the actual share returns of this month (period from t to t+1) are multiplied by the specified weight vectors to obtain the real portfolio return of every strategy for this month.

[27] The first optimization already proceeds on 01/01/1997. The optimized portfolios are kept fixed for one month and the first excess return is determined on 02/01/1997.

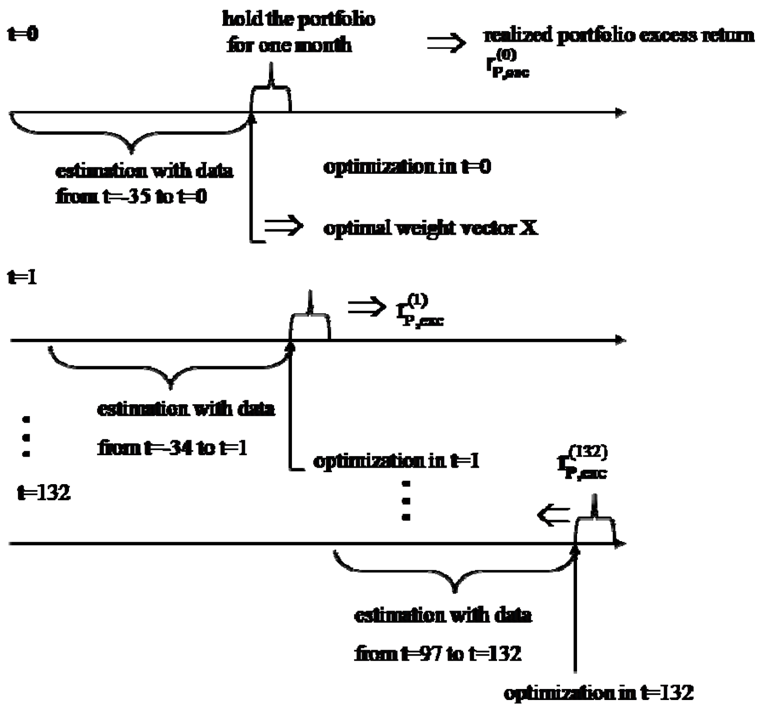

Figure 4.9: Rolling Optimization Procedure for the Measurement of Performance

The riskless interest rate is subtracted from this value, in order to identify the real excess portfolio return for the period from t to t+1. The resulting return is also known as the out-of-sample return, as for every optimization only past and current information is used and in the next step the performance of the strategies is measured outside the estimation period. We choose the Sharpe ratio, Jensen's alpha, Treynor ratio and the certainty equivalent from (1) to compare and assess the different strategies. The performance measures are appropriate for several situations according to the intention of portfolio composition.[28]

The optimizations are accomplished with different restrictions, unconstrained (optimization 1) as well as with short sales constraints (optimization 2). However, asset weights greater than one are allowed. The portion which is invested in the riskless asset amounts to $x_0 = 1 - \sum_{i=1}^{N} x_i$ in both optimizations. A negative x_0 indicates a debt position in the riskless interest rate. The results of the unconstrained optimization with $\tau = 0.21$ and $\lambda = 2$ are presented in Table 4.1.

[28] Cf. Breuer et al. (2004), pp. 374.

#	strategy	Sharpe ratio	rank	certainty equivalent	rank
1	dividend discount model	0.2135	4	-0.3640	8
2	BL: number of analyst forecasts	0.2615	1	-0.0614	5
3	BL: historical	0.1927	5	0.0120	1
4	BL: He-Litterman	0.2228	2	-0.0096	4
5	BL: Monte Carlo	0.2206	3	-0.3001	7
6	historical	0.1733	6	-1.8707	9
7	Bayes	0.1166	9	-0.0810	6
8	market	0.1267	8	0.0030	3
9	equally weighted	0.1584	7	0.0060	2
#	**strategy**	**Jensen's alpha**	**rank**	**Treynor ratio**	**rank**
1	dividend discount model	0.1103	2	0.0314	3
2	BL: number of analyst forecasts	0.0827	4	0.0372	2
3	BL: historical	0.0091	7	0.0174	7
4	BL: He-Litterman	0.0376	5	0.0239	5
5	BL: Monte Carlo	0.1032	3	0.0311	4
6	historical	0.2386	1	0.1600	1
7	Bayes	0.0234	6	0.0206	6
8	market	0.0000	9	0.0089	9
9	equally weighted	0.0025	8	0.0122	8

Table 4.1: Performance of the Strategies without Short Sale Restrictions with $\tau = 0,21$ and $\lambda = 2$

Three BL-approaches achieve the highest Sharpe ratios. Of these, the approach with the number of analysts' forecasts for the determination of the view confidence matrix has the highest Sharpe ratio (0.2615) by far. No strategy attains a negative Sharpe ratio, this means that the realized portfolio return on average achieves at least the riskless interest rate.[29] According to the certainty equivalent, a BL-approach outperforms all other strategies too (historical variance-covariance matrix) and the equally weighted portfolio and market portfolio are on ranks two and three. If the strategies are arranged on the basis of Jensen's alpha, the pure historical portfolio outperforms all other strategies and the pure dividend strategy is ranked second. Only then the BL-approaches achieve ranks three, four and five. As Jensen's alpha is manipulable (by means of borrowing the alpha could be enhanced), the Treynor ratio is more appropriate, as it cannot be influenced by borrowing. According to the Treynor ratio, the portfolio with historical expected returns attains the greatest risk premium per accepted systematic risk. The BL-approach with the number of analysts' forecasts is again on rank two, above the pure dividend approach.

[29] The riskless interest rate is represented through the interest rate of government bonds with a maturity of one year. The data are available in the internet at http://www.bundesbank.de/statistik/statistik_zeitreihen.-php?lang=de&open=zinsen&func=list&tr=www_s300_it03a.

#	strategy	1	2	3	4	5	6	7	8	9
1	dividend discount model		0	0	0	0	0	20	0	0
2	BL: number of analyst forecasts			0	0	0	0	5	20	20
3	BL: historical				0	0	0	0	0	0
4	BL: He-Litterman					0	0	20	20	0
5	BL: Monte Carlo						0	20	0	0
6	historical							0	0	0
7	Bayes								0	0
8	market									0
9	equally weighted									

Table 4.2: Significance Levels of the Sharpe Ratios According to Memmel With Short Sales and $\tau = 0,21$ and $\lambda = 2$

Table 4.2 shows the levels of significance, on which the Sharpe ratios of the strategies differ, according to the test of Memmel (2003). His test rectified the former test of Jobson and Korkie (1981). 5 %, 10 % and 20 % are chosen for the level of significance, as the power of the test is weak. The lower triangular matrix is symmetric to the upper one and is thus not presented. The Sharpe ratios of the BL-approach with the number of analysts' forecasts and the Bayesian portfolio are different on a significance level of 5 %. Furthermore, it is remarkable that the statistically different Sharpe ratios primarily appear with the BL-approaches in comparison to other portfolio strategies. A significantly different Sharpe ratio also arises between the dividend strategy and the Bayesian portfolio. The reason could be that the first five strategies are forward-looking procedures with analysts' forecast whereas the Bayesian estimator relies on historical data. The market portfolio and the equally weighted portfolio (strategies 8 and 9) are calculated with the current market capitalizations or are determined ad hoc without estimations. Hence, these two methods are neither based on past nor future.

	↓ **wins against** →	1	2	3	4	5	6	7	8	9
1	dividend discount model	-	0	63	0	0	110	110	110	110
2	BL: number of analyst forecasts	110	-	110	110	110	110	110	110	110
3	BL: historical	47	0	-	33	37	66	77	77	72
4	BL: He-Litterman	110	0	77	-	109	110	110	110	110
5	BL: Monte Carlo	110	0	73	1	-	110	110	110	110
6	historical	0	0	44	0	0	-	110	110	110
7	Bayes	0	0	33	0	0	0	-	0	0
8	market	0	0	33	0	0	0	110	-	0
9	equally weighted	0	0	38	0	0	0	110	110	-

Table 4.3: Comparison of the Strategies for all Parameter Constellations - Unconstrained

Table 4.1 and Table 4.2 display the performance of the strategies for a specified parameter constellation. Of course, with modified parameter constellations the results could be different. For this reason, we analyze several parameter constellations for the risk aversion parameter λ and the input parameter for the BL model τ. We apply common risk aversions from 0.5 to 3.5 in steps of 0.3 (11 values) and in order to establish small values for τ, we choose τ from 0.01 to 0.46 in steps of 0.05 (10 values). Thereby 110 parameter constellations result. Table 4.3 contains direct comparisons of how many times strategy A wins against strategy B for all possible 110 parameter constellations. The table has to be read the following way: the strategy of a certain row wins against the strategy in a certain column in x out of 110 parameter constellations. Thus the BL-approach with the number of analysts' forecasts wins in all parameter constellations against all other strategies. The BL-approach according to He-Litterman is the second best approach, as it wins in more than half of the constellations against other strategies (except for strategy 2). In at least 77 of 110 parameter constellations, it achieves a better performance than the BL-approach with the historical implementation. The Bayesian portfolio performs worst for all parameter constellations. The market portfolio and the equally weighted portfolio follow directly. The performances of the nine strategies without short sales is presented in Table 4.4.

#	strategy	Sharpe ratio	rank	certainty equivalent	rank
1	dividend discount model	0.1440	7	-0.1307	8
2	BL: number of analyst forecasts	0.1495	5	-0.0334	5
3	BL: historical	0.2071	1	0.0134	1
4	BL: He-Litterman	0.1613	3	-0.0152	4
5	BL: Monte Carlo	0.1469	6	-0.1115	7
6	historical	0.2052	2	-0.2482	9
7	Bayes	0.0354	9	-0.0439	6
8	market	0.1267	8	0.0030	3
9	equally weighted	0.1584	4	0.0060	2
#	**strategy**	**Jensen's alpha**	**rank**	**Treynor ratio**	**rank**
1	dividend discount model	0.1103	2	0.0314	3
2	BL: number of analyst forecasts	0.0827	4	0.0372	2
3	BL: historical	0.0091	7	0.0174	7
4	BL: He-Litterman	0.0376	5	0.0239	5
5	BL: Monte Carlo	0.1032	3	0.0311	4
6	historical	0.2386	1	0.1600	1
7	Bayes	0.0234	6	0.0206	6
8	market	0.0000	9	0.0089	9
9	equally weighted	0.0025	8	0.0122	8

Table 4.4: Performance of the Strategies With Short Sale Restrictions With $\tau = 0.21$ and $\lambda = 2$

Now, the BL-approach with the historical variance-covariance matrix outperforms all other strategies according to the Sharpe ratio and the certainty equivalent. Regarding Jensen's alpha and the Treynor ratio again the pure historical strategy performs best. As can be seen in Table 4.5 there are not as much significant outperformances according to the Sharpe ratio as without short sale constraints. The smaller variance of the portfolio weights with restrictions affects the realized portfolio returns and Sharpe ratios, thereby it is more difficult to reach a significant outperformance.

#	**strategy**	1	2	3	4	5	6	7	8	9
1	dividend discount model		0	0	20	0	0	0	0	0
2	BL: number of analyst forecasts			0	0	0	0	0	0	0
3	BL: historical				0	0	0	20	10	0
4	BL: He-Litterman					0	0	0	0	0
5	BL: Monte Carlo						0	0	0	0
6	historical							10	0	0
7	Bayes								0	0
8	market									0
9	equally weighted									

Table 4.5: Significances of the Sharpe Ratios According to Memmel Without Short Sales and $\tau = 0,21$ and $\lambda = 2$

Table 4.6 shows comparisons of the strategies for all parameter constellations for the case with short sale constraints. The pure historical strategy performs best on average. In 73 of 110 constellations this strategy attains a higher Sharpe ratio as the historical BL-approach. For $\lambda = 2$ and $\tau = 0.21$, one of the 37 other cases occurs, as the BL-approach outperforms the pure historical portfolio (see Table 4.4). All in all, the BL-approach with the historical variance-covariance matrix achieves the second rank and the BL-approach according to He-Litterman is ranked on position three.

	↓ wins against →	1	2	3	4	5	6	7	8	9
1	dividend discount model	-	0	0	0	22	0	110	110	0
2	BL: number of analyst forecasts	110	-	6	1	110	0	110	110	8
3	BL: historical	110	104	-	104	110	37	110	110	104
4	BL: He-Litterman	110	109	6	-	110	0	110	110	110
5	BL: Monte Carlo	88	0	0	0	-	0	110	99	0
6	historical	110	110	73	110	110	-	110	110	110
7	Bayes	0	0	0	0	0	0	-	0	0
8	market	0	0	0	0	11	0	110	-	0
9	equally weighted	110	102	6	0	110	0	110	110	-

Table 4.6: Comparison of the Strategies for all Parameter Constellations –Constrained

5 Conclusion

For the application of the Black-Litterman model, own views about the expected returns of assets are necessary, in order to deviate from the market weights. If there are no views, the procedure does not provide an opportunity to execute active portfolio management. Furthermore, analysts do not forecast in the way which is required for the implementation of the Black-Litterman model. In this contribution, the views for the Black-Litterman model are generated on the basis of the dividend discount model with the help of analysts' forecasts. We suggested four possibilities to compute expected returns with the Black-Litterman model. Two of these methods are described and examined for the first time. The confidence in the specified views are determined both on the basis of the number of analysts' forecasts and by applying a Monte-Carlo simulation on the basis of the distribution of analysts' forecasts. Thus, we contribute to the literature on a quantitative forecast model for the application of the Black-Litterman approach.

In our empirical examination, expected returns and optimal portfolio weights were calculated with real capital market data on the basis of a number of strategies: market portfolio, historical estimation, BL with number of analysts' forecasts, BL with Monte-Carlo simulation, BL with historical variance-covariance matrix, BL according to He-Litterman, dividend discount model, Bayesian estimator, and equally weighted portfolio. The effect of the different methods for the determination of the views on the portfolio weights after the application of the Black-Litterman model was analyzed. Finally, an out-of-sample performance analysis over a period of 132 months was implemented. This is, to our best knowledge, the first out-of-sample capital market study of the Black-Litterman model. Our implementation of the Black-Litterman model based on the number of analysts' forecasts outperforms all other strategies by far regarding the Sharpe ratio, if no constraints are included in the optimization. Within the constrained optimization, the Black-Litterman approach with the historical variance-covariance matrix provides the second best Sharpe ratio and the pure historical approach wins. Generally, the several Black-Litterman approaches show the best performances. Thus we recommend using the Black-Litterman model and give advice how to implement it.

6 Appendix

To calculate an entry ω_{jj} of view confidence matrix Ω, which refers to the conficence in the j-th view, the probability for the view specified by the investor is used. It is assumed, for instance, that the investor believes with a probability of 70 % that the difference in expected returns between portfolio A and portfolio B lies between 5 % and 7 %. To this end, the 85 % quantile[30] of the standard normal distribution is determined, as the probability of 70 % is distributed symmetrically around the expected return and the right boundary of the confidence region has the same x-coordinate as the 85 % quantile of the standard normal distribution (see Figure 6.1). Hence, the limiting values a and b of the standard normally distributed random variable Z with $P(a \le Z \le b) = 70\%$ are obtained.

The value of the 85 % quantile amounts to 1.036, thereby

$$P(1.036 \le Z \le 1.036) = 70\% . \tag{27}$$

The return forecast r^P of the investor could be formalized in the following way:

$$P\left(0,05 \le r^P \le 0,07\right) = 70\%. \tag{28}$$

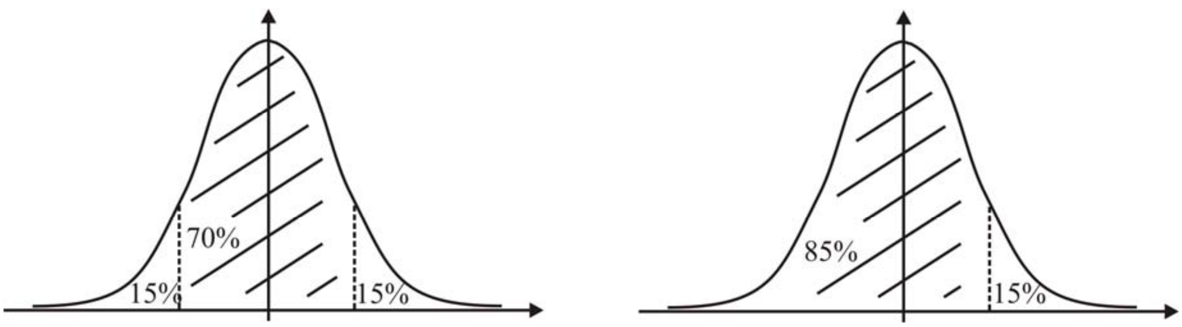

Figure 6.1: Quantiles of the Normal Distribution

This normally distributed random variable has to be linked to the standard normally distributed random variable Z for the determination of the view variance. A standard normally distributed random variable results from a normally distributed random variable $r^P \sim N\left(\mu_r, \sigma_r^2\right)$ with the following transformation:

$$Z = \frac{r^P - \mu_r}{\sigma_r} \sim N\left(0,1\right). \tag{29}$$

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[30] The α-quantile identifies the x-coordinate, where the distribution function of the standard normal distribution amounts to α. See Poddig et al. (2001), p. 185.

Solving (29) for r^p and inserting the resulting term in (28), yields:

$$
\begin{aligned}
& P\left(0,05 \le Z \cdot \sigma_r + \mu_r \le 0,07\right) = 70\,\% \\
\Leftrightarrow\ & P\left(\frac{0,05-\mu_r}{\sigma_r} \le Z \le \frac{0,07-\mu_r}{\sigma_r}\right) = 70\,\% \\
\Leftrightarrow\ & P\left(\frac{-0,01}{\sigma_r} \le Z \le \frac{0,01}{\sigma_r}\right) = 70\,\%.
\end{aligned} \tag{30}
$$

Comparing the last term with (27), the standard deviation of the return forecast follows directly:

$$\frac{0,01}{\sigma_r} \overset{!}{=} 1,036 \Leftrightarrow \sigma_r = 0,00965. \tag{31}$$

To insert the variance on the diagonal of matrix Ω, the squared term of (31) is calculated.

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