

Hedging Performance of the Libor Market Model: The Cap Market case

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Abstract

This paper aims to investigate the hedging performance of the Libor market model as well as the need to use models that incorporate explicitly volatility specific factors to better the hedging results. We compare the hedging performance of a standard Libor market model to that of a CEV Libor market model and find that although the volatility risk is not completely removed by a hedge portfolio that is composed only of bonds, results show that using a standard Libor market model is adequate to obtain high hedging performance in the cap market.

JEL Classification : G12, G13, G19.

Keywords : interest rate caps, Libor market model, Constant Elasticity of Variance, hedging.

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1 Introduction

Most empirical papers focus on assessing the pricing performance of interest rate models with the aim of picking out *the model* that best fits market prices. However, as pointed out by [Longstaff et al., 2001] this approach can be dangerously misleading.

In recent years, some researchers have examined the issue of model performance from a hedging perspective. The *best* interest rate model is the one that offers the *best* hedging performance. [Driessen et al., 2003], for example, analysed the hedging performance of HJM and BGM like models of caps and swaptions. They used 1, 2 and 3-factor models and concluded that, in the case of factor hedging, 3-factor models give better results. [Fan et al., 2003] use swaptions and reach the same conclusion, that is 3 or 4-factor models give better hedging performance than 1 and 2-factor models. Also, [Gupta and Subrahmanyam, 2002] tested the appropriate number of factors necessary to hedge both caps and floors. They compared, however, one- to two-factor models and concluded 2-factor models are better to achieve satisfactory hedging performance of caps and floors.

Therefore, high hedging performance of vanilla interest rate derivatives is achieved by means of higher dimensional interest rate models. . The underlying assumption here is that price changes are affected only by yield factors, that is factors that derive innovations in the yield curve.

However, including sloly yield factors is challenged by [Collin-Dufresne and Goldstein, 2002] and [Heidari and Wu, 2003] who demonstrate the need to use *volatility specific factors* (VSF) and thus use options along with bonds as hedge instruments. So an interest rate derivatives trader has to specify not only the number of factors in her model but also the nature of factors. A high hedging performance should rely on a model in which common factors as well as volatility factors are specified.

[Heidari and Wu, 2003] show the existence of factors that are independent of the yield curve. They identify three common factors which explain 99.5% of the variations of the yield curve. Nevertheless, these factors turn out to be insufficient since they explain only 59.48% of changes of swaptions implied volatilities. The conclusion is that there is a need to incorporate additional factors. In order to identify these independent factors, [Heidari and Wu, 2003] regress swaption implied volatilities on the common factors and then identify, from the residuals, three additional factors. These factors are *volatility specific factors*. The authors show that it is necessary to use models that incorporate explicitly volatility factors in addition to common factors hence accurate hedging of swaptions can be achieved. The hedging portfolio has to include, in addition to natural instruments, contingent claims such as options.

[Collin-Dufresne and Goldstein, 2002] obtain the same results for caps and floors. They argue that volatility risk cannot be hedged by means of a portfolio composed exclusively of bonds. Their empirical findings show weak correlation between changes in swap rates and yields of at-the-money cap and floor straddles. Relying on linear regressions, the authors show that, in some cases, as little as 10% of straddle yields can be due to changes in the term structure. Running a principal component analysis on the residuals reveals that an additional state variable can explain 85% of the rest of the variation. Therefore, they conclude that accurate hedging has to rely on a class of models that generate the *unspanned stochastic volatility* (USV). In this class of models, volatility related factors are explicitly specified.

Many authors dispute these results and claim that there is no necessity to incorporate *VSF* in standard models since the hedging performance of these models are already satisfactory enough. The value added of such models are not worth it.

Examples of these studies are those of [Gupta and Subrahmanyam, 2002] and [Fan et al., 2003] who investigate the issue of hedging caps and floors and swaptions respectively. In addition, [Gupta and Subrahmanyam, 2002] assert that using hedge portfolios composed exclusively of caps (floors) and euro-dollar Futures contracts yields satisfactory hedging results for caps (floors). They point out the fact that if there was a need to include explicitly volatility factors then none of the models under examination would have yielded high hedging performance of caps and floors. They conclude that a model that uses common factors can hedge sufficiently well caps and floors. Similarly, [Fan et al., 2003] reach the same conclusion in the case of swaptions. They use straddles to investigate the presence of independent factors and run a principal component analysis on regression residuals to find that there is no dominant factor left in the residuals.

Recently [Li and Zhao, 2003] use quadratic term structure models to investigate the presence of volatility factors in the cap market. They conclude that USV is present in the cap market.

I propose in this paper to re-examine the issue of the need to use VSF to achieve high hedging performance. For this purpose, I take a different approach : instead of focusing sloly on the study of the hedging performance of standard models (models that do not incorporate explicitly VSF). I examine, as well, the hedging performance of non-standard models (models that generate USV). Then I compare both performances.

I focus on the study of standard version and its extended Constant Elasticity

of Variance (CEV) of the now-widely used Libor market model¹ If the CEV Libor market model does not yields better hedging results than the standard Libor model then it is not necessary to include *volatility specific factors* to hedge efficiently caplet prices.

In the next section, I briefly review the Libor market model, its calibration given a specific instantaneous volatility form and its extended CEV version. The third section introduces the data used as well as the implementation procedure. Hedging results of both models are presented and discussed in section four.

2 The Libor Market Model : a review

[Brace et al., 1997], [Miltersen et al., 1997] and [Jamshidian, 1997] derived a new class of arbitrage-free interest rate models in which the underlying variable, that is the forward Libor rate, is an observable financial quantity and that for the first time.² Thus these models are called Libor Market Model (hereafter LMM). In the LMM, forward Libor rates follow lognormal processes. This assumption is already used by practitioners since they rely on the Black model ([Black, 1976] to price caps and floors. The forward Libor rate satisfies the following relationship :

Let

L_i be the forward Libor rate maturing at time T_i ; $i = 0, 1, \dots, N$; N forward Libor rates ;

δ_i be the accrual period for the forward rate L_i . $\delta_i = T_{i+1} - T_i$;

$B(t, T_i)$ be the zero-coupon bond price maturing at time T_i .

$$1 + \delta_i L_i(t) = \frac{B(t, T_i)}{B(t, T_{i+1})}; \quad t \leq T_N \quad (1)$$

Consider $N+1$ bonds available in the Libor market. So a set of bond maturities is given :

$$T_0 = 0 < T_1 < T_2 < \dots < T_{N+1}$$

The SDE satisfied by forward Libor rates under the spot measure³

¹The Libor market models which have gained wide interest among academics and practitioners.

²Previous interest rate models used for example the spot rate, the long rate and the instantaneous forward rate as underlying variables. See [Hull and White, 1990], [Brennan and Schwartz, 1982] and [Heath et al., 1992] as examples.

³Under the spot measure the numeraire $N(t) = \frac{B(t, T_i)}{\prod_{j=0}^{i-1} B(t_j, T_{j+1})}$. Thus, $N(t)$ is the price process obtained by rolling over one-period zero-coupon bonds (see [Rutkowski, 1999] for more details).

$$\frac{dL_i(t)}{L_i(t)} = \sum_{j=\eta(t)}^i \frac{\delta_j L_j(t) \sigma_j(t) \sigma_i(t)}{1 + \delta_j L_j(t)} dt + \sigma_i(t) dW(t) \quad (2)$$

where :

$\eta(t)$ is the index for the next reset date at time t . (this means that $\eta(t)$ is the smallest integer such that $t \leq t_{\eta(t)}$)

dW is a wiener process

and $\sigma_i(t)$ is the instantaneous forward rate volatility.

One of the main advantages of the class of Libor Market Models is that it provides a theoretical justification of the market's use of the Black model to price caplets. Consider a caplet with strike X , a reset rate the Libor rate L_i and a tenor δ . The payoff of this contract at time T_{i+1} is

$$\max((L_i - X), 0)$$

The market valuation formula for this caplet is :

$$c_i(t) = \delta B(t, T_{i+1})(L(t, T_i)N(d1) - XN(d2)) \quad (3)$$

where :

$$d_1 = \frac{\log L(t, T_i)/X + 0.5 \sigma_i^{market}(T_i - t)}{\sigma_i^{market} \sqrt{T_i - t}}$$

and

$$d_2 = d_1 - \sigma_i^{market} \sqrt{T_i - t}$$

where :

$L(t, T_i)$ is a forward Libor rate seen at time t for the period $[T_i; T_{i+1}]$

The cap market does not quote caplet volatilities but rather cap volatilities. Therefore Caplet volatilities are extracted from the quoted volatilities by means of an algorithm as described in [Avellaneda and Laurence, 2000] and [Alexander, 2003].

Another property of LMM is that the calibration to cap volatilities (Actually caplet volatilities as explained earlier) is straightforward by means of the following relationship :

$$(\sigma_i^{market}(t))^2 T_i = \int_0^{T_i} (\sigma_i^{inst}(t))^2 dt \quad (4)$$

The instantaneous volatility can be piecewise-constant or has a parametric form. We choose to use Rebonato's parametric form of the instantaneous volatility (see [Rebonato, 1999]). The instantaneous volatility is specified as follow :

$$\sigma_i^{inst}(t) = \lambda_i[(a + b(T_i - t)) \exp(-c(T_i - t)) + d] \quad (5)$$

where :

λ_i is a scaling factor that makes the calibration perfect.

To this end, each λ_i has to be almost equal to one. The four parameters a, b, c, and d have to be computed first using (Eq. 5) without taking into account the scaling factors. This is done through a non-linear minimization procedure, i.e. a Levenberg-Marquardt algorithm. Then each scaling factor is obtained by means of (Eq. 6) in order to ensure exact calibration to Black caplet volatilities.

$$\lambda_i^2 = \frac{(\sigma_i^{market}(t))^2 T_i}{\int_0^{T_i} ((a + b(T_i - t)) \exp(-c(T_i - t)) + d)^2 dt} \quad (6)$$

This parametric specification of the instantaneous volatility, in contrast with a piecewise-constant form, preserves the qualitative shape of the implied volatility observed in the market, i.e. a hump at around two years (see Table 2).

2.1 An extended version : a CEV LMM

The LMM can be extended to a CEV model in order to take into account the volatility skew⁴ present in the cap market. The underlying volatility factor can be captured by the Constant Elasticity of Variance parameter α . [Andersen and Andreasen, 1998] were the first to use such an extended version of the standard model. A CEV process for the forward Libor rates is of the form :

$$dL_i(t) = L_i^\alpha(t) \sum_{j=\eta(t)}^i \frac{\delta_j L_j^\alpha(t) \sigma_j(t) \sigma_i(t)}{1 + \delta_j L_j(t)} dt + \sigma_i L_i^\alpha(t) dW(t) \quad (7)$$

Where :

α is a positive constant.

3 Data and Implementation procedure

The data are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10-month libor rates and 1, 2, 3, 4, 5, 6, 7 and 10-year swap rates. These rates are used to construct the Libor yield curve through a stripping method. In addition, weekly data of at-the-money-forward caps volatilities as well as strike 2.5%, 3%, 3.5%, 4% and 4.5% of maturities of 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10-year are used. Tables 1 and 2 present summary statistics on the forward Libor rates across maturities and implied volatilities across strikes and maturities respectively.

⁴Implied market volatilities and strike prices evolve in opposite direction

[TAB. 1 about here.]

[TAB. 2 about here.]

Caps are resetted quartely. Since a complete set of cap market volatilities is available, it is possible to back out caplet volatilities. We derived weekly price changes from january 16, 2002 to june 18, 2003. So we obtain 75 weekly observations for each strike level. However, since the data available do not include the prices of caplets over their lifes we need to extrapolate the price for a given caplet one week after its inception. To perform this, we use the following procedure : in week i we have the caplet volatility σ_i of maturity T . One week later we have at our disposal caplet volatilities at different maturities. We use this data to interpolate σ_i of maturity $T - 1 week$.

3.1 Implementation of a 3-factor LMM

A principal component analysis is carried out in order to investigate the common factors that derive innovations in the Libor term structure. The first three factors that explain most of the variation (99.28%) are shown in Table 3. The first component explains 97.70 % of changes in the term structure, the second and third factors account for 1.24 % and 0.34% respectively. These results are in line with previous findings such as in [Litterman and Scheinkman, 1991].

[TAB. 3 about here.]

As a result a 3-factor LMM captures most of the variation that occurs in the libor curve. Its SDE is :

$$\frac{dL_i(t)}{L_i(t)} = \sum_{j=\eta(t)}^i \frac{\delta_j L_j(t) \sum_{p=1}^s \gamma_{j,p}(t) \gamma_{i,p}(t)}{1 + \delta_j L_j(t)} dt + \sum_{p=1}^s \gamma_{i,p}(t) dW_p(t) \quad (8)$$

where :

s is the number of factors that is 3.

and

$\gamma_{j,p}(t)$ is the p^{th} instantaneous volatility component of the j^{th} forward Libor rate. In addition, it must satisfies the equality below :

$$(\sigma_i^{inst})^2(t) = \sum_{p=1}^s \gamma_{i,p}(t)^2 \quad (9)$$

$\gamma_{j,p}(t)$ is determined by using the eigenvalues and eigenvectors of the PCA⁵. The LMM can be calibrated to caplet volatilities and historical correlation matrix of forward Libor rates (see Figure 1).

⁵see [Hull and White, 2000]

[FIG. 1 about here.]

So it is possible to write

$$\Delta L_i = \sum_{p=1}^N \alpha_{i,p} x_i \quad \text{where } i = 1, \dots, N$$

$\alpha_{i,p}$ is the factor loading (eigenvector) for the i^{th} forward Libor rate and the p^{th} factor. x_i is a random variable with mean 0 and variance (eigenvalue) v_i^2 . Since we chose a number of factors $s < N$, $\gamma_{i,p}$ is obtained by means of the equality :

$$\gamma_{i,p} = \frac{\sigma_i v_p \alpha_{i,p}}{\sqrt{\sum_{p=1}^s v_p^2 \alpha_{i,p}^2}} \quad (10)$$

Note that since the first three components explain 99.28% of the variation, we expect the model correlation matrix to still be similar to the market correlation matrix.

Implementating the LMM requires the use of Monte Carlo simulation. First we use a Euler discretization scheme for equation (8). Although this discretization is not well suited, as pointed out by [Glasserman and Zhao, 2000] since the prices would not be arbitrage free, it still appropriate as long as Δt is small enough. Using Equation (8) and applying Ito's lemma to $\ln L_i(t)$ we obtain :

$$L_i(t_{k+1}) = L_i(t_k) \exp \left[\left(\sum_{j=k+1}^i \frac{\delta_j L_j(t_k) \sum_{p=1}^s \gamma_{j,p} \gamma_{i,p}}{1 + \delta_j L_j(t_k)} - \sum_{p=1}^s \frac{\gamma_{i,p}^2}{2} \right) \Delta t + \sum_{p=1}^s \gamma_{i,p} \epsilon_p \sqrt{\Delta t} \right] \quad (11)$$

In addition, we can discretize the 3-factor version of Equation (7) as follows ⁶ :

$$Q_i(t_{k+1}) = Q_i(t_k) + \Delta t \sum_{j=k+1}^i \left[\frac{\delta_j L_j^\alpha(t_k) \sum_{p=1}^s \gamma_{j,p} \gamma_{i,p}}{1 + \delta_j L_j(t_k)} - \sum_{p=1}^s \frac{\alpha L_i^{\alpha-1}(t_k) \gamma_{i,p}^2}{2} \right] + \sum_{p=1}^s \gamma_{i,p} \epsilon_p \sqrt{\Delta t} \quad (12)$$

where :

$$Q_i(t) = \frac{1}{1 - \alpha} L_i^{1-\alpha}(t)$$

We run thousands of Monte Carlo simulations with antithetic variates in

⁶see [Hull and White, 2000]

order to compute caplet prices using both processes in Equations (11) and (12) through a standard LMM and a CEV LMM respectively. These prices are used to assess the hedging performance of both model as described in the next section.

4 Hedging performance

Figure 2 shows the three principal components that affect significantly changes in the Libor curve. The first component corresponds to parallel shifts. This means that a shock at any point will affect almost uniformly the whole curve. In contrast, a shock to the second factor causes the short and long rates to move in opposite direction. This corresponds to the slope of the curve. And finally, the third factor is a curvature factor since short and long rates move in the same direction whereas intermediate maturity rates move in the opposite one.

[FIG. 2 about here.]

Given this 3-factor model, a delta-based hedge portfolio is composed of three hedge instruments that are common to all caplets. The natural hedge instruments used here are the zero-coupon bonds. The first hedge instrument is a short-maturity bond (3 months). The second instrument is a spread of bonds consisting of short position of short-maturity bond and a long position of long-maturity bond (10 years). Finally, a butterfly composed of long positions of short- and long-maturity bonds and a short position of intermediate-maturity bond (5.5 years) is used to hedge against the curvature risk. We applied the same hedging strategy to both models standard LMM and CEV LMM. The Delta ratio is computed in both cases through Monte-Carlo simulation.

The hedge performance of a model is measured through a ratio of the standard deviation of the hedge portfolio and that of the unhedged position. Table 4 shows the hedging performance of the standard LMM across strikes. We notice that, on the one hand, the LMM hedges efficiently nearly 85% to 90% of the variation of the Libor curve. These results confirm that a high dimensional model yields efficient hedging results. This is in line with previous empirical work as discussed in section 1. On the other hand, the CEV LMM gives almost similar hedging results. For ATMF caplets, the CEV model's performance is even lower than that of the standard LMM. Thus there is no need to use USV model in the cap market since the add-value of such models with respect to hedging performance is not significant.

[TAB. 4 about here.]

Moreover, in order to investigate the presence of USV in the cap market, we run multiple regressions of changes in market implied volatilities on the

three hedge instruments. Then we run principal component analysis on the regressions residuals. Table 5 reports the most significant components in the residuals. We observe that there is at least two dominant factors at each strike level. Therefore, we can argue that there are additional factors that can affect the evolution of the volatility surface. This result confirms previous findings as in [Collin-Dufresne and Goldstein, 2002] and [Li and Zhao, 2003].

[TAB. 5 about here.]

5 Conclusion

This paper aims at investigating two issues in interest rate hedging. First, we examine the hedging performance of a 3-factor Libor market model. We find that the model hedges efficiently interest rate caps. In addition, a CEV LMM does not yield higher hedging results so it is not necessary to use such non-standard models to better hedging performance in the cap market. Second, we investigate the issue of USV. Results show the existence of factors that are specific to volatility and which are not eliminated when using a hedge portfolio composed exclusively of bonds. Therefore, while VSF exist, high hedging performance of caps is achieved by means of a model that do not incorporate them explicitly.

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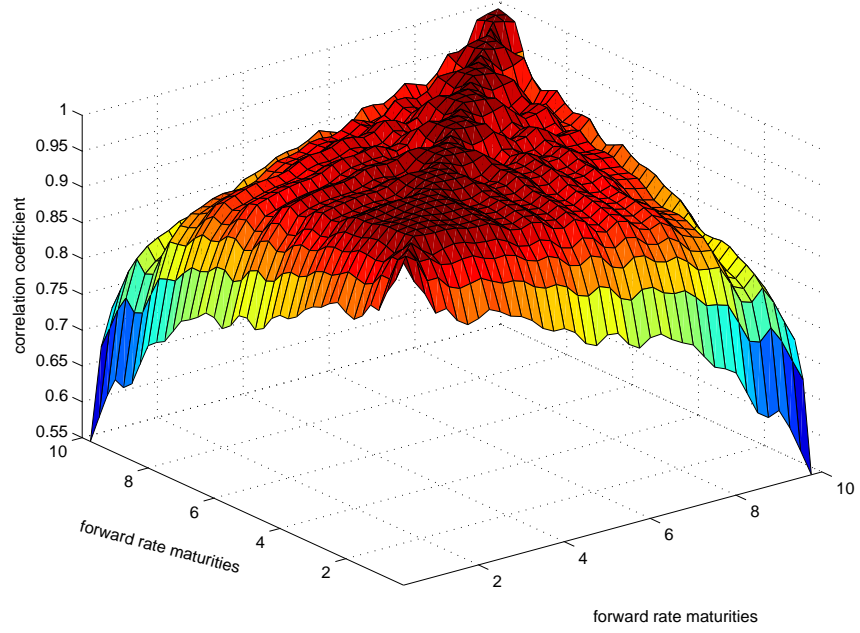


FIG. 1 – Historical correlation matrix of forward Libor rates

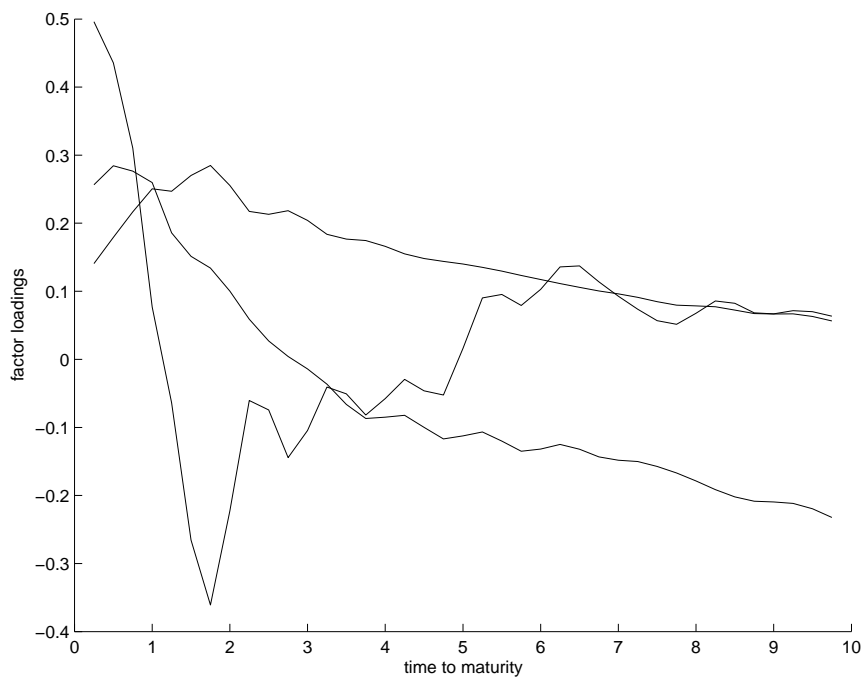


FIG. 2 – The three most significant principal components

	Maturity (years)				
	1	2	3	4	5
Mean	3.056%	3.719%	4.231%	4.645%	4.943%
Std. dev.	0.106%	0.045 %	0.006%	0.009%	0.008%

	Maturity (years)				
	6	7	8	9	10
Mean	5.219%	5.408%	5.536%	5.616%	5.646%
Std. dev.	0.009%	0.007%	0.006%	0.004%	0.012%

TAB. 1 – Statistics of forward Libor rates

Maturity (years)	Strike level					
	ATMF ^a	2.5 %	3.0%	3.5%	4.0%	4.5%
1	23.18% (3.50%)	28.07% (3.26%)	25.09% (2.04%)	23.03% (2.73%)	22.81% (3.60%)	23.41% (3.85%)
2	23.94% (4.59%)	29.63% (2.00%)	26.64% (2.38%)	24.50% (3.14%)	23.39% (4.04%)	22.71% (4.96%)
3	22.38% (4.59%)	28.97% (1.73%)	25.83% (2.00%)	23.50% (2.37%)	21.85% (2.91%)	20.60% (3.27%)
4	20.80% (4.05%)	28.08% (1.42%)	24.96% (1.52%)	22.49% (1.61%)	20.62% (1.92%)	19.23% (2.18%)
5	19.49% (3.39%)	27.30% (1.22%)	24.24% (1.25%)	21.78% (1.22%)	19.81% (1.40%)	18.33% (1.60%)
6	18.55% (2.98%)	26.62% (1.10%)	23.65% (1.08%)	21.21% (0.98%)	19.21% (1.06%)	17.69% (1.19%)
7	17.68% (2.55%)	26.00% (1.00%)	23.14% (0.98%)	20.76% (0.83%)	18.73% (0.84%)	17.19% (0.91%)
8	17.03% (2.26%)	25.48% (0.95%)	22.70% (0.90%)	20.37% (0.74%)	18.34% (0.71%)	16.84% (0.77%)
9	16.44% (1.97%)	25.01% (0.91%)	22.31% (0.85%)	20.03% (0.69%)	18.03% (0.64%)	16.49% (0.64%)
10	15.94% (1.81%)	24.61% (0.91%)	21.99% (0.92%)	19.72% (0.66%)	17.73% (0.59%)	16.21% (0.58%)

^aAt-The-Money-Forward

TAB. 2 – Statistics of Cap Implied Volatilities across maturities and strikes

This table reports Means and Standard Deviations (in parantheses) statistics of implied volatilities across strikes and maturities.

Eigenvalue	Percentage (P)	Cumulative percentage (CP)
λ_1	97.70 %	97.70%
λ_2	1.24 %	98.94%
λ_3	0.34 %	99.28%

TAB. 3 – The first three principal components

	ATMF	2.5%	3%	3.5%	4%	4.5%
LMM	8.94%	14.67%	14.31%	13.75%	12.52%	11.35%
CEV LMM	10.62%	17.14%	16.16%	12.20%	11.83%	11.27%

TAB. 4 – **Hedge performance ratio**

Strikes	ATMF		2.5 %		3%	
Eigenvalues	P	CP	P	CP	P	CP
λ_1	57.53%	57.53 %	78.11 %	78.11 %	80.55%	80.55 %
λ_2	21.73%	79.27%	13.17%	91.28 %	18.09 %	91.73%
λ_3	8.66%	87.92 %	5.32%	96.60%	5.70%	97.43%
Strikes	3.5 %		4 %		4.5 %	
Eigenvalues	P	CP	P	CP	P	CP
λ_1	80.55 %	80.55 %	70.53%	70.53%	67.91%	67.91%
λ_2	12.18%	92.73 %	20.05 %	90.58%	22.73%	90.63%
λ_3	5.85%	98.58%	6.72%	97.31%	6.97%	97.60%

TAB. 5 – **Most significant eigenvalues in the multiple regressions residuals**

This table reports most significant eigenvalues obtained through a two-step procedure. First we run separate multiple regressions of implied volatilities across strikes on the three hedge instruments. Then, we apply PCA on the residuals in order to investigate hidden significant factors.