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Risques et Applications


## Regime Switching Models: Real or Spurious Long Memory?


**Dominique GUEGAN*    Stéphanie RIOUBLANC****



**Abstract**

In this paper, we are interested in the possible confusion between long memory and structural breaks of Markov switching models. We want to understand which are the real causes of the autocorrelation functions' slow decay observed on many economic and financial time series. We investigate more particularly the influence of the means values and the occurrence of states changes, on the long or short memory behavior of Markov switching models.



*JEL classification:* C22.

*Keywords:* Markov switching models, Autocorrelation function, Long memory, Short memory.



*Ecole Normale Supérieure, Cachan, Senior Academic Fellow de l'IEF, Département d'Economie et Gestion, 61, avenue du Président Wilson, 94230 Cachan, Tél. : + 33 (1) 47 40 55 75 - Fax : + 33 (1) 47 40 23 48, guegan@ecogest.ens-cachan.fr

**Ecole Normale Supérieure de Cachan and Banque de France, 31 rue Croix des Petits Champs, 75001 Paris, Tél. : + 33 (1) 42 92 49 95, stephanie.rioublanc@banque-france.fr.

# 1 Introduction

Structural breaks have been observed in many economic and financial time series. A large literature concerning models able to take into account such changes of state exists nowadays. One of the most popular of these models is the Markov Switching process introduced by Hamilton (1988). Nevertheless, there has been a considerable interest in recent years in the possible long memory behavior of structural breaks models, see also Chan (1990), Diebold and Inoue (2001), Breidt and Hsu (2002) and Dufrénot, Guégan and Peguin-Feissolle (2003) for a discussion on existence of this kind of behavior in models with jumps, for instance. This long memory behavior property in the data due to structural breaks or regime switches is called "spurious long memory". We are particularly interested in this subject because a right understanding of the asymptotic behavior of such models would have an impact on the estimation and forecast of time series with structural breaks. Indeed, in two previous papers, we showed that the transition probabilities $p_{11}$ and $p_{22}$ and the means' parameters $\mu_i$, $i = 1, 2$ influence the autocorrelation function's behavior of the model (1) (see Guégan and Rioublanc (2003)) and that simulations seem to create spurious long memory behavior (see Guégan and Rioublanc (2004)) inside this model. Nevertheless, the way those parameters act on this asymptotic behavior as well as the reason why we empirically obtain a long memory behavior (whereas the model (1) exhibits theoretically a short memory behavior), were not completely clear and need to be more investigated. Here, we are particularly interested in the fact that this spurious long memory in the data may be due to level shifts as well as the occurrence in states changes, which is obviously linked to the transition probabilities. Thus, the slow decay of autocorrelation functions would not be real long memory. That is why a better understanding of the autocorrelation function's behavior of model (1) could have an impact on the estimation and forecast of time series with structural breaks. In case of spurious long memory, interpreting the slow decay of the autocorrelation functions as long memory would be a mistake and could have repercussions on time series forecasting. Indeed, long memory implies that we need a long period of observations to forecast the future behavior of the time series. Long memory supposes indeed that far observations still have an impact on the present dynamic of our process. Whereas in case of spurious long memory, a past short sample would be necessary for the forecasts. Thus confusing long memory and structural change would have an important impact on forecasting for instance. In the following, we will note $n_{s_t}$ the number of changes during the sample.

## 2 Real or spurious long memory?

There exists different criteria to define the long memory behavior (see Guégan (2004) for a survey on theses different criteria). The definition we use characterizes the long memory in terms of asymptotic decay of the autocorrelation function.

**Definition 1** *Let $(X_t)_t$ be a stationary process for which the following holds. There exists a real number $d \in ]0, 1/2[$ and a constant $C > 0$ such that the autocorrelation function $\rho(k)$ verifies*

$$\lim_{k \to \infty} \rho(k) = Ck^{2d-1}$$

*Then $(X_t)_t$ is called a stationary process with long memory behavior.*

Thus, the asymptotic decay of the autocorrelation function is hyperbolic in the case of long memory processes. That is the reason why we will speak about the slow decay of the autocorrelation function. This latter converges exponentially towards zero in case of short memory processes. In this article, we will consider that if its empirical autocorrelation function has some lags larger than 5 significant, the series has a long memory behavior, otherwise it has a short memory behavior.

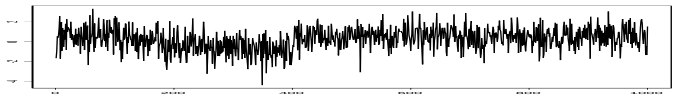
For this study, we consider the basic Markov switching model defined by the following equation:

$$X_t = \begin{cases} \mu_1 + \varepsilon_t & \text{if } s_t = 1, \\ \mu_2 + \varepsilon_t & \text{if } s_t = 2. \end{cases} \quad (1)$$

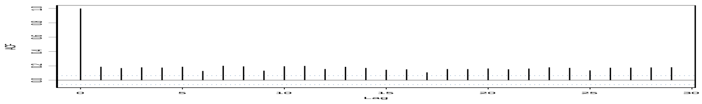
The process $(s_t)_t$ is a hidden ergodic Markov chain, characterized by its transition matrix $P$, whose elements are the fixed transition probabilities $p_{ij}$, defined by:

$$p_{ij} = P[s_t = j | s_{t-1} = i], \quad 0 \leq p_{ij} \leq 1, \quad \sum_{j=1}^{2} p_{ij} = 1. \quad (2)$$

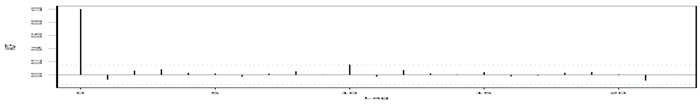
The process $(\varepsilon_t)_t$ is a centered Gaussian strong white noise with variance one. We simulate the model (1) for two different pairs of levels: $(\mu_1,\ \mu_2) = (5, -5)$ and $(\mu_1,\ \mu_2) = (0.5, -0.5)$. The sample size is $T = 1000$. The plan of our simulations is to increase the parameter $n_{s_t}$, the number of times for which the series changes of states, by playing on the transition probabilities. Then, we investigate the autocorrelation functions behavior of the simulated series in order to study if the number $n_{s_t}$ has an incidence on the asymptotic behavior of Markov switching models.



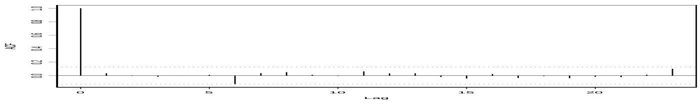
(a) Trajectory of the simulated series.



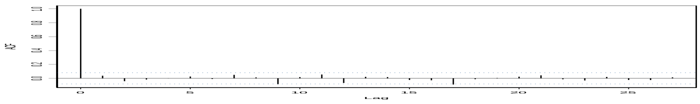
(b) Autocorrelation function of the whole series.



(c) Autocorrelation function of the series' first part ($s_t = 1$).



(d) Autocorrelation function of the series' second part ($s_t = 2$).



(e) Autocorrelation function of the series' third part ($s_t = 1$).

Figure 1: Model (1) simulated with $(\mu_1,\ \mu_2) = (0.5, -0.5)$, $(p_{11}, p_{22}) = (0.999, 0.999)$ and $T = 1000$. $n_{s_t} = 2$.

The underlying idea to this paper is that the larger $n_{s_t}$ is, the quicker the decay of the autocorrelation function is. Thus, we start by simulating the model (1) for a small number $n_{s_t}$. We study the autocorrelation function of the whole sample but also on each different part. Because $n_{s_t}$ is small, we are expecting a slow decrease of the autocorrelations, but some behaviors similar to short memory processes on each single part. Such behaviors would be a preliminary evidence of spurious long memory created by level shifts. The trajectory of the simulated series is represented on Figure 1 (a). First, the process is in state 1, then get to state 2 and finally get back to the initial state. Thus, for that simulation, $n_{s_t} = 2$. Its autocorrelation function represented on Figure 1 (b) exhibits a long memory behavior. Nevertheless, on each single part of the series, we can observe a short memory behavior, see Figure 1 (c)-(e). Thus, this example shows that the slow decay observed on the autocorrelation function of some series issued from Markov switching models does not really exhibit long memory, but is created by level shifts. Thus, the phenomenon observed on the autocorrelation functions of such series is some spurious long memory.

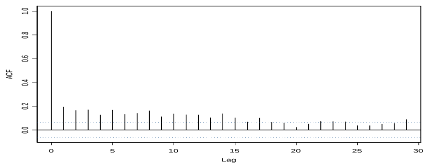
# 3 Role of $n_{s_t}$.

In the previous section, we have seen that a two-state Markov switching model can exhibit spurious long memory behavior. Nevertheless, we know that such models present theoretically a short memory behavior (see Guégan and Rioublanc (2003)). In this section, we investigate how the number $n_{s_t}$ could influence the asymptotic behavior of this model. More precisely, we want to know if a two-state Markov switching model can exhibit spurious long memory behavior for some $n_{s_t}$ and short memory for some others, according to this number $n_{s_t}$.

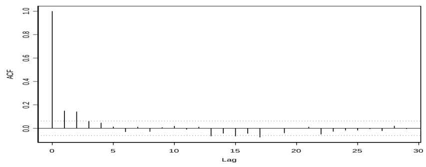
We simulate several series of the model (1) in such way that $n_{s_t}$, the number of states changes, increases. In our simulations, $n_{s_t}$ raises from 2 to 828. For each $n_{s_t}$, we consider the same Gaussian noise $\varepsilon_t$ for the two pairs of levels, $(\mu_1,\ \mu_2) = (5, -5)$ and $(\mu_1,\ \mu_2) = (0.5, -0.5)$. In most of cases, we consider $p_{11} \simeq p_{22}$. We also investigate the levels part on the autocorrelation function's behavior of the model (1).

First, we consider the case $(\mu_1,\ \mu_2) = (0.5, -0.5)$. When $n_{s_t}$ is smaller than 105, we clearly observe a slow decrease of the autocorrelation function, and thus some long memory behavior, see Figure 2 (a) for instance.

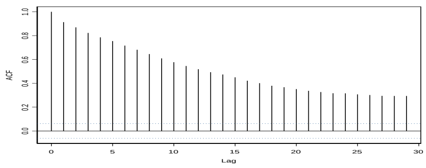
Then from $n_{s_t} = 112$, the autocorrelation functions of the simulated series behave almost like those of short memory behavior, although some lags remain slightly significant (see Figure 2 (b)). Here, we will thus consider that the model (1) exhibits a long memory behavior when the process changes of states less than 110 times and a short memory behavior when $n_{s_t}$ is larger.



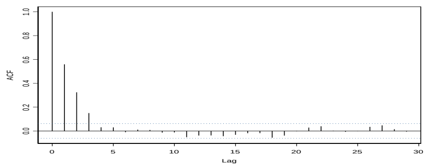
(a) $(\mu_1,\ \mu_2) = (0.5, -0.5)$, $n_{s_t} = 23$



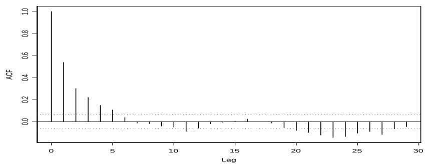
(b) $(\mu_1,\ \mu_2) = (0.5, -0.5)$, $n_{s_t} = 132$



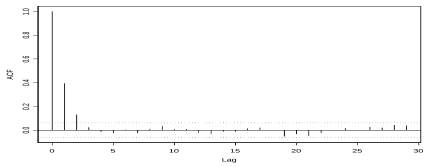
(c) $(\mu_1,\ \mu_2) = (5, -5)$, $n_{s_t} = 23$



(d) $(\mu_1,\ \mu_2) = (5, -5)$, $n_{s_t} = 208$



(e) $(\mu_1,\ \mu_2) = (5, -5)$, $n_{s_t} = 214$



(f) $(\mu_1,\ \mu_2) = (5, -5)$, $n_{s_t} = 299$

Figure 2: Autocorrelation functions of some simulated series issued from model (1).

Then, we consider the couple $(\mu_1,\ \mu_2) = (5, -5)$. We are expecting a similar result. For fixed transition probabilities $p_{ii}, i = 1, 2$, we observe a much slower decrease of the autocorrelation functions than those obtained for $(\mu_1,\ \mu_2) = (0.5, -0.5)$, compare Figure 2 (a) and Figure 2 (c) for instance. The only difference between the two series stands in the mean values, because both are obtained from the same states' vector $s_t$, simulated from $(p_{11},\ p_{22}) = (0.975, 0.975)$. Thus, the levels values seem to have an impact on the convergence's speed of the autocorrelation function of model (1). The autocorrelation functions decrease always slowly until $n_{s_t} = 192$, see Figure 2 (c) for instance. Then, from $n_{s_t} = 208$ to 285, various cases arise. Although most of the autocorrelation functions decrease quickly towards 0, some others still decline slowly, see Figures 2 (d) and (e) for an illustration. From $n_{s_t} = 247$ to 285, the autocorrelation function behavior is close to that of short memory behavior, but we only observe an real quick decrease from $n_{s_t} = 299$, see Figure 2 (e). Here, we consider that before $n_{s_t} = 214$, we are in presence of long memory, then a transition's period occurs between $n_{s_t} = 219$ and 285 and from $n_{s_t} = 299$, the model (1) exhibits a short memory behavior.

We put evidence on the important part of the means values $\mu_i, i = 1, 2$ on the asymptotic behavior of model (1). Indeed, for $(\mu_1,\ \mu_2) = (0.5, -0.5)$, the empirical autocorrelation functions decrease exponentially towards 0 from $n_{s_t} = 112$ , whereas for $(\mu_1,\ \mu_2) = (5, -5)$, we have to wait until $n_{s_t} = 299$ to observe a such decrease. By evolving $n_{s_t}$ in our simulations, we have also pointed out that the occurrence of the states changes has an impact on the autocorrelation function behavior. Indeed, the more the process changes of states, the quicker the decay of the autocorrelations is.

**Remark:**

In most of simulations, we have considered $p_{11} \simeq p_{22}$. Nevertheless, the same number of states changes $n_{s_t}$ can be obtained from different transition probabilities. More precisely, during our simulations, for $T = 1000$, we have obtained $n_{s_t} = 9$ for $(p_{11}, p_{22}) = (0.99, 0.99)$ but also for $(p_{11}, p_{22}) = (0.999, 0.005)$. The trajectories of the two simulated series are represented on Figures 3 and 4.

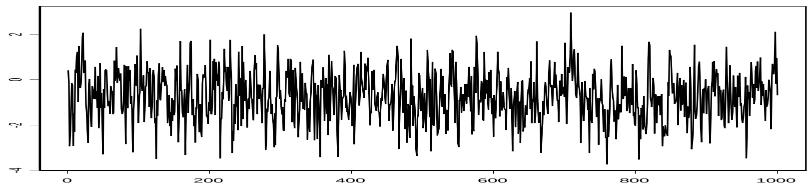


Figure 3: Simulated series issued from model (1) with $(\mu_1,\ \mu_2) = (0.5, -0.5)$, $(p_{11}, p_{22}) = (0.999, 0.005)$ and $T = 1000$. $n_{s_t} = 9$.

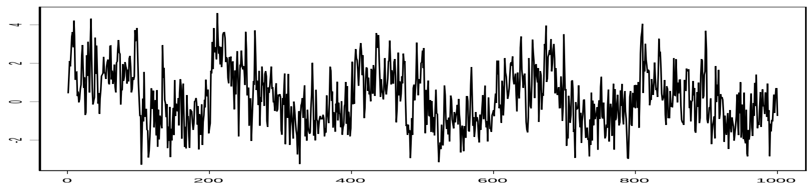


Figure 4: Simulated series issued from model (1) with $(\mu_1,\ \mu_2) = (0.5, -0.5)$, $(p_{11}, p_{22}) = (0.99, 0.99)$ and $T = 1000$. $n_{s_t} = 9$.

The states changes occur obviously differently in the two simulated series. Will the autocorrelation functions of the respective series behave nevertheless similarly? Figures 5 and 6 prove that not necessarily. In the first case, the probability $p_{11}$ is very high whereas $p_{22}$ is very small.

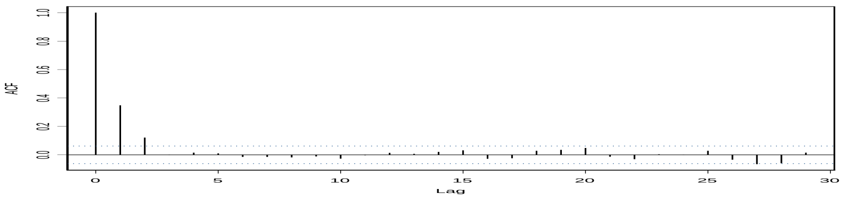



Figure 5: Autocorrelation function of the series represented on Figure 3.

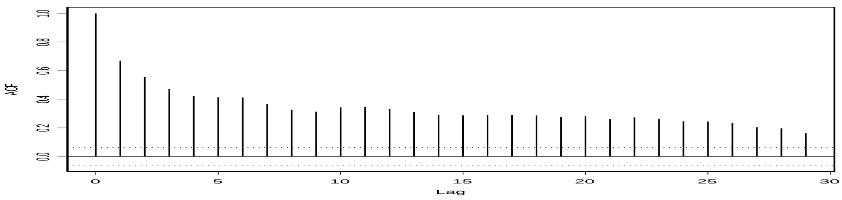



Figure 6: Autocorrelation function of the series represented on Figure 4.

Thus, even if the system goes into the second state, it immediately get back in the first state. The process observed on Figure 3 behave like an autoregressive model of order 1. Thus, it was expected that its autocorrelation function represented on Figure 5 decrease exponentially towards zero, like a short memory process' ones. The two states of the second series reported on Figure 3 are much more persistent, that is why the autocorrelation function represented on Figure 6 decrease slowly. Thus, the number of times where the process gets from one state to the other is not the only responsible for the behavior of the autocorrelation function of a two-state Markov switching model. This example shows that the way the changes take place into the sample act also on this behavior. Now, we better understand the link between long or short memory inside this model and the transition probabilities, since we simulate the states' process $s_t$ from these latter. The notion of states' duration seems to be very an important notion in the asymptotic behavior of the model (1).

## 4 Conclusion

In the literature, many studies have shown that stationary processes with structural breaks can produce slowly decaying autocorrelations, and are thus similar to those of long memory process. This phenomenon is observed in many economic and financial time series. Nevertheless, we show that for a two-state Markov switching, this decay of the autocorrelations is not real long memory but some spurious long memory created by the mean structural change. Indeed, with a simple example, we have shown that the slow decay observed on some empirical autocorrelation functions of the model (1) is due to regime switches, and thus represents the property of spurious long memory. This is in accord with the theoretical result previously demonstrated, which proves that Markov switching models exhibit asymptotically a short memory behavior, with respect to the behavior of their autocorrelation function, see Guégan and Rioublanc (2003). From simulations, we show that the autocorrelation functions of Markov switching models exhibit long memory when the shifts are rare. More precisely, we show that the frequency of switching is partly responsible for the behavior of short or long memory of those models. The more switching there exists, the quicker the decay of the autocorrelations is. With two simple simulated series issued for model (1)), we observed that the way the states changes intervene in the sample is important too. Now, we better understand how the transition probabilities $p_{ii}, i = 1, 2$ play a prominent role on the autocorrelation function's convergence's speed of Markov switching models. Nevertheless this problem needs to be more investigated in order to know if the means values are actually responsible for this behavior, or if it can not rather be the difference $\mu_1 - \mu_2$.

Moreover, we have restricted the study to the case $p_{11} \simeq p_{22}$, but it will be very interesting to consider many pairs of transition probabilities in order to better understand how those parameters really intervene in the autocorrelation function behavior of the model (1).

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