

# Times-to-Default : Life Cycle, Global and Industry Cycle Impacts

F. Couderc\*, O. Renault†

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## ABSTRACT

This paper studies times-to-default of individual firms across various risk classes. Using Standard & Poor's ratings database we explore the behaviour of instantaneous default probabilities conditioning on micro- and macro-economic variables and identify their impacts. We show that both financial markets and the global business cycle matter for explaining times-to-default. More importantly past macro-economic and financial information is also instrumental in modelling large variations in intensities. Short term determinants and long term trends are therefore relevant for capturing patterns of default probabilities. In a semi-parametric setup we document limitations of traditional credit risk models. We also show how parsimonious covariate choices or lagged information can improve the performance of simple models, and introduce industry contagion factors.

\*FAME & University of Geneva.

†Associate Fellow, FERC, Warwick Business School, Coventry CV4 7AL, UK.

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## Introduction

The academic literature on credit risk has historically been more focused on the pricing of credit risk rather than on risk management issues, and more specifically on single-name asset pricing. As pricing relies on the dynamics of credit variables (in particular default probabilities and spreads) under the risk neutral measure and not under the objective probability measure, relatively little is known about intensities (or instantaneous default probabilities) under the objective measure. Yet historical default and migration probabilities constitute critical inputs of popular commercial credit risk portfolio models as well as defaultable asset pricing models such as the Jarrow Lando & Turnbull (1997) specification. Even for pricing purposes, information arising from historical probabilities may be relevant, as shown by Bielecki, Jeanblanc & Rutkowski (2004) in an incomplete market setting. Recently Fledelius, Lando & Nielsen (2004) and Couderc (2004) have studied objective intensities of default in non-parametric frameworks and documented cyclical effects and other non Markovian patterns. The latter paper shows that calendar time effects account for a large part of deformations in the term structure of default probabilities, thus indicating the need to condition on economic indicators. Duffie & Wang (2004) proposed an empirical investigation of parametric log-linear intensities using accounting covariates and the so-called distance-to-default. Half-way between these two approaches, this paper extends both analyses by putting more structure on the intensity model and provides insights into the determinants of intensities. We perform a detailed analysis of explanatory variables through semi-parametric and parametric specifications. We identify impacts of the business cycle and the financial market, and study whether lagged information conveys additional explanatory power on the behaviour of default probabilities. The semi-parametric models are particularly relevant diagnostic tools. They allow us to check misspecifications of the implicit assumption of conditional exponentiality used in log-linear models, and to assess the performance of the covariates in explaining times-to-default. The paper also contributes to further the knowledge on market perception of default risk. There is still a debate on the global shape of intensities of default with respect to ageing effects. Madan & al (2004) have proposed an investigation of hazard rates under the risk-neutral measure from a pool of credit default swap on large companies. They find strong support for increasing hazard rates. However, as they underline, market insights concerning default may largely differ from physical phenomenon because of collateral effects. By separating business conditions from ageing effects our semi-parametric specifications exhibit physical shapes of intensities.

In credit risk measurement, it is customary to oppose structural models (firm-value based) and reduced-form models (intensity-based). The former are more intuitive from an economic standpoint as they model default as the first time at which firm asset values fall below liabilities. The intensity based approach offers better calibration and easier extensions to multiple assets (e.g. see Duffie and Singleton (2003)) but ignore what the default mechanisms are. In this paper we apply the intensity-based technology to estimate default probabilities on Standard & Poor's ratings database, while keeping track of default determinants by incorporating observable factors. Our approach is closely related to factor-models and our results should be relevant to this broad class of models that are widely used by practitioners. However our modelling setting is richer as we do not only focus on the calendar time evolution of the default distribution across firms. We employ a duration framework to be able to assess credit risk both in terms of

default probabilities and times-to-default. This methodology can accommodate possible "life cycle" effects, i.e. changes in default probabilities arising from the "age" of a company (or the time it has spent in a given risk class). In retail credit both age and calendar time effects are routinely used, but the literature on corporate credit often ignores those effects. Besides factor-models mainly decompose the default risk in common determinants (systematic risk) and idiosyncratic terms. The common part is often reduced to one or two non observable factors. While a lot of effort has been put on the study of firm specific determinants of default as well as on correlation between firms, we concentrate on observable common determinants. Not relying on latent covariates is key to enhancing forecast accuracy of times-to-default. In addition the identification of systematic component is essential for understanding the risks of baskets of defaultable assets and of complex credit derivatives. Therefore the common factors we test in this paper can then naturally be used to study common default dependencies and to extend the methodology related to portfolio risk activities. Duration frameworks and more particularly Cox-type semi-parametric specifications such as that proposed in this study have already been used in finance. For instance Lunde, Timmermann & Blake (1999) applied the methodology to study the performance of mutual funds. Bandopadhyaya (1994) studied the behaviour of firms under the Chapter 11 protection. Wheelock & Wilson (1995) (2000) explored the reasons underlying bank failures or acquisitions, and Shumway (2001) investigated bankruptcy from the equity market, also using a similar setup.

The contributions of the paper are fourfold. First, we provide extensive empirical analysis of the behaviour of default probabilities conditioning on financial markets, the business cycle and the credit market. We explore the determinants of default and their horizons. Second, we test the appropriateness of some log-linear duration models relying solely on financial information to explain times-to-default across the rating spectrum. For that purpose, a Cox-type semi-parametric framework based on the Gamma kernel is proposed. The use of a Gamma kernel estimator is crucial to capture variations in intensities of default and deficiencies of models. Contrary to standard estimators such as the Ramlau-Hansen (1983) estimator, it is free of boundary bias<sup>1</sup>, corrects their tendency to oversmooth hazard rates and allows for the evaluation of probabilities in the short run. Third, we show how careful choices of variables and lags can substantially enhance the performance of log-linear models. Finally we explore industry contagion effects which may capture the remaining persistency. We use autoregressive models that are more traditionally applied to high frequency trade data. Our results deliver guide lines for future research, and should prove useful to improve the fit and accuracy of current pricing and risk measurement models.

The paper is organised as follows. In the first Section we briefly present the ratings data and discuss structural factors included in our analysis. Section 2 studies the determinants of default through log-linear models conditional on our set of economic and financial covariates. We consider a broad range of specifications, including lagged information, and discuss extensively empirical results. It includes a more thorough analysis of the determinants of default probabilities and tests of "life cycle" effects within a given risk class. Estimators of default

<sup>1</sup>This feature has already been widely documented in the case of density function estimation with semi-finite support. For instance see Chen (2000).

intensities in a semi-parametric setting are proposed in Section 3 in order to assess the performance of log-linear models. This class of models, which is widely used in risk management, appears to perform poorly when calibrated on financial covariates. We propose simple refinements of these models based on macroeconomic and credit market variables, as well as on a careful choice of lags, which considerably increase explanatory powers over all rating classes. In Section 4 we show how industry factors could be instrumental in further improving intensity models. We point out that some industries may be forerunner of the global business cycle whereas others just suffer from its consequences.

## I. Potential Determinants of Default

### A. Ratings and Duration Data

Our ratings data was extracted from Standard & Poor's Credit Pro 6.6 database. This database contains S&P's rating histories for 10439 companies over the period Jan. 1981 to Dec. 2003. Overall 33044 rating migrations are recorded in CreditPro as well as 1386 defaults and default rate ranges from 3% to 29% across industries. The Credit Pro database as already been used and extensively described by Bangia & al. (2002) over the period Jan. 1981 to Dec. 1998. Rating events require careful treatment as three sources of censoring are present in the database. Left truncation arises from the fact that 1371 issuers already had a rating before they were included in the database (i.e. before Jan. 1981). We do not have information about when the first rating was published and therefore for robustness checks we run all estimations both on the full sample and on the reduced sample excluding left-censored data (the reduced sample contains 9068 companies and 25993 rating migrations). Obviously, right censoring is also an inherent feature of any ratings database as most companies survive after the end of the recordings. A specific type of right censoring requires special consideration. Some companies leave the ratings process and fall into the not-rated (NR) category. Several reasons may explain this fact: the rated company may be acquired by another firm or may simply decide no longer to be rated by S&P. The database has the nice feature to identify firms that migrated to NR and subsequently defaulted. therefore the NR class is not a complete loss of information: although there is no longer any indication of credit quality, a NR firm is a non defaulter.

Within our sample, firms are classified by industrial groups and each of them has been refined by subindustry criteria. Practically, we have at our disposal 13 industries or 526 distinct subindustries distributed among 93 countries. But 6897 firms or 66 % are US ones. Moreover, S&P attributes 25 distinct ratings plus the NR one, but we aggregate the data coming from a grade and its plus/minus modifiers because of minimal population requirements. Besides, all grades below B- have been put in the CCC class.

The database allows to consider two types of durations. On the one hand we look at times-to-default from entry in a risk class up to the last available observation. On the other hand, we examine times-to-default conditional on staying in a given risk class up to the default time.

## B. Default Drivers

In this Section we present potential determinants of default intensities that will be used to calibrate log-linear models. Several authors have calibrated mainstream models on financial variables : interest rates for reduced form models and equity information for structural models. For instance Duffee (1999), Driessen (2002) or Collin-Dufresne & al. (2001) examined impacts of selected financial covariates on credit spreads. The business cycle has also been factored in other studies (e.g. see Koopman & Lucas (2004)). To our knowledge there exists no systematic study of the determinants of default including both financial and non financial variables. Given that our rating and default sample is primarily American, we use US explanatory variables. Many of these variables are redundant and will be eliminated at the estimation stage in multi-factor analysis. Our data was extracted from the Federal Reserve of St. Louis website and Bloomberg. The first two variables are key factors of structural models.

- Return on S&P500: Short and mid term economic performance should be positively correlated with S&P500's returns and we expect a negative impact on default intensities. Furthermore, an increase in equity prices tends to decrease firm leverage and therefore also push down default probabilities.
- Volatility of S&P500 returns: In a traditional Merton (1974)-type model, the two drivers of default probability are leverage and the volatility of firms' assets. The volatility of equity returns is often used as a proxy for the latter and we expect it to have a positive impact on default intensities. We use the realized annualized volatility computed over the last 60 trading days.
- 10 yr. treasury yield: Higher interest rate levels imply payments of higher coupons on corporate bonds which may entail difficulties for borrowers. Hence, this variable could impact positively on default probabilities. However interest rates tend to be lower in contraction periods and higher in expansions. Thus the ultimate impact on intensities is uncertain and may depend on issuer quality.
- Slope of term structure (10 yr. rate minus 1 yr. rate): Steep term structures of interest rates are usually associated with strong growth prospects and we expect this variable to impact negatively on mid- to long-term intensities.
- Real GDP growth: As a signal of current macro-economic conditions this variable should be negatively correlated with short term probabilities.
- Industrial production growth: This is an alternative growth measure which should have a similar impact as that of GDP growth. Interest lies in the more frequent update of these series.
- Personal Income Growth: Same expected impact as previous two variables.
- CPI growth: Inflation is again a general indicator of economic conditions. We expect to observe a negative correlation with short term default probabilities, as high inflation has often been associated with growth.

Beside general economic variables, more specific credit factors may be instrumental in explaining default intensities. Some of them are used to derive implied probabilities from reduced-form models.

- Spread of long term BBB bonds over treasuries: Spreads should reflect default probability as well as expected recoveries and should therefore be positively correlated with default intensities.
- Spread of long term BBB bonds over AAA bonds: This variable factors in the risk aversion of investors and may be a measure of their risk forecast. Furthermore, an increase in the relative spread may reflect an increase in firms' asset volatilities (see Prigent, Renault & Scaillet (2001)). We therefore expect default intensities to increase with this variable.
- Net issues of Treasury securities: This indicator should positively impact short term probabilities of default as higher deficit and borrowing is an indicator of economic difficulties. Furthermore, high public sector borrowing may crowd out private borrowers and lead to increased financial difficulties for firms. However, if borrowing is used for investments, an increase in Treasury issuance may be linked to stronger growth in the long term and decreasing probabilities of default.
- Money lending (M2-M1) and bank credit growth: These factors measure credit liquidity and should be associated with default intensities.

Before proceeding with regressions of intensities on the above variables, we ran a principal component analysis (PCA) on this set of macro-economic factors to determine how many factors were necessary to explain most of the variations in intensities. Using the eigenvalue criteria, we found that five significant factors were explaining a cumulated percentage of 71% of the total variations captured by this information set. PCA analysis on non-parametric estimates of default intensities based on the Gamma kernel (rather than on the raw data) also suggest that four to five factors are relevant and account for 73% to 94% of the changes in intensities<sup>2</sup>. The relatively low explanatory power obtained with 5 factors in the PCA analysis indicates that it is unlikely that we will be able to explain more than 75% of variations in default probabilities using the set of variables presented above but we can expect to reach a higher figure than the 25% reported in empirical applications as Collin-Dufresne & al. (2001) for the credit cycle.

Our dataset includes both forward looking and current information. In particular, stock market components, CPI and personal income growth deliver snapshots of current global business conditions whereas interest rate based measures also contain information on future economic conditions. Default is not an exogenous process but often the result of renegotiations between the firm and its creditors. Good economic prospects should induce investors to renegotiate contracts rather than trigger liquidation. This should be reflected in the significance of some "forward looking" explanatory variables. As we underlined our database not only

<sup>2</sup>The number of factors and cumulated explanatory power depend on the risk class considered, i.e. whether one uses the entire sample, or only non Investment grades, BB, B or CCC firms. The non-parametric PCA inputs are estimated increments in intensity from the Gamma kernel intensity estimator (see Section A).

contains information on durations to default but also rating migrations. Ratings are usually regarded as opinions on the creditworthiness of obligors over the medium term. Their predictive power should be valuable in explaining medium-term default probabilities. Consequently we believe that the set of variables described above should be expanded with rating based variables. Kavvathas (2000) used as explanatory variables the weighted<sup>3</sup> log upgrade-downgrade ratio and the weighted average rating of new issuers. He actually only took into account the first PCA factor of these variables, but other variables may also be relevant. The average rating of financial institutions may be of primary interest in describing the short term trend of the global economy (in terms of credit crunch for instance). This trend can also be captured by the ratio of downgrades over all non-stayer transitions. Thus we include the following rating-based variables:

- IG and NIG<sup>4</sup> upgrade rates : both variables should include information on economic health.
- IG and NIG downgrade rates : downgrades should be higher in bad conditions.

One may also want to add firm specific factors such as leverage, cash flows or size which constitute the main determinants of bankruptcy as pointed out by Lennox (1999). In particular, historically the size of the firms seems to induce very different behaviours. Moreover, these factors may be introduced at aggregate levels, for instance to provide a measure of the solvency of new issuers. Recently, Duffie & Wang (2004) used the earnings ratio and firm size as specific factors, jointly with a measure of the distance-to-default. However ratings should constitute stable and good proxies for firm-specific components and a fair alternative as specific variables are not always available. From an accounting perspective, default cannot realistically be initiated by small changes in earnings, leverage or any balance sheet information but rather by negative trends or by unexpected large changes in cash flows. Any negative trend should have been incorporated in issuer ratings. Furthermore as pointed out by Collin-Dufresne & al. (2001) who used leverage, idiosyncratic factors do not represent the dominant factor in credit risk changes and seem to exhibit lower explanatory power than common components.

[INSERT TABLE I HERE]

Table I presents basic statistics on the set of retained factors. Obviously some of the above variables are highly correlated, which would deteriorate statistical significance on the full set of variables. However our main purpose consists of identifying the relevant factors and their relationship with default probabilities. Therefore, we will concentrate on univariate and parsimonious multivariate analysis.

<sup>3</sup>with respect to the distance between the old and the new rating.

<sup>4</sup>The investment grades class (IG) gathers together the AAA, AA, A and BBB classes whereas BB, B and CCC classes are collected in the non investment grades (NIG) class.

## II. Predictors and Indicators of the Default Cycle

The analysis of the impacts and explanatory powers of the potential determinants of default discussed above requires to define a basic framework. In finance a simple and traditional practice to study the effects of structural factors on stock returns consists of running regressions of returns on the explanatory variables. We propose to use an analogous technique in order to examine sensitivities of default intensities on our information set. As intensities have to be positive, we look for "regressions" of log-intensities. These models are known as log-linear duration models. We briefly recall the basics of these models and then turn to estimations. We start with time-dependent covariates which capture instantaneous impacts of economic conditions on intensities for various lags. Then we consider time-independent covariates which aim at capturing potential effects of initial conditions (e.g. the state of the economy at a firm's entry in the risk class). We refer to this last phenomenon as life cycle effects or time profiles since a factor is likely to modify the risk of default of firms over their whole life in a given risk class.

### A. Log-Linear Models

Form a firm  $i$ , let  $D_i$  denote the uncensored duration up to default and  $C_i$  the censored duration.  $U_i = \min(C_i, D_i)$  is the time at which the firm leaves the class either because of censoring ( $C_i$ ) or default ( $D_i$ ). The  $U_i$  are the true observations, jointly with indicators of censoring. We also let  $\mathbf{Z}$  denote a vector of explanatory variables. We consider intensities as exponential affine functions of factors which remain constant between two observations of the factors. Hence, conditional on the realization of the covariates, durations are piecewise exponential :

$$\lambda^i(u, t_i) = \exp(\gamma + \beta' \mathbf{Z}(t_i, u + t_i)) \quad \forall i. \quad (1)$$

where  $\mathbf{Z}$  can include a mixture of time-dependent and time-independent covariates. The exponentiality assumption could be relaxed by replacing the constant  $\exp(\gamma)$  by another formulation. For instance one could impose a conditional Weibull hazard where the covariates may be time-dependent (through  $u + t_i$ ) and/or time-independent (through  $t_i$ ).

Our chosen parametric framework allows to use efficient and tractable estimation techniques, namely maximum likelihood<sup>5</sup>. The estimation works in the following way. Assuming that structural variables dynamics are independent, the likelihood is separable into two terms, one related to the dynamics of covariates and the other one dealing with conditional durations. Therefore if we are not interested in factors dynamics, we can ignore this part and focus purely on durations. For a given firm  $i$ , the likelihood  $l$  of observed duration  $u_i$  can be written conditionally on factors realizations at firm's "death or exit" but the whole construction of the risk classes information set has to be known :

$$l(u_i) = l_1(u_i | \mathcal{F}_{t_i+u_i}^Z) \times l_2(\mathcal{F}_{t_i+u_i}^Z)$$

<sup>5</sup>See Duffie & Wang (2004) for a clear presentation of maximum likelihood in this case.



where  $l_1$  is the univariate likelihood of conditional durations and  $l_2$  the likelihood associated with the dynamics of covariates. From that point, letting  $L_1$  and  $L_2$  denote the multivariate counterparts of  $l_1$  and  $l_2$ , the multivariate likelihood function for a sample of  $n$  firms observed up to time  $t = \max_i \{t_i + u_i\}$  is defined by

$$L(u_1, \dots, u_n) = L_1(u_1, \dots, u_n | \mathcal{F}_t^Z) \times L_2(\mathcal{F}_t^Z)$$

with

$$L_1(u_1, \dots, u_n | \mathcal{F}_t^Z) = \prod_{i=1}^n \exp\left(-\int_0^{u_i} \lambda^i(s, t_i) ds\right) \left(\mathbb{I}_{(u_i > c_i)} + \lambda^i(u_i, t_i) \mathbb{I}_{(u_i^- < c_i)}\right) \quad (2)$$

where  $c_1, \dots, c_n$  are realizations of censoring variables  $C_1, \dots, C_n$ .

The estimation of this model is therefore quite straightforward and both censored and default durations contribute to the likelihood. The main task is the selection of appropriate explanatory variables. Empirical results on this specification are provided in next sections.

## B. Economic Shocks over Time

In this section, we determine whether intensities are sensitive to each factor identified previously. We also explore the necessity to lag factors to extract more information. Surprisingly, the issue of lagged information has been ignored in most papers, although Koopman & Lucas (2004) and Kwark (2002) have reported lagged effects between the market and the credit cycle. We analyse the explanatory power of each factor performing maximum likelihood estimations. For each covariate we run 11 estimations corresponding to 11 different lags. In all cases, we look at 95% and 99% confidence tests, and likelihood ratio. The alternative model of the likelihood ratio test corresponds to unconditional exponentiality, i.e. the case of constant intensity. We also break our dataset into several samples, namely investment grade (IG), non investment grade (NIG), AA, A, BBB, BB, B and CCC samples. For each of these risk classes we look both at durations to first exits from the risk class and durations to last days of observation. Further robustness checks are performed by leaving out left truncated firms and, focusing on the US subsample.

Tables III, IV and V present results on the IG and NIG samples<sup>6</sup> respectively over financial, business and credit indicators. Table II reports estimates of sensitivities with respect to upgrade and downgrade rates over rating classes. We have found that the sample used makes little differences to the results. For example, considering durations up to the first exits keeps sensitivities almost unchanged and only lower the significance of parameters. Focusing on the US subsample does not modify estimates more than 10% on average, and does not alter signs. Such robustness could be expected as risk classes are quite stable and the whole sample is made of 66% US firms.

<sup>6</sup>Further results are available on request.

From a general perspective all variables are significant and one could be puzzled by the fact that estimated sensitivities are almost identical or even higher for the IG class. However, with an average intercept around -13.1 for IG and -8.8 for NIG, the explanatory power is weaker for IG. The exponential form indeed smooths variations as the average decreases. This is confirmed by likelihood ratios<sup>7</sup> which are between four to ten times higher for NIG than for IG. In addition, these global findings do not change across ratings. In particular it is worth mentioning that due to the exponential specification, only signs, significance levels and likelihood ratios can provide insights, whereas possible differences in sensitivity levels across ratings cannot be interpreted.

Looking carefully at signs for lags up to two years, we observe that the probability of default covaries with the predicted signs. Increases in the market index decrease the probability of default while increases in volatility push up probabilities. Economic expansion indicators provide good news for default but credit crunch amplifies defaults from the money lending variable. Increases in interest rates are good news because they reflect anticipation of growth. On the contrary decreases in short term yield increase default probabilities as low rates are strongly correlated with recessions. The slope of the term structure of riskless rates enters the models with either a positive or a negative sign, depending on the lag. A steep contemporaneous or recent slope (with less than one year of lag) tends to be associated with higher intensities of default while past steep slopes (over one year lags) tend to decrease intensities. The only exception to this short term/long term split is for the CCC class, for which a steep slope is always associated with lower intensities, irrespective of the lags. This can be explained in two ways. First, low short term interest rates can indicate a slowing down of economic activity and it increases competition in the corporate bond market. Second, increases in long term interest rates are often interpreted as expectation of higher growth. Future growth may be dominant effect for junk issuers, as these companies are highly levered and require strong business conditions to move up the rating ladder.

Three year lagged information provides new insights. Sensitivities to covariates are either constant, increasing or decreasing as lags increase but we found that some factors impact differently on default probabilities in the long run. Some variables, such as the market index, the term structure slope, the real GDP growth or net treasury issues appear to be leading indicators of future peaks of the default cycle. For example early repayments and small levels of issues by US Treasury signaled the peak of the US cycle which was later followed by a major default crisis over the sample window. Hence negative net treasury issues increase future default probability at a three year horizon in our sample. Note that these lagged effects could also be significant because the default process is time consuming from its origination as reported by Altman (1989). From that perspective, the default cycle can also persists after economic recoveries, generating explanatory power for lagged covariates.

Furthermore, looking at likelihood ratios (LR), we observe that the default cycle and the credit cycle are not necessarily synchronous. Estimated sensitivities exhibit the best LR using forward credit covariates (IG spread, BBB yield). Similarly some factors like the personal income growth are not appropriate as they seem to lag the default cycle rather than lead it. Causality analysis in that direction would be particularly relevant for future research. Finally remark that rating classes are much more sensitive to downgrade rates than to upgrades.

<sup>7</sup>Not reported here.

[INSERT TABLE II HERE]

In Tables III, IV and V we can see that lagged covariates are significant at all stages. This finding could support the hypothesis that firms' default time profiles matter within rating classes. We now investigate this issue.

[INSERT TABLE III HERE]

[INSERT TABLE IV HERE]

[INSERT TABLE V HERE]

### C. Factors as Determinants of Time profiles

In this Section we test whether economic conditions could determine firms' default time profiles within rating classes and if those profiles provide additional explanatory power. Macroeconomic variables are intuitive candidates for the explanation of the shape of intensities over distinct vintages (see Couderc (2004)). For instance one can observe an initial fall in the intensity followed by a sharp increase for the 1986 pool, which was a year of robust economic growth. In order to address this hypothesis, we rely on the time-independent covariates setup.

Considering intensities up to the last day of observation from entry in the rating process, estimates  $\hat{\beta}$  do not show any evidence that some covariates have significant impacts over the whole intensity curve whatever the rating class we consider. We can reasonably believe that if a company faces credit difficulties when it enters into a specific risk class, either these difficulties should be absorbed or at least diluted after a while, or the company should default. In the latter case, the company should be quickly downgraded. Similarly, strong business growth in a sector may vanish quickly as that sector is likely to become more competitive, and also because any worthwhile project has limited duration.

As a consequence, instead of looking for impacts on full time profiles, we focus on short and mid term horizons within rating classes. Practically, we ran maximum likelihood estimations on subsamples imposing additional right censoring and left truncation on durations. We considered durations, up to 1 year, 3 years, 5 years, between 1 and 3 years, and finally between 3 and 5 years. Such cut-outs are suggested from Table III. As before we looked at factors' impact individually (results not reported here). All empirical results confirmed the findings of Tables III and II in the following way. If economic conditions do not affect the whole default time profile, the duration of a company within a rating class is influenced by conditions at entry in the class.

Using time-independent covariates, we find that factor signs are the same as those we obtained previously in the case of time-dependent covariates with lags from 0 to 2 years. This implies that the significance of the impacts of the real GDP, the S&P 500 returns, net treasury issues and the slope of the term structure, for lags longer than 2 years are weaker than those obtained for shorter lags. Shorter lags are intuitively those that are most relevant for the explanation of default intensities.

The above results suggest that, from a modelling standpoint, observed decreases in intensities in the very short run could easily be handled through conditioning on information at the firm's entry in the risk class. This would be especially relevant in short term risk management. Moreover it implies that, at the time of a rating change, short term risk predictions (less than 2 years) can be proxied through factors at that date and do not necessary require the prediction of factors.

### III. Tests and Improvements of Explanatory Factors Efficiency

We now turn to the assessment of log-linear models and, more generally, to the efficiency of the information set. Kavvathas (2000) and Aunon-Nerin & Burkhard (2003) show that economic variables do not explain a huge part of transition probability changes. Closer to our study, Collin-Dufresne & al. (2001) also report 25% to 30% of explanatory power by macro-economic factors on spread changes. These authors report that their residuals are highly cross correlated and that there is only one significant underlying factor which cannot be explained by their state variables. Such a factor will be examined later in Section IV. The explanatory variables used in Collin-Dufresne & al. included interest rates indicators, changes in volatility, expected recovery rates and leverage. One noticeable result is that, contrary to structural model predictions, systematic factors seem to be more important than firm-specific ones in explaining spread changes. The authors argue that the failure of their state variables to capture a large amount of the systematic part of spreads should be due to the strong impact of local demand/supply shocks<sup>8</sup>. We now bring complementary answers to these issues. In the following we propose a semi-parametric framework which allows us to extract variations in default intensities which are not explained by a given factor model. By doing so, we are able to study misspecifications of parametric models and to assess the performance of the covariates. In particular we show that explained variations in intensities could be widely underestimated because of inappropriate choices in the information set. Then looking carefully at models relying on financial market information, we show how traditional models can be enhanced while remaining parsimonious.

#### A. A Semi-Parametric Framework for Intensities

In this section we develop a semi-parametric framework to model default intensities and test the impact of the covariates presented above. We start from a fully non-parametric estimator of default intensities based on the Gamma kernel. We also consider parametric log-linear models using Cox proportional hazard methodology (Cox (1972) (1975)), a well known tool in biostatistics, both with time-independent and time-dependent covariate specifications. The

<sup>8</sup>For instance, some fund managers systematically rule out non investment grade corporate bonds from their portfolios: at the time of a downgrade, numerous funds close their positions. In addition, identified demand and supply phenomena such as the "flight-to-quality", amplify the credit cycle, especially during recessions. Note that all these phenomena are closely linked to credit events, and as a consequence should be captured, at least partially, by our information set.

baseline hazard is estimated using the GRHE estimator<sup>9</sup> (Gamma Ramlau-Hansen Estimator) while the parametric part is estimated by partial likelihood. A complete survey of related models, estimation techniques and asymptotics can be found in Andersen & Gill (1982) or Andersen & al. (1997).

### The GRHE estimator

The GRHE estimator has been introduced by Couderc (2004). It is based on a Gamma kernel smoother of the cumulative hazard rate belonging to the popular Nelson-Aalen class<sup>10</sup>. The use of asymmetric kernels is crucial in credit risk. Indeed as duration increases, the number of firms under observation tends to zero. Standard smooth estimators with finite and symmetrical support need large bandwidths (smoothing parameters) to provide estimations on the long run. Therefore they suffer from oversmoothing, which reinforces the boundary bias inherent to these kernels (e.g. see Bouermani & Scaillet (2001) for the properties of asymmetric kernels on density function estimations). Couderc [2004] has shown that the GRHE is free on boundary bias and is able to capture changes in intensities on the short run (which may cover up to 5 years) as well as subsequent deformations, whereas Fledelius & al. (2004) obtained flat intensities using the standard Epanechnikov kernel. Empirical applications hereafter prove that not using an unbiased estimator would lead to inaccurate assessments of log-linear models. The necessary conditions ensuring the consistency of the estimator are assumed to be met. Accordingly, all firms in a given risk class are supposed to be homogenous and independent. Censoring mechanisms which may prevent from observing firms up to their default time are random and independent from the default process. For a given firm, these mechanisms are reported through the process  $Y^i(u)$ . The most important building block of the GRHE lies in the following assumption :

**Assumption III.1** *The intensity of individual firms satisfies the Multiplicative Intensity Model:*

$$\lambda^i(u) = \alpha(u) Y^i(u) \quad (3)$$

where  $\alpha(u)$  is deterministic and called the hazard rate whereas  $Y^i(u)$  is a predictable and observable process.

Then the estimator is specified as :

**Definition III.1** *The gamma kernel estimator  $\alpha(u)$  of the hazard rate (Gamma Ramlau-Hansen Estimator, GRHE) is defined by*

$$\alpha(u) = \int_0^{\infty} \frac{1}{Y(s)} \frac{s^{u/b} e^{-s/b}}{b^{u/b+1} \Gamma\left(\frac{u}{b} + 1\right)} dN_s. \quad (4)$$

<sup>9</sup>See below.

<sup>10</sup>See Andersen & al. (1997) (1982) for details on the Nelson-Aalen estimator and its properties.

where  $dN_s$  counts the number of default at time  $s$  and  $Y(u)$  is computed as the number of firms for which the last time of observation is greater than  $u$ .  $Y(u)$  is usually described as the risk set and handles censoring.  $b$  is a smoothing parameter, the so-called bandwidth. The intuition behind this non-parametric estimator is as follows: the probability of default over the next infinitesimal time step is estimated as a weighted average of past, current and subsequent instantaneous default rates. The weights are determined by the choice of the kernel, by the bandwidth as well as by the durations between default events. Default rates are computed as the ratio of the number of firms defaulting at the same time over the number of firms in the sample which survived up to that time.

The restrictions imposed on the process  $Y(u)$  are sufficiently weak to permit more complex specifications of this process. In what follows we rely on the multiplicative intensity model, to enrich the firm-specific part of the intensity specification by introducing covariates. As a consequence, one should think about this non-parametric estimator as a useful diagnostic tool to model the unexplained baseline hazard.

### Time-Independent Covariates

Let  $\mathbf{Z}$  denote a vector of covariates. The case of time-independent covariates corresponds to the following assumption on the behaviour of intensities.

**Assumption III.2** *The hazard functions conditional on structural factors are proportional to a class baseline hazard  $\lambda^\circ(u)$  representing the common intensity shape:*

$$\begin{aligned}\lambda^i(u, t_i) &= \lambda^\circ(u) \exp(\boldsymbol{\beta}'\mathbf{Z}(t_i)) & \forall i \\ &= \alpha^\circ(u) Y^i(u) \exp(\boldsymbol{\beta}'\mathbf{Z}(t_i)) & \forall i\end{aligned}\tag{5}$$

where  $\mathbf{Z}(t_i)$  is the set of structural variables taken at the date of entry  $t_i$  of the firm  $i$  in the class, and  $\boldsymbol{\beta}$  is the vector of sensitivities associated with a given risk class.

In this framework, provided that structural variables dynamics are not explosive, the Gamma kernel estimator of the baseline intensity becomes:

**Corollary 1** *Under assumptions III.1 and III.2, a semi-parametric estimator of the baseline hazard function is given by*

$$\hat{\alpha}^\circ(u) = \int_0^\infty \frac{1}{\hat{Y}(s)} \frac{s^{u/b} e^{-s/b}}{b^{u/b+1} \Gamma(\frac{u}{b} + 1)} dN_s\tag{6}$$

$$\hat{Y}(s) = \sum_i Y^i(s) \exp(\hat{\boldsymbol{\beta}}'\mathbf{Z}(t_i))\tag{7}$$

A convenient feature of this model is that an estimate  $\widehat{\beta}$  of the sensitivities  $\beta$  can be derived separately from the baseline intensity through Cox partial likelihood:

$$\widehat{L} = \prod_{k=1}^n \frac{Y^k(u_k) \exp(\beta' \mathbf{Z}(t_k))}{\sum_i Y^i(u_k) \exp(\beta' \mathbf{Z}(t_i))} \quad (8)$$

where  $u_k$  is the observed duration of firm  $k$  and  $t_k$  is its date of entry in the class. This powerful methodology allows for instance to identify industrial impacts. This can be achieved by introducing dummy variables for the various industrial sectors in the set of explanatory variables  $\mathbf{Z}$ . Furthermore this two-stage estimation technique does not affect the non-parametric estimation of the baseline intensity as the speed of convergence of the partial likelihood estimator is of order  $\frac{1}{\sqrt{n}}$  and therefore higher than that of the kernel estimator. In particular, confidence intervals on the baseline estimator are not affected by the estimation of  $\beta$ .

### Time-Dependent Explanatory Variables

The case of time-dependent covariates is quite similar.

**Assumption III.3** *The hazard functions conditional on structural factors are proportional to a class baseline hazard  $\lambda^\circ(u)$  representing the common intensity shape:*

$$\lambda^i(u, t_i) = \lambda^\circ(u) \exp(\beta' \mathbf{Z}(u + t_i)) \quad \forall i \quad (9)$$

where  $\mathbf{Z}(t)$  is the set of structural variables taken at the calendar date  $t$ , corresponding to the duration  $u = t - t_i$  for the firm  $i$  in a given risk class, and  $\beta$  denotes the sensitivities associated with this risk class.

In this formulation covariate values at calendar time  $u + t_i$  impact the default probability of firm  $i$  at that time, which has been in the risk class for  $u$  days. The baseline estimator becomes:

**Corollary 2** *Under assumptions III.1 and III.3, a semi-parametric estimator of the baseline hazard function is given by*

$$\widehat{\alpha}^\circ(u) = \int_0^\infty \frac{1}{\widehat{Y}(s)} \frac{s^{u/b} e^{-s/b}}{b^{u/b+1} \Gamma\left(\frac{t}{b_{c_j}} + 1\right)} dN_s \quad (10)$$

$$\widehat{Y}(s) = \sum_i Y^i(s) \exp\left(\widehat{\beta}' \mathbf{Z}(t_i + s)\right) \quad (11)$$

and the partial likelihood function from which the estimate  $\widehat{\beta}$  of  $\beta$  can be obtained by

$$\widehat{L} = \prod_{k=1}^n \frac{Y^k(u_k) \exp(\beta' \mathbf{Z}(t_k + u_k))}{\sum_i Y^i(u_k) \exp(\beta' \mathbf{Z}(t_i + u_k))} \quad (12)$$

One can naturally extend the set of variables further by considering both time-dependent and time-independent covariates. It is technically not more complicated but one has to be careful with the selected covariates, in order not to introduce identification problems. It then corresponds to the log-linear case from equation (1) where the constraint  $\lambda^\circ(u) = \exp(\gamma)$  has been imposed. Corollaries 1 and 2 are direct consequences of Andersen & al. (1997) and Couderc (2004).

## B. Economic Factors Inefficiency

The semi-parametric framework presented above allows us to test the usefulness of our set of factors. This is achieved by comparing fully non-parametric specifications and semi-parametric models. The time-dependent covariate framework also allows to assess the quality of log-linear models, i.e. parametric models. For each semi-parametric model we can associate a log-linear counterpart. Therefore, even if the estimation process is not the same, sensitivities to covariates should be equivalent<sup>11</sup>.

We checked this last point and found that all factors keep the same signs at the same horizons when switching from a log-linear to a semi-parametric model. Only small variations in magnitude can be observed. On the IG and NIG samples significant coefficients do not change. On rating subsamples, some coefficients become insignificant. However the main issue consists in the capacity of economic shocks to capture real intensities of default. If log-linear models are appropriate representations of default intensities, then the estimated baseline intensity  $\widehat{\alpha}^\circ(u)$  should be close to a constant function. Tables VI and VII present estimates of sensitivities  $\widehat{\beta}$  for different specifications over rating classes. Figure 1 plots the corresponding estimated baseline functions for IG and NIG classes.

[INSERT TABLE VI HERE]

Table VI stages three models fitted on stock market and interest rate information for various rating classes. For robustness checks, we included a dummy indicating non-US firms. Sensitivities to this non-US indicator were not significant. Likelihood ratios select the joint model as the best one, whereas interest rates alone provide the poorest fits. These results confirm findings of Driessen (2002) or Janosi & al. (2002) on credit spreads. Interest rates appear to be unhelpful explanatory covariates of default, whereas stock market information brings significant explanatory power. This is also true for the broader IG and NIG classes. Figures 2(a) and 2(c) display baseline hazard rates for non-parametric, semi-parametric and

<sup>11</sup>As we already underlined, the difference only lies in the slower rate of convergence of the semi-parametric specification to its asymptotic distribution with respect to the parametric model.



parametric specifications. In the latter, the baseline intensity is constant. It is very clear from the graph that the constant intensity does not represent the data accurately, in particular in the NIG class. Deviations from the constant (i.e. under the hypothesis that the equivalent log-linear model correctly represents the data) are reduced for the IG category when stock market variables are included. Nonetheless they remain significant and short term default probabilities are completely overstated. Deviations are not significantly reduced in the NIG class, and default probabilities are overestimated on a larger part of the debt life. In an attempt to capture effects of the 2001 default peak (responsible to a large extend for the first hump of the non-parametric estimates), the model grants too much weight to covariations between the stock market and defaults. In particular for all classes but the CCC, the model overestimates default probabilities on the short run (the dashed blue line is higher than the solid blue line up to 2 to 4 years)<sup>12</sup>.

Interestingly, the market volatility exhibits (through the S&P500) a striking feature in Table VI. Its relative impact with respect to other factors varies a lot over rating classes and corresponding coefficients are insignificant in fifty percent of the cases. Once again previous studies on credit spreads under the risk-neutral measure observed this lack of power of the volatility. Such a finding is highly challenging for structural models as the volatility determines the dynamics of equities. However the main impact of volatility may not immediate but may arise at a longer horizon: default is usually a progressive and lengthy process.

Factor models, such as the log-linear specification employed here have now become a market standard to model default probabilities. However their empirical performance has been challenged in many occasions. According to Yu (2002), factor models are not doomed to fail. They provide tractable and simple framework which can also be extended to model correlation arising from observed jumps in intensity at default times. In the remainder of the paper, we will focus on improving the empirical performance of factor models taking into account the above results.

### C. Choices of Parsimonious Information Sets

Our results suggest that a way to improve traditional log-linear models would be to include a variable reflecting economic conditions at the firm's entry in the rating class. This term would be specified such that its effect would vanish with time, i.e. initial conditions would progressively become irrelevant. The resulting improvement would however highly depend on the specification of the vanishing term, and in particular on its decay rate. We do not explore this further but focus on an alternative approach to improve the performance of log-linear models and to illustrate that information may be poorly exploited by traditional models.

[INSERT TABLE VII HERE]

<sup>12</sup>This may be partially corrected specifying Weibull baseline hazard instead of exponential hazard.

We select the most successful factors from our univariate analysis<sup>13</sup> according to their likelihood ratios. As a consequence, the model including the six factors outperforms models from stock market and interest rate information from LR tests. Most of the selected factors are not highly correlated, except the GDP and the S&P500 return which exhibit a correlation of 36%. This explains the fading of weights on stock market in this 6-factors model as the GDP offers a higher univariate LR (Table VII). All covariates bring additional information and should be kept. They all enter the model with the predicted signs. Figures 2(b) and 2(d) show that the baseline hazard is strongly shifted downwards and that distortions have been reduced, compared to Figures 2(a) and 2(c). This manifests that the new factors are instrumental in explaining times-to-default. This improvement comes from GDP and NIG downgrade rates, indicating a strong impact of the economic cycle on the default cycle, as well as a persistence in erosion of firms' quality.

[INSERT FIGURE 1 HERE]

Yet the humps are once again present which implies that financial market and business cycle variables do not capture all joint movements of default. We already identified one source of the phenomenon : the misspecification arising from ruling out effects of starting conditions on the short term. The presence of humps suggests that the default cycle is longer than the business cycle and that some persistency has to be incorporated. Furthermore, the tendency to overestimate default probabilities on low grades is due to incapacity of traditional factors to explain the huge number of defaults observed in the latest default crisis. As correlations of default seemed to be particularly high during this period, we may believe that this failure could be reduced by modelling snowball effects, for instance through contagion as we will see later.

#### D. Improvement of Traditional Models Through Lagged Covariates

We showed earlier that a careful choice of the set of explanatory factors greatly increases model performance. So far however, all selected variables have been contemporaneous, but potential lead-lag effects from financial markets or the business cycle on the default cycle could also be considered. Looking back at Table III, in a univariate setting, we can observe that lagged variables may have more explanatory power than current economic/market conditions, and some variables lead the default cycle by an average of three years. Besides lagged information could capture parts of persistency patterns. Therefore, we adapted our "stock market" model which is the most akin to structural credit risk models, and added lagged volatility and stock market return. Estimated parameters using our semi-parametric estimator are provided in Table VIII.

[INSERT TABLE VIII HERE]

[INSERT FIGURE 2 HERE]

<sup>13</sup>shown in Tables III and II.

The main insight from Table VIII is that lagged information is relevant in addition to contemporaneous one. While current volatility is not statistically significant for some classes, lagged volatility always is, except for the AA class. Both current and lagged volatilities are associated with higher default rates, as expected. The results on the lagged returns are more surprising, as they imply that high equity returns tend to be associated with higher default probabilities three years later. We can propose several explanations for this finding. First, it may reflect some cyclicalities in equity returns. We have found that high current equity returns tend to be associated with low current default rates. If there is cyclicalities in equity returns, with a peak-to-trough time of approximately 3 years, it is plausible that high returns will be associated with high default rates 3 years on. However we have found no evidence of such a cyclicalities. An alternative explanation would be that in good times (when the equity market is performing well) companies can afford to raise large amounts of debt while preserving acceptable levels of leverage. Several years later, this level of debt may become unsustainable for some firms, thereby raising the default rate.

[INSERT FIGURE 3 HERE]

Whatever the reason, including lags substantially improves the models from LR tests. Figures 3(a) and 3(b) show significant decreases in baseline hazards and as well as in deviations from the constant, compared to Figures 2(a) and 2(c). The improvement is even more dramatic than that achieved by additional factors for the NIG class (Figure 2(d)). We can observe a global levelling down of the errors. The latter result may be strongly dependent on our sample which consists of the past 23 years. However, Altman (1989) among others already suggested lagged dependencies would be useful because of the lag between the time at which a firm starts experiencing difficulties and its default time. We argue that lags should be retained because they capture at least parts of the persistency in the default process. Finally, let us notice that all figures strongly support the use of Weibull intensities as common baseline instead of the exponential distribution. The Weibull distribution produces monotonically increasing or decreasing intensities exactly as we obtained. Among all rating classes but the B and CCC we found that default intensities globally increase with durations (Figure 3 present estimations on the BBB to CCC classes). Economic conditions enter the default problem producing shocks along with the intensity trajectory. Hence our results show that conditional on the realization of the factors, the Weibull independence assumption should be preferred to the usual exponential independence of intensities. These results are in line with findings of Madan & al. [2004] under the pricing measure. Junk issuers exhibit a globally decreasing intensity implying that as time elapses, their financial standing should improve. Junk issuers can be seen as "do or die" firms. They will either default quickly or, given their high level of leverage and firm risk, they may be very successful in the longer term. Therefore, conditional on surviving the first few years, their default probability should fall substantially over the longer term. Most startups would fall in this category but they are not captured in our sample as very few of them are rated. Non-junk issuers exhibit an increasing hazard rate, reflecting increasing uncertainty in the longer term.

## IV. Modelling Dependence through Industry Contagion

In the previous section, we observed that a significant part of variations in NIG intensities is still unexplained by covariates. We pointed out the huge number of defaults that occurred during and after the 2001 recession and the difficulty for factor-models to fit this peak. So far we have ignored contagion effects. Classical reduced form models relying on structural factors have often been criticised for failing to replicate empirically the observed default correlation, as well as factor models. Jarrow & Yu (2001), Yu (2002) and Gagliardini & Gouriéroux (2003) documented the fact that when a firm defaults in a portfolio, other firms' intensities may jump and generate substantial default correlation. Contagion models (e.g. Davis (1999), Davis & Lo (2001a) (2001b), Schoenbucher & Schubert (2001)) are able to replicate some of these effects but they are often difficult to calibrate. Kyiotaki & Moore (1997) have shown through a theoretical equilibrium model that business cycle may only be a contagion vehicle. "Disease" starts from local changes in the credit cycle (roughly among an industry) and leads to global shocks in defaults. Therefore, in such a context it will be impossible to design a consistent model using only calendar time dependent factors and leaving aside pure default information and industrial factors. Koopman & Lucas (2004) studied this cyclicity using the well-established machinery of VAR models and factoring in cycles. Using GDP, bankruptcy rates and credit spreads as respective proxies for the business cycle, global credit cycle and pure default cycle, they show that co-movements between economic conditions and defaults may arise on the long term. However, as in previous studies, no general pattern can be extracted from the data but their findings support the idea that parts of the credit and default cycles contain their own dynamics. In this section we further document this phenomenon which may be instrumental in explaining the high level of defaults observed in the last recession.

In a two-state hidden Markov chain model, Crowder, Davis & Giampieri (2003) showed that adverse economic conditions do not affect all industries in the same way. There is evidence of sector-specific crisis, such as that affecting the energy sector in the mid-eighties or the telecom crisis of the early 2000s. Therefore, the number of defaults occurring in a given industry and in a given time step may represent a good indicator of the health of this industry and be useful in predicting default probabilities over the next time step. If the business cycle is really a contagion vehicle, such factors may efficiently enlarge the information set we previously used.

In order to test this, we rely on a class of autoregressive models that have been introduced to study the durations between trades in microstructure econometrics. These models are called Autoregressive Conditional Duration (ACD) models. More specially we focus on a log-ACD specification (see Bauwens & Giot (2000) and Engle & Russel (1998)). The intuition is the following. If one observes short times between defaults in a given sector, it probably means that the sector faces a crisis and therefore that one can expect the next default to occur shortly. In terms of intensities, it implies that the intensity of a firm in a given sector should be inversely related with past durations between two defaults in that sector. ACD models allow to take those effects into account by assuming that the expected duration until the next default is a function of past durations. Hence, defaults will tend to cluster. We control for sector size because, even if probabilities of default remain constant over time, variations in the sector size

will artificially create clusters in the sector intensity of default. Obviously large sectors will face a higher number of defaults than smaller ones for equivalent default probabilities.

For a given risk class  $c$ , we consider an aggregate counting process  $N_t^c$ . We introduce additional left-censoring for each firm  $i$  defining  $S_i = \max\{\tau_{j+1}; \tau_j < t_i\}$  where  $\tau_k$  denotes the  $k^{th}$  jump time of the process  $N_t^c$ . This censoring scheme is designed to take into account only firms which were already in the class  $c$  before the last observed default time in this class. Thus the process  $N_t^c$  can be written as  $N_t^c = \sum_{i \in c} \mathbb{I}_{(t \geq S_i; D_i + t_i \leq t; D_i \leq U_i)}$ , and  $\Delta N_{\tau_i}^c = \sum_{i \in c} \mathbb{I}_{(\tau_i \geq S_i; D_i + t_i = \tau_i; D_i \leq U_i)}$ .<sup>14</sup> We now specify the intensity of the process  $N_t^c$ .

**Assumption IV.1** *Durations between two jumps of the counting process  $N_t^c$  follow a log-ACD(1,1) model :*

$$\tau_k - \tau_{k-1} = \psi_c(k) \epsilon(k) \quad (13)$$

$$\begin{aligned} \log(\psi_c(k)) &= w_c + a_c \log(\tau_{k-1} - \tau_{k-2}) + b_c \log(\psi_c(k-1)) \\ &= w_c + a_c \log(\epsilon(k-1)) + (a_c + b_c) \log(\psi_c(k-1)) \end{aligned} \quad (14)$$

where  $\epsilon(k)$  are independent unit exponential variables<sup>15</sup>.

$\psi_c(k)$  is known right after the  $(k-1)^{th}$  default and represents the expected duration up to the  $k^{th}$  default given the population under observation at time  $\tau_{k-1}$ . In other words, conditionally on the past, durations between defaults are exponentially distributed and we assume that both right truncation and left censoring are uninformative. We now extract the relevant information for firms in the simplest way<sup>16</sup>:

**Assumption IV.2** *The intensity of default within the risk class  $c$  affects all firms in the same way, and intensity  $\lambda^i$  of firm  $i$  is given by*

$$\lambda^i(u, t_i) = \lambda^\circ(u + t_i)$$

Therefore, the intensity associated with the log-ACD model is given by

$$\lambda_c(k) = \frac{1}{\psi_c(k)} = \sum_{i \in c \text{ at } \tau_{k-1}} \lambda^\circ(k)$$

If  $X(k)$  is the number of firms under observation for the  $k^{th}$  duration, it simply states that the common intensity  $\lambda^\circ$  of firms which belong to the class  $c$  is given by  $\lambda^\circ(k) = \frac{1}{\psi_c(k) X(k)}$ . This

<sup>14</sup>Remark that by considering durations between default times, we do not focus on the complete natural filtration generated by all firms as we do not take entry dates into account.

<sup>15</sup>In order not to introduce bias,  $\tau_0$  will be the date at which the first default has been observed in the risk class.

<sup>16</sup>Assumption IV.2 could be enriched by conditioning on business and financial covariates.

last statement says that durations between consecutive default are also inversely proportional to the number firms in the risk class.

[INSERT TABLE IX HERE]

Table 6 provides the estimated parameters for the above model on 11 broad industry categories defined by Standard & Poor's. We find high levels of persistency for most industries. Implied intensities provide the most interesting results. Figures 4, 5 and 6 show that the macro-economic cycle does not have the same impact on default intensities in all industries, as found by Crowder, Davis & Giampieri (2003) in a simpler framework. For instance Figure 6(a) shows that the telecommunication sector was not affected by the 1990-1991 downturn but was the most hit by the 2001 recession. 1986-1987 appears to have been a crisis period for the energy sector, while other sectors were little affected.

[INSERT FIGURE 4 HERE]

[INSERT FIGURE 5 HERE]

[INSERT FIGURE 6 HERE]

Several other phenomena can be identified in these pictures. First, the persistency of the default cycle can be observed: inter-default intensities remain high several years after the peak of a recession. Second, we can see that some sectors appear to be forerunners of economic downturns, while others seem to follow recessions. As a consequence, information relating to sectors whose default cycle leads the economic cycle could prove valuable for credit risk management. The default rate in these industries may be a good variable to forecast the aggregate default rate in the economy. This is left for further research. Remark that we ran estimations on BB, B and CCC rating classes too. Obviously levels of implied intensities were found to be increasing with decreasing rating quality but variations and log-ACD(1,1) coefficients do not display different patterns. Therefore default rates in various rating classes cannot be used to forecast default rates in other classes. However it gives support to the fact that migrations should be mainly driven by only one underlying factor.

## Summary

In this paper we study times-to-default in the S&P rated universe. We develop a semi-parametric framework that enables us to analyze times-to-default conditional on various economic and market factors. Our empirical results show that changes in intensities can be attributed in part to the global economic cycle and can also be explained by financial variables such as equity returns and their volatility as well as interest rates. The economic conditions prevailing at a firm's entry in a rating category also play a significant role in explaining its short term default probability. We show that physical intensities of default are globally increasing with duration for all rating classes but the B and CCC classes. These results echo findings of Madan & al. (2004) on risk-neutral intensities who explained such a result by an over-exposure to innovation for established firms. Increasing exposures to managerial inefficiency and agency conflicts can more generally induce increasing likelihood of default for large companies. Therefore the market's perception of the evolution of default probabilities seem to coincide with physical phenomena.

The significant explanatory variables found in our study can be used to improve traditional credit risk models. In particular, a set of carefully selected factors including lagged variables can substantially enhance the performance of log-linear models. Traditional models mainly capture some short term determinants of the default probabilities but leave aside long term business trend effects. However our results evidence that default is triggered by their joint impact, indicating that efficient models should incorporate both aspects. Intuitively, corporate defaults may be induced by large changes in local or global business conditions but also by successive declines in the company's performance. The legal process may also delay the default event which in turn might not be explained anymore by financial or economic conditions at that date. We consequently argue that past information constitutes a crucial component of adequate modelling.

Importantly, macro-economic variables are insufficient to fully capture the dynamics of default probabilities because the default cycle exhibits persistency, and business factors should be added to financial information to improve predictive power. Loosely speaking the default cycle is slower than the economic cycle. As a consequence, models relying solely on the business cycle will tend to overstate real default probabilities in expansions and stable periods, and will undershoot default peaks during and following recessions. The procyclicality between outputs of credit risk management models and the business cycle is an outstanding issue. If the models used to assess default probabilities overestimate the default risk during recessions, banks will need to increase their reserves in order to meet capital requirements and will be forced to curtail their lending, thereby reinforcing the credit crunch phenomenon. Our results show that assessing default probabilities through duration models would not produce procyclical behaviours. This is mainly due to the persistency of the default cycle and to the fact that it slightly lags the business cycle.

In an analysis of durations between consecutive defaults within industrial classes we show that strong differences prevail between sectors. Some industries lead the global default cycle while others maintain high levels of defaults during economic recoveries. This suggests the existence of bidirectional contagion between the default cycle and the business cycle. Therefore,

instead of attempting to continuously improve the selection of macro factors proxying the health of industrial sectors, we suggest that more predictive proxies could be endogenously extracted from the default process itself.

From a risk management perspective, our results show that ratings constitute consistent proxies of firm-specific factors over the medium term. This mid-term view explains the stability of ratings and justifies their usual qualification as "through-the-cycle". Global economic conditions and aggregate default processes are useful complements of ratings as determinants of default probabilities of individual firms. We believe that our results are relevant for the design of structured credit products such as basket credit derivatives and CDOs and for more traditional credit risk portfolio management, in particular to improve the performance of factor-models.



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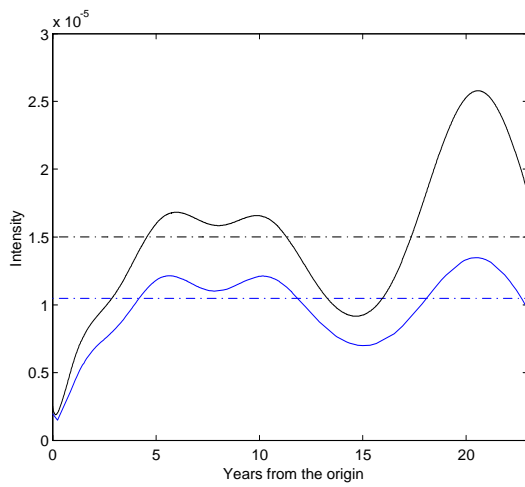
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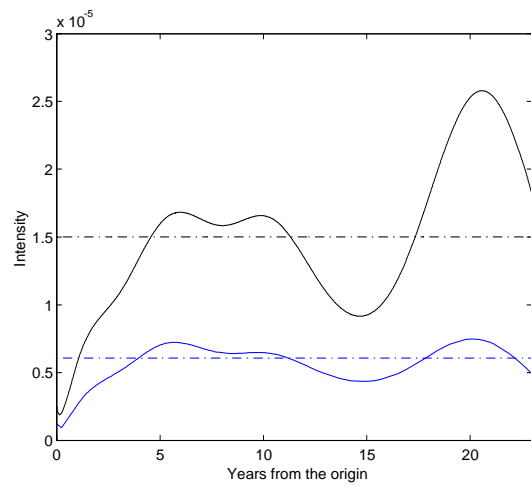
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**Figure 1**  
**Baseline Hazards of Multivariate Models**

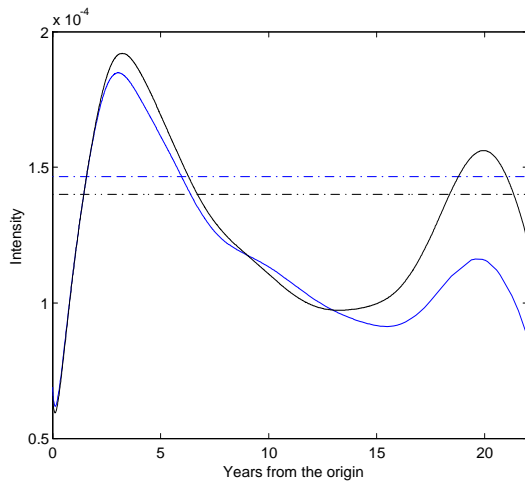
Estimated non-parametric baseline hazard rates  $\alpha^\circ(u)$  and corresponding means over Investment Grades and Non Investment Grades. Black lines denote the full non-parametric model ( $\alpha(u, t_i) = \alpha^\circ(u)$ ) and blue lines show semi-parametric specifications ( $\alpha(u, t_i) = \alpha^\circ(u) \exp(\beta' \mathbf{Z}(u + t_i))$ ). Dashed lines represent averages of baselines - they are not statistically different from the estimated constants  $\exp(\gamma)$  of log-linear model counterparts ( $\alpha(u, t_i) = \exp(\gamma + \beta' \mathbf{Z}(u + t_i))$ ). The "Stock Market" model uses the contemporaneous return and volatility on the S&P500. The "Best Six" model includes in addition the US Term Structure Slope, the real GDP growth, the BBB spread and the NIG downgrade rate.



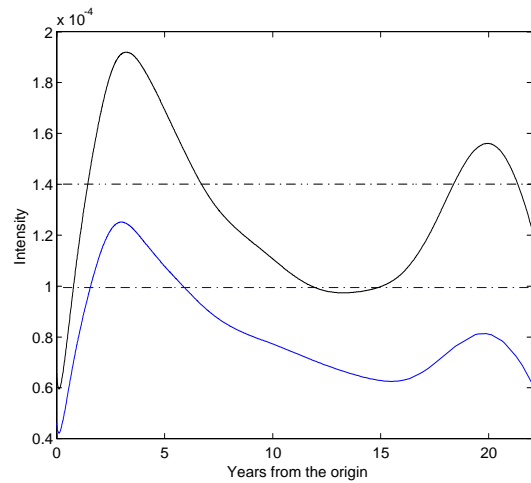
(a) IG, "Stock Market" model



(b) IG, "Best Six" model



(c) NIG, "Stock Market" model

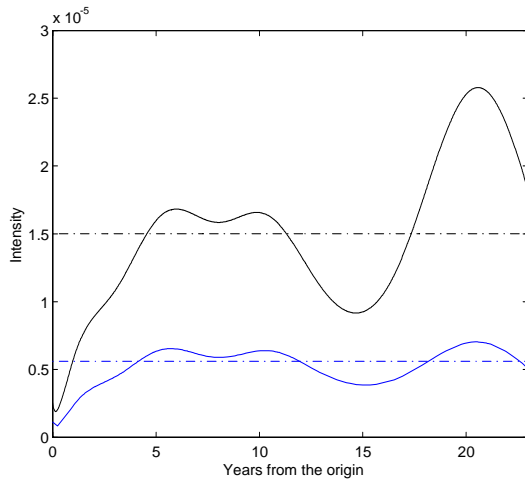


(d) NIG, "Best Six" model

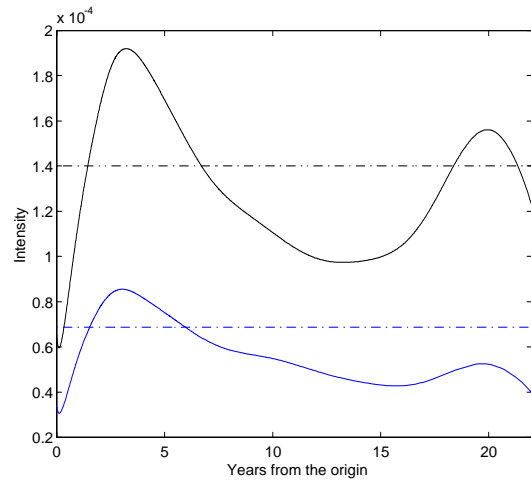
**Figure 2**

**IG and NIG Baseline Hazards of Improved Multivariate Models**

Estimated non-parametric baseline hazard rates  $\alpha^\circ(u)$  and corresponding means over Investment Grades and Non Investment Grades. Black lines denote the full non-parametric model ( $\alpha(u, t_i) = \alpha^\circ(u)$ ) and blue lines show semi-parametric specifications ( $\alpha(u, t_i) = \alpha^\circ(u) \exp(\beta' \mathbf{Z}(u + t_i))$ ). Dashed lines represent averages of baselines - they are not statistically different from the estimated constants  $\exp(\gamma)$  of log-linear model counterparts ( $\alpha(u, t_i) = \exp(\gamma + \beta' \mathbf{Z}(u + t_i))$ ). The improved "Stock Market" model uses the contemporaneous return and volatility on the S&P500 as well as their three year lags.



(a) IG, improved "Stock Market" model

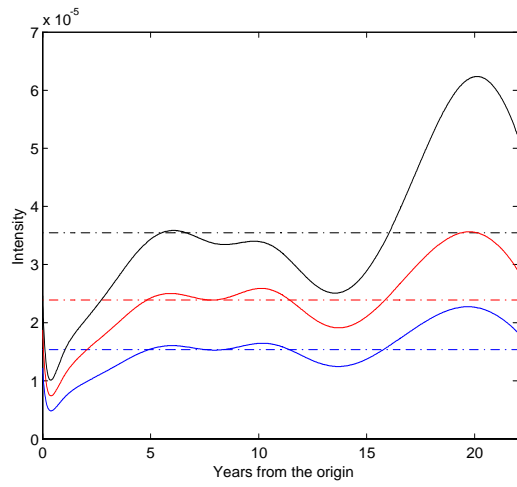


(b) NIG, improved "Stock Market" model

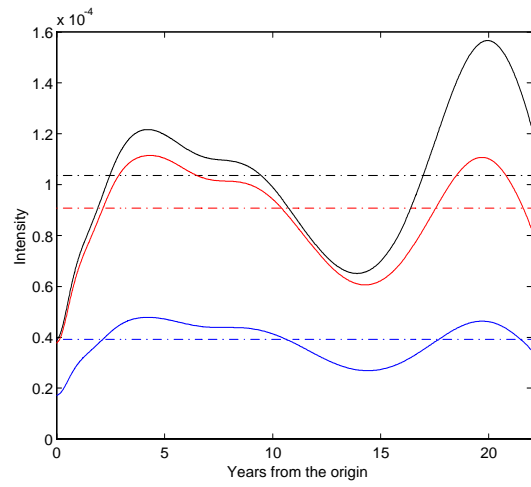
**Figure 3**

**Ratings Baseline Hazards of Improved Multivariate Models**

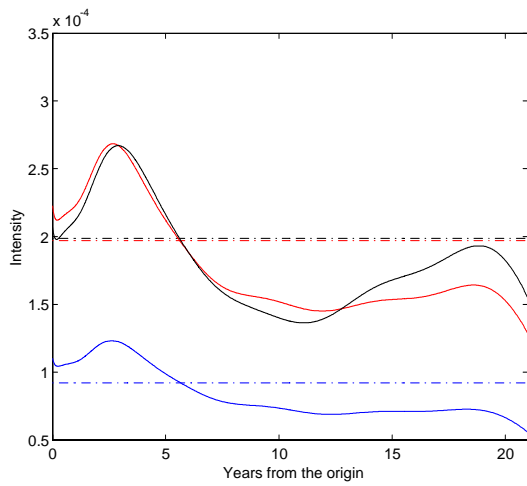
Estimated non-parametric baseline hazard rates  $\alpha^\circ(u)$  and corresponding means over Investment Grades and Non Investment Grades. Black lines denote the full non-parametric model ( $\alpha(u, t_i) = \alpha^\circ(u)$ ), blue and red lines show semi-parametric specifications ( $\alpha(u, t_i) = \alpha^\circ(u) \exp(\beta' \mathbf{Z}(u + t_i))$ ). Dashed lines represent averages of baselines - they are not statistically different from the estimated constants  $\exp(\gamma)$  of log-linear model counterparts ( $\alpha(u, t_i) = \exp(\gamma + \beta' \mathbf{Z}(u + t_i))$ ). The "Stock Market" model (red) uses the contemporaneous return and volatility on the S&P500. The improved "Stock Market" model (blue) adds the three year lags.



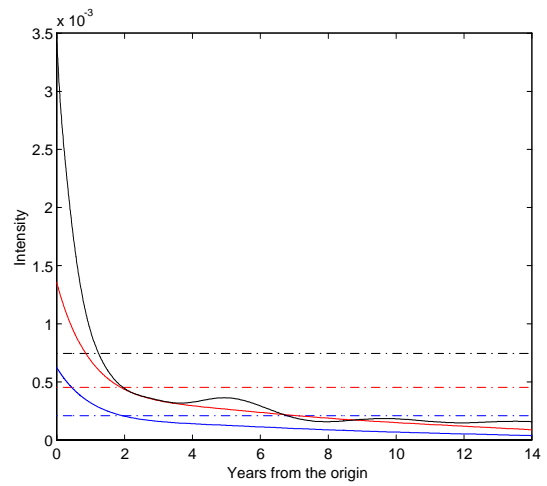
(a) BBB class



(b) BB class



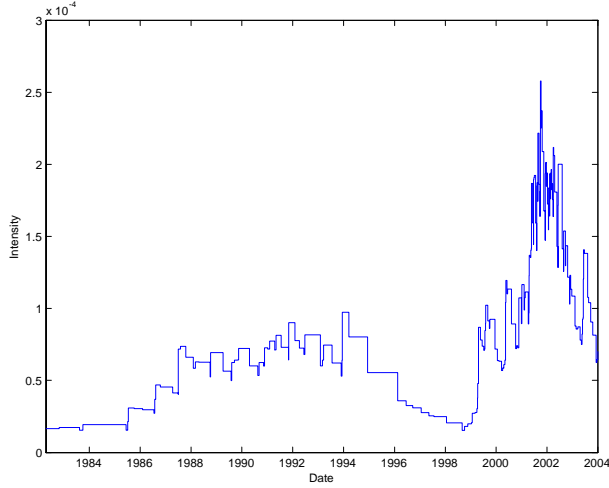
(c) B class



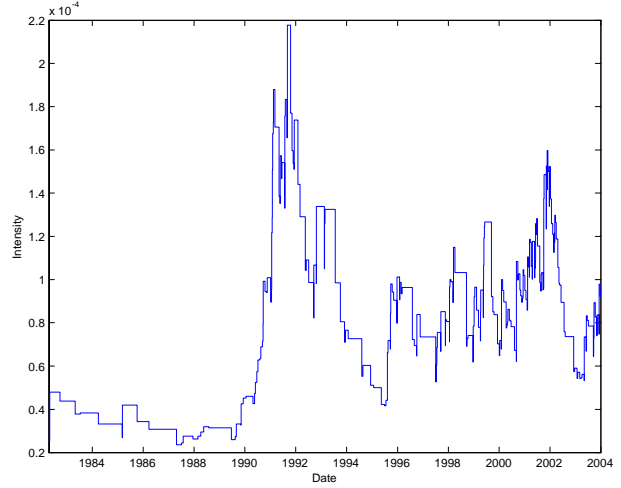
(d) CCC class

**Figure 4**  
**Intra-Industry Implied Hazard Rates**

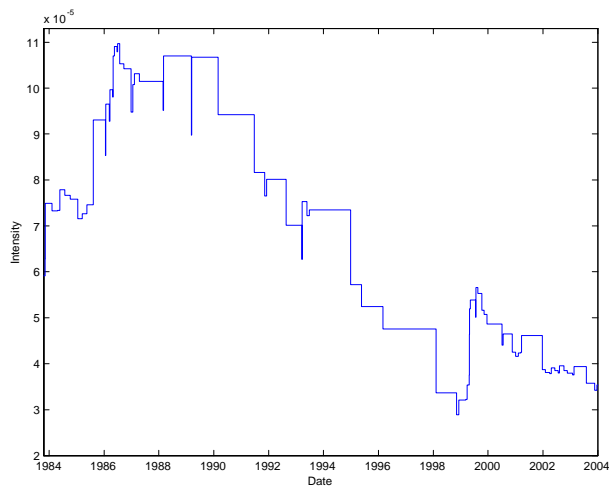
Predicted intensity of default  $\lambda^\circ$  for different industries using a log-ACD(1,1) model on inter-default durations within sectors.



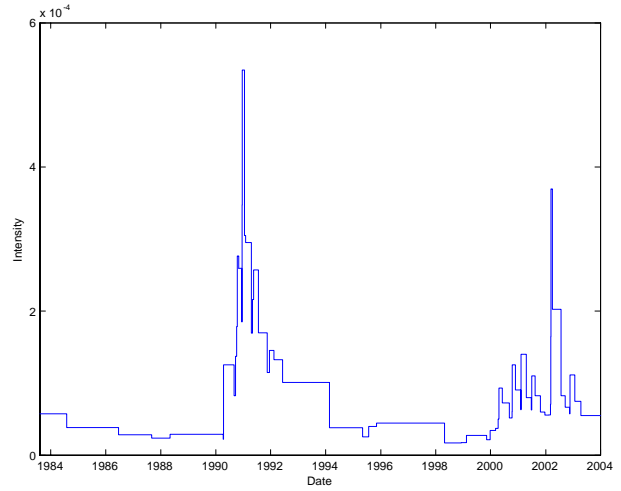
(a) Aero./Auto./Capital Goods/Metal



(b) Consumer/Service



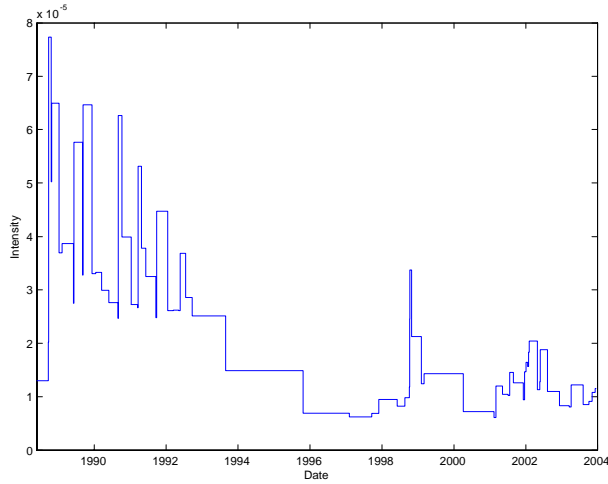
(c) Energy/Natural Resources



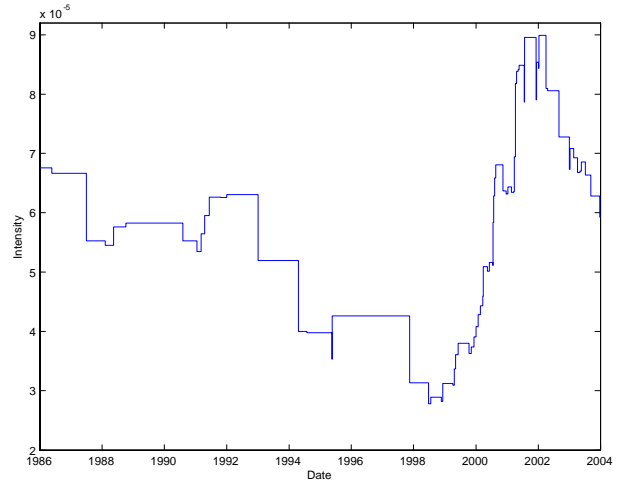
(d) Forest Product/Building

**Figure 5**  
**Intra-Industry Implied Hazard Rates**

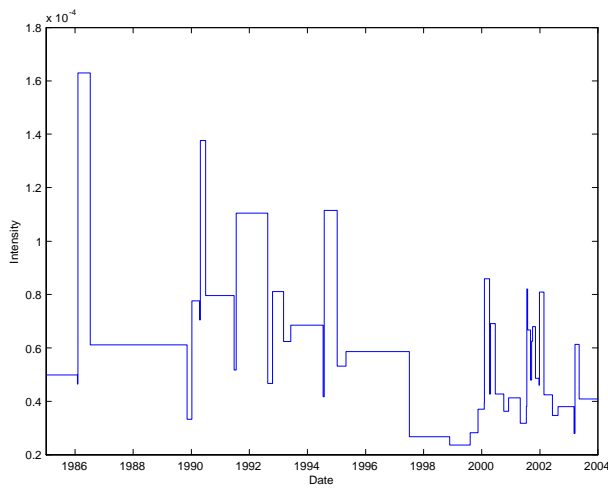
Predicted intensity of default  $\lambda^\circ$  for different industries using a log-ACD(1,1) model on inter-default durations within sectors.



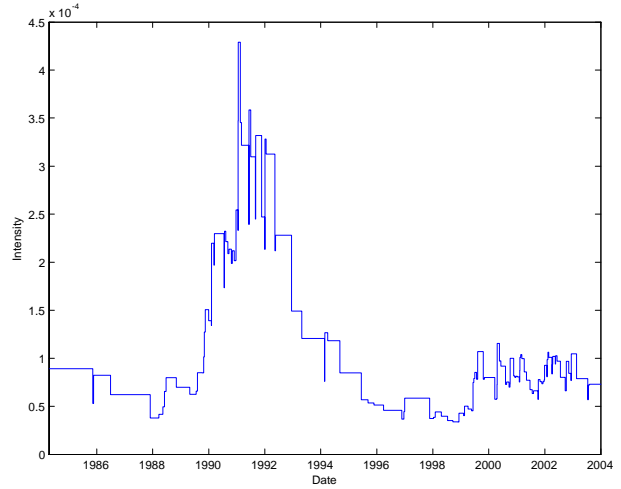
(a) Financial Institutions



(b) Health Care/Chemicals



(c) High Tech./Office Eq.

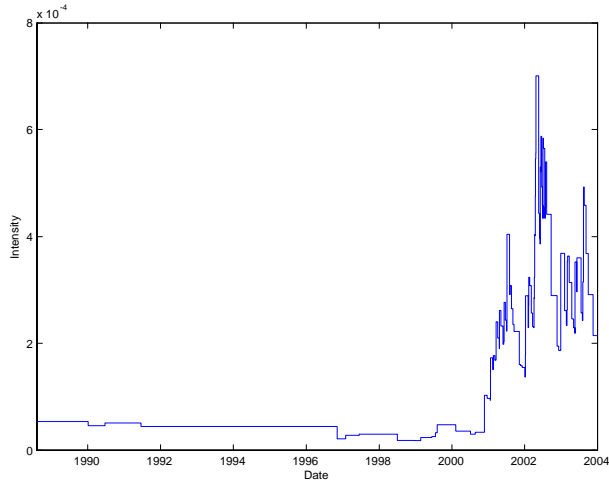


(d) Leisure/Time/Media

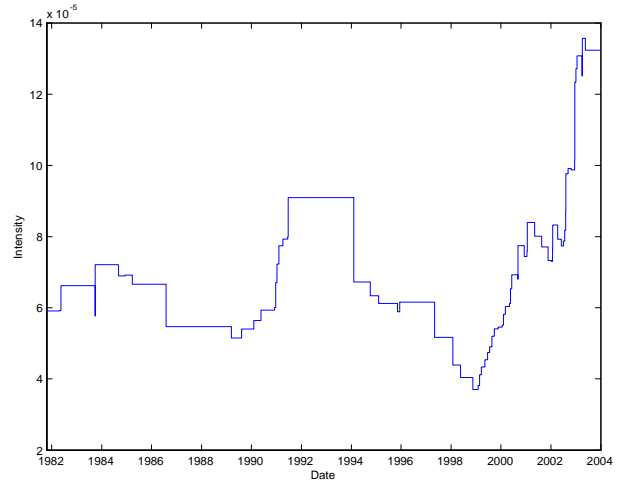


**Figure 6**  
**Intra-Industry Implied Hazard Rates**

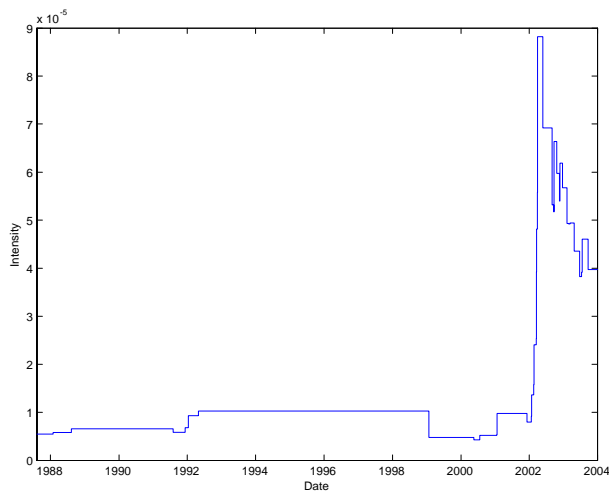
Predicted intensity of default  $\lambda^\circ$  for different industries using a log-ACD(1,1) model on inter-default durations within sectors.



(a) Telecommunication



(b) Transports



(c) Utility

**Table I**  
**Statistics on Covariates**

Basic statistics on retained factors. Figures are given on an annual basis. All variables but upgrade and downgrade rates are US indicators.

	Mean	Min	Max	Volatility	3 Year Autocorrelation
S&P500 Return	0.093	-0.324	0.439	0.158	-0.12
S&P500 Vol.	0.154	0.063	0.628	0.073	0.08
Treas. 10 yr. Yield	0.079	0.033	0.153	0.028	-0.23
Term Struc. Slope	0.013	-0.021	0.033	0.011	-0.08
Real GDP Growth	0.030	-0.028	0.081	0.019	0.17
Des. Ind. Prod. Growth	0.002	-0.018	0.020	0.006	0.22
CPI Growth	0.035	0.011	0.118	0.019	0.60
Pers. Inc. Growth	0.060	0.015	0.134	0.023	-0.32
BBB Yield	0.100	0.062	0.172	0.027	0.62
BBB Spread	0.022	0.013	0.038	0.006	0.09
BBB-AAA Spread	0.011	0.006	0.027	0.004	0.44
Treas. Net Issues	0.144	-0.391	0.748	0.163	-0.12
Money Lending Growth	0.062	-0.039	0.128	0.039	0.16
IG Upgrade Rate	0.005	0	0.020	0.003	0.09
NIG Upgrade Rate	0.010	0	0.109	0.009	0.06
IG Downgrade Rate	0.011	0	0.031	0.006	0.04
NIG Downgrade Rate	0.017	0	0.131	0.013	-0.05

**Table II**  
**Sensitivities to Aggregate Default Indicators**

Estimations of log-linear intensities  $\lambda^i(u, t_i)$  with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities  $\beta$  from univariate specifications  $\lambda^i(u, t_i) = \exp(\gamma + \beta' Z(u + t_i))$  where the default arrival is assumed to be piecewise exponential conditional on factor realizations. Constants  $\gamma$  are not reported. \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level.

Default Factors \ Ratings	AA	A	BBB	BB	B	CCC
IG Upgrade Rate	-6.339	-16.691	-38.853	-42.941**	-68.568**	-70.051**
NIG Upgrade Rate	-5.054	-7.651	-22.777	-28.416	-33.657**	-41.229**
IG Downgrade Rate	38.517	53.027**	64.619**	59.399**	50.417**	34.746**
NIG Downgrade Rate	22.426	24.379**	23.822**	25.408**	26.206**	23.990**

**Table III**  
**Sensitivities w.r.t Financial Markets Information**

Estimations of log-linear intensities  $\lambda^i(u, t_i)$  on IG and NIG with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities  $\beta$  from univariate specifications  $\lambda^i(u, t_i) = \exp(\gamma + \beta' Z(u + t_i))$  where the default arrival is assumed to be piecewise exponential conditional on factor realizations. We consider lags from five years backward to two months forward. Constants  $\gamma$  are not reported. \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level. For each factor the lag offering the highest likelihood ratio has been stressed in italics.

Investment Grades				
Lag	S&P500 Return	S&P500 Vol.	Treas Yield	Term Str.Slope
-2M	-2.19**	2.92**	-15.87**	<i>42.31**</i>
-1M	-2.25**	3.36**	-15.98**	40.69**
0M	-2.72**	3.80**	-15.33**	38.44**
1M	<i>-2.85**</i>	<i>3.99**</i>	-15.35**	33.11**
2M	-2.77**	3.65**	-14.57**	29.59**
3M	-2.61**	3.50**	-13.90	28.29**
6M	-2.40**	3.11**	-11.84**	18.84**
1Y	-1.96**	2.59**	-11.07**	-3.56
2Y	-.49	3.85**	-9.08**	-28.18**
3Y	1.33**	3.85**	-11.75**	-20.84**
5Y	1.12**	3.54**	-10.79**	-7.55

Non Investment Grades				
Lag	S&P500 Return	S&P500 Vol.	Treas Yield	Term Str.Slope
-2M	-1.99**	1.78**	-10.88**	24.83**
-1M	-1.89**	1.81**	-10.91**	22.54**
0M	-1.96**	2.10**	-10.41**	19.66**
1M	<i>-2.05**</i>	<i>2.39**</i>	-10.44**	16.93**
2M	-2.02**	2.47**	-10.45**	13.53**
3M	-2.01**	2.47**	-10.03**	10.98**
6M	-1.76**	2.51**	-8.57**	.45
1Y	-1.02**	1.99**	-7.05**	-19.00**
2Y	.16	2.84**	-6.53**	<i>-29.72**</i>
3Y	1.66**	<i>3.60**</i>	<i>-12.21**</i>	-21.08**
5Y	1.28**	2.49**	-7.64**	-12.30**

**Table IV**  
**Sensitivities w.r.t. Business Cycle Information**

Estimations of log-linear intensities  $\lambda^i(u, t_i)$  on IG and NIG with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities  $\beta$  from univariate specifications  $\lambda^i(u, t_i) = \exp(\gamma + \beta' Z(u + t_i))$  where the default arrival is assumed to be piecewise exponential conditional on factor realizations. We consider lags from five years backward to two months forward. Constants  $\gamma$  are not reported. \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level. For each factor the lag offering the highest likelihood ratio has been stressed in italics.

Investment Grades				
Lag	Real GDP Growth	Ind. Prod Growth	CPI Growth	Pers. Inc Growth
-2M	-17.70**	-21.54	-16.36**	-25.96**
-1M	-17.47**	-48.37**	-15.85**	<i>-27.13**</i>
0M	-19.23**	-38.40**	-15.08**	-26.74**
1M	-20.81**	-46.55**	-12.77*	-25.14**
2M	-19.97**	-20.68	-11.57*	-23.49**
3M	<i>-21.91**</i>	<i>-57.27**</i>	-9.85**	-22.16**
6M	-19.84**	-46.64**	-4.47	-18.09**
1Y	-12.62**	-47.77**	-2.76	-9.72**
2Y	3.29	-24.67*	-3.91	3.13**
3Y	15.32**	5.99	-16.07**	-4.0
5Y	17.42**	23.95	<i>-22.23**</i>	-4.42

Non Investment Grades				
Lag	Real GDP Growth	Ind. Prod Growth	CPI Growth	Pers. Inc Growth
-2M	-21.27**	-35.17**	-9.09**	<i>-20.58**</i>
-1M	-21.44**	-39.38**	-7.87**	-19.82**
0M	-22.66**	-36.20**	-5.87**	-18.78**
1M	<i>-23.01**</i>	-50.57**	-4.47*	-17.26**
2M	-22.32**	-37.65**	-3.56**	-15.11**
3M	-22.58**	<i>-54.06**</i>	-2.02	-13.16**
6M	-17.57**	-53.61**	1.83	-7.93**
1Y	-6.78**	-35.55**	.53	-4.5
2Y	10.27**	-4.75	-2.95**	5.29**
3Y	11.71**	13.65**	<i>-15.50**</i>	1.34
5Y	11.28**	15.10**	-11.96**	-3.07

**Table V**  
**Sensitivities w.r.t. Credit Markets Information**

Estimations of log-linear intensities  $\lambda^i(u, t_i)$  on IG and NIG with time-varying covariates over the whole sample up to last days of observation. The table displays sensitivities  $\beta$  from univariate specifications  $\lambda^i(u, t_i) = \exp(\gamma + \beta' Z(u + t_i))$  where the default arrival is assumed to be piecewise exponential conditional on factor realizations. We consider lags from five years backward to two months forward. Constants  $\gamma$  are not reported. \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level. For each factor the lag offering the highest likelihood ratio has been stressed in italics.

Investment Grades					
Lag	BBB Yield	BBB Spread	IG Spread	Treas. Issues	Money Lending
-2M	-10.28**	73.55**	50.50**	1.70**	2.05
-1M	<i>-10.23**</i>	75.08**	<i>50.38**</i>	1.55**	2.54
0M	-9.65**	74.35**	44.63**	.99**	3.39
1M	-9.39**	<i>76.89**</i>	41.38**	.90*	4.45*
2M	-9.05**	72.86**	34.49**	.98**	4.78**
3M	-8.38*	72.70**	35.73**	1.09**	5.32**
6M	-6.95**	65.61**	25.75	.24	6.33**
1Y	-7.51*	49.11**	-7.02	-.75*	9.34**
2Y	-6.24*	44.19**	-19.64	<i>-2.43**</i>	9.02**
3Y	-8.60	41.57**	-13.48	-2.41**	<i>12.09**</i>
5Y	-9.84**	-4.38	-52.73**	-3.39**	6.13**

Non Investment Grades					
Lag	BBB Yield	BBB Spread	IG Spread	Treas. Issues	Money Lending
-2M	-5.62**	<i>62.97**</i>	<i>53.08**</i>	.59**	4.56**
-1M	-5.76**	62.07**	51.84**	.46**	4.88**
0M	-5.54**	59.09**	45.95**	.14	5.41**
1M	-5.45**	60.12**	39.87**	-.02	6.00**
2M	-5.42**	60.30**	31.96**	-.22	6.34**
3M	-5.02**	59.68**	28.85**	-.26*	6.56**
6M	-4.24**	51.92**	7.58*	-.71**	7.10**
1Y	-4.94**	28.91**	-13.84**	-1.61**	7.55**
2Y	-5.42**	18.04**	-23.09**	<i>-2.11**</i>	7.17**
3Y	<i>-9.70**</i>	30.16**	-15.08**	-1.76**	<i>11.01**</i>
5Y	-7.91**	-27.57**	-47.58**	-2.15**	4.71**

**Table VI**  
**Multivariate Log-Linear Models**

Estimations of log-linear intensities  $\lambda^i(u, t_i)$  with time-varying covariates over rating classes for durations up to the first exits and all countries. The table displays sensitivities  $\beta$  from multivariate specifications  $\lambda^i(u, t_i) = \exp(\gamma + \beta' \mathbf{Z}(u + t_i))$  where the default arrival is assumed to be piecewise exponential conditional on realizations of covariates. We focus on financial market information as used by several studies. Constants  $\gamma$  are not reported. \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level.

Model	Stock Market	Interest Rates	Both	Stock Market	Interest Rates	Both
Factors \ Class		AA			A	
S&P500 Return	-3.169**		-3.033*	-2.027**		-1.674*
S&P500 Vol.	0.029		0.097	0.349		0.350
Treas. Yield		-10.548	-5.704		-9.951**	-6.675*
Term. Str. Slope		19.72	12.008		27.326**	19.683
		BBB			BB	
S&P500 Return	-2.112**		-1.441**	-1.920**		-1.436**
S&P500 Vol.	2.132**		2.552**	0.756**		1.15*
Treas. Yield		-4.467	-1.899		-3.055*	-1.137
Term. Str. Slope		33.254**	25.661**		25.799**	17.109**
		B			CCC	
S&P500 Return	-1.965**		-1.687**	-1.273**		-1.272**
S&P500 Vol.	0.329		0.166**	0.972*		0.351
Treas. Yield		-8.364**	-4.535**		-14.611**	-10.252**
Term. Str. Slope		16.932**	4.913**		-12.851*	-10.981**

**Table VII**  
**Parsimonious Multivariate Proportional Hazard Models**

Estimations of semi-parametric models of default intensities with time-varying covariates over IG and NIG classes for durations up to the first exits and all countries. The table displays sensitivities  $\widehat{\beta}$  from multivariate specifications  $\lambda^i(u, t_i) = \lambda^\circ(u) \exp(\beta' \mathbf{Z}(u + t_i))$ . \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level.

Model	Stock Market	Interest Rates	Both	Best Six	Stock Market	Interest Rates	Both	Best Six
Factors \ Class	IG				NIG			
S&P500 Return	-2.26**		-1.39**	-0.62	-1.87**		-1.62**	-0.23*
S&P500 Vol.	2.35**		2.53*	2.99**	0.50*		0.33*	0.66**
Treas. Yield		-10.67**	-4.37			-7.97**	-4.21	
Term. Str. Slope		32.22**	24.87**	26.91**		15.52**	4.09*	4.18**
GDP				-10.15*				-14.02**
BBB Spread				8.06				12.16**
NIG Down. Rate				14.92**				15.86**



**Table VIII**  
**Improved Multivariate Proportional Hazard Models**

Estimations of semi-parametric models of default intensities with time-varying covariates over rating classes for durations up to the first exits and all countries. The table displays sensitivities  $\widehat{\beta}$  from multivariate specifications  $\lambda^i(u, t_i) = \lambda^\circ(u) \exp(\beta' \mathbf{Z}(u + t_i))$ . \* (resp. \*\*) stands for significance at 95% (resp. 99%) confidence level.

Factors \ Class	AA	A	BBB	BB	B	CCC	IG	NIG
S&P500 Return	-2.74*	-1.23*	-1.64**	-1.08**	-1.25**	-0.653**	-1.62*	-1.15**
-3Y S&P500 Return	1.39	0.94	0.45	1.56**	1.66**	2.49**	0.83	1.65**
S&P500 Vol.	-0.84	0.25	2.28**	0.62*	0.65*	0.77*	2.52**	0.31
-3Y S&P500 Vol.	1.94	3.54**	2.12**	3.65**	3.11**	2.41**	2.76**	3.13**

**Table IX**  
**Intra-Industry Default Behaviour**

Log-ACD(1,1) estimates on inter-default durations within various industry categories. Ljung-Box Q-test and Arch-test on residuals including successively 1, 5, 10 and 20 lags (figures correspond to 20 lag) were found to be insignificant. \* (resp. \*\*) denotes significance at the 95% (resp. 99%) confidence level. Other sectors, namely Insurance and Real Estate, were too sparse to run estimations.

Risk Class	Size	$w_c$	$a_c$	$b_c$
Aerospace				
Automotive	232	0.1327	0.1356*	0.8434**
Capital Goods				
Metal				
Consumer	296	0.1560*	0.1077**	0.8627**
Service Sector				
Energy and	88	0.7581	0.0494	0.7938**
Natural Resources				
Financial	89	2.1295*	0.2483**	0.2964
Institutions				
Forest Products	68	0.9152*	0.3383**	0.5042**
and Building				
Health Care	81	0.1459	0.0423	0.9284**
and Chemicals				
High Tech	54	3.7032*	0.2779**	0.1516**
and Office Eq.				
Leisure Time	162	0.4072	0.1421*	0.7745**
and Media				
Telecommunications	141	0.2056*	0.1894**	0.7730**
Transports	79	0.0654	0.0422	0.9449**
Utility	57	0.1671	0.1203**	0.8532**