

The Strategic Interaction between Committing and Detecting Fraudulent Misreporting*

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ABSTRACT

The paper considers an agency model of fraudulent misreporting which implies a rich set of relationships between the commission of fraud, the observation or detection of fraud, economic performance, and the compensation policy of the firm. The paper develops a number of testable empirical implications and highlights several interesting phenomena, including implications on exogenous variables that can cause an increase in the amount of fraud committed but a decrease in the amount of fraud being observed (and visa versa). Thus, empirical studies that seek to identify the firm or managerial characteristics associated with the commission of fraud cannot infer a relationship by simply examining how the amount of observed fraud varies with these characteristics. In addition, the paper also shows that an increase in an industry's growth potential can cause that industry to fall from a high-productivity pooling equilibrium (with high levels of incentive compensation and effort and, as a result, many high-productivity firms) to the lower-productivity mixed-strategy equilibrium (with lower levels of incentive compensation and effort and, as a result, fewer high-productivity firms), resulting in a drop in economic performance.

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1. Introduction

The revelation of fraudulent misreporting in numerous high-profile cases in the United States around the start of the twenty-first century (e.g., Adelphia, Enron, Global Crossing, Tyco, Waste Management Inc., and Sunbeam) resulted in a substantial loss in market value.¹ It is unclear, however, whether these cases represent isolated instances of lapses in corporate ethical judgment or whether they indicate a general degradation in corporate morality and/or an increase in the incentive to commit fraud. Clearly indicating a belief in a systemic source, the U.S. Congress enacted the Sarbanes-Oxley Act in 2002 in an effort to rein in managers in what was feared to be a pervasive “fast and loose with the facts” opportunistic corporate culture. Yet, it is still unclear what social or economic forces changed to cause the increase in fraudulent misreporting. In addition, without knowing the cause, it is also unclear whether Sarbanes-Oxley will be an effective counter-measure (especially given the time-series and cross-sectional variation in the economic conditions firms face). In fact, the recent arrests of two Bear Sterns hedge fund managers and the Securities and Exchange Commission investigations of dozens of corporate fraud cases related to sub-prime mortgage securities would seem to raise doubt.

The above issues are difficult to address because the amount of fraud committed is not directly observable; we only observe the amount of fraud that is detected, which is jointly determined by the amount of fraud actually being committed and the probability of getting caught given the extent to which fraudulent activities are investigated. To the extent that environmental influences may affect the commission and investigation of fraud differently, there

¹ According to Cornerstone Research, 231 fraud lawsuits in the year 2002 alone resulted in a total disclosed dollar loss of \$203 billion in market capitalization. From 1996 to 2004, on average there were 195 lawsuits per year with a total disclosed dollar loss of \$127 billion per year; typically around 80% of these lawsuits involve misrepresentation of financial statements. When looking into these misrepresentation cases, one can find that almost all of them involve earning inflation of some sort.

may not be a one-to-one correspondence between the amount of fraud observed and the amount committed. Thus, the fact that fraudulent behavior is not observable makes it difficult to discover and document links between potential environmental influences and fraudulent behavior. Instead, we must develop theoretical models which provide testable implications with respect to observable phenomena and, when these models are supported by the data, infer relationships among non-observable variables based on the implications of such models. In order to provide such structure, this paper develops a theoretical model of fraud with two critical features: (1) there is a strategic interaction between the commission and detection of fraud (which allows us to develop conditional statements on what can and cannot be inferred about the commission of fraud from the observation of fraud), and (2) the extent of fraud committed and investigated varies with the economic environment (which allows us to develop time-series and cross-sectional implications on the amount of fraud and the effectiveness of regulation).

Specifically, we develop an agency model in which managers are induced via an equity-based compensation (EBC) contract to exert personally costly effort that increases the expected returns of the firm. In the model, the realized return of the firm is not observable by the market; rather the manager must report its value. Similar to Goldman and Slezak (2006), EBC provides the manager the incentive to exert effort – but also the incentive to upwardly bias reports. The regulatory agency, seeking to minimize the deadweight loss associated with fraud, is responsible for detecting fraud and imposing penalties; it bases its investigation strategy on the manager's equilibrium fraud commission strategy in order to optimally trade off the benefit of reducing fraud against the agency's detection cost.

To capture both cross-sectional and time-series variation (high growth versus old-economy firms and recession versus expansion), we assume that after the initial stage, a

potential new investment project arrives with a certain probability (proxying for growth potential). After privately observing the project's expected profitability, the manager either adopts or rejects it. In order to obscure past fraud, fraudulent managers have the incentive to over-invest (i.e., invest in new projects that have negative expected NPV). This overinvestment results in the deadweight loss associated with fraud.²

Depending upon the parameters characterizing the regulatory and contracting environment, three potential types of equilibrium may obtain: truthful separating equilibrium in which each manager truthfully reports their realized return, pooling equilibrium in which all poorly-performing managers mimic the reports of highly-performing managers but the regulatory agency does not monitor to verify reports, and a mixed strategy equilibrium in which poorly-performing managers commit fraud with an equilibrium probability while the regulatory agency randomly audits those firms that report high earnings.

The paper provides a number of empirical implications regarding the commission and detection of fraud, incentive contracts, and economic performance. First, the model implies that fraudulent reporting activities will be concentrated in high-growth industries. This result is similar to the theory predictions of Wang (2006) and consistent with the empirical evidence in Wang (2005) and Johnson, Ryan and Tian (2003).³ Second, the model implies that while the fraud incentive is strongest in good times, fraud commission and detection are more likely to

² The exact nature of the deadweight loss is not critical for most of our results; all that is needed is that there be some benefit to reducing fraud (in terms of more efficient production) so that the regulator must balance the benefit of reduced fraud against the implementation costs associated with monitoring fraud. However, the paper does develop some implications related to the specific form of fraud inefficiency we assume.

³ Wang (2005) provides empirical evidence that firms engaging in fraudulent reporting tend to overinvest relative to their peers. Johnson, Ryan and Tian (2003) find that their sample of exposed fraudulent reporting firms are not random draws from all possible industries, but rather demonstrates a statistically significant industry concentration, with the concentrated industries having significantly higher than average growth potential.

occur when the high growth industries fall into downturns.⁴ Specifically, when the parameters are such that the pooling equilibrium obtains, then there will be relatively high levels of EBC, high average short-term performance, and no fraud being exposed (although fraud is committed). In contrast, when the parameters are such that the mixed strategy equilibrium obtains, the equilibrium will have low EBC, low average short-term performance, and fraud will be exposed. Thus, consistent with the empirical evidence in Johnson, Ryan and Tian (2003), these results imply that exposed fraud will occur in periods with relatively weak economic performance.⁵

Third, the model implies that an increase in growth potential, which is typically good news (i.e., implies higher future profitability and, as a result, increased firm value), can strikingly have a negative impact on value and economic performance. Specifically, we show that, by altering the incentives to commit and investigate fraud, an increase in growth potential alone can cause an industry to fall from a high-productivity pooling equilibrium (with high levels of EBC and effort and, as a result, many high-productivity firms) to the lower-productivity mixed-strategy equilibrium (with lower levels of EBC and effort and, as a result, fewer high-productivity firms), resulting in a drop in economic performance. We show that, given the strategic interaction between the incentives to commit and investigate fraud, this drop in economic performance will be accompanied by an increase in the amount of exposed fraud. These results imply that, while innovation may be beneficial to economic growth by generating increased future growth opportunities, innovation can also have a dark side when fraud is possible. In fact, there are many examples of innovations that were accompanied by fraud scandals: financial innovation in mortgage derivatives, product innovation in

⁴ Its primary focus, the model in Povel, Singh, and Winton (2007) generates similar “boom-and-bust” results. As discussed below, the two models and the mechanisms by which these boom-and-bust results obtain differ.

⁵ Their results show that both the exposed fraud firms and their (industry- and size-matched) control firms significantly underperformed the overall stock market during the fraudulent reporting periods.

telecommunications, and innovation created by the deregulation of energy markets, to name a few.

Fourth, the model implies that the extent of detected fraud need not be indicative of the extent of fraud committed. This is in contrast to the signal jamming models of fraud (see, for example, Goldman and Slezak (2006)) in which the equilibrium probability of committing fraud is one. In these types of models, an increase in the probability of detection will necessarily result in an increase in the incidence of observed fraud. In our model, however, the equilibrium probability of committing fraud can be inversely related to the probability of observing detected fraud (in the mixed strategy equilibrium), leading to potential ambiguity in the statistical relationship between the amount of fraud detected and committed.

Similar to our model, there is a strategic interaction between the commission and detection of fraud in Povel, Singh, and Winton (2007), hereafter PSW, and Wang (2006); in both of these models the extent to which managers are monitored depends upon the information content of the managers' equilibrium reports and the extent to which managers commit fraud (i.e., bias and reduce the informativeness of reports) depends upon the likelihood of being monitored. In both models, monitoring serves to reduce adverse selection caused by fraud. In Wang (2006), managers commit fraud on behalf of current equity holders who benefit from a lower cost of capital stemming from inflated equity prices caused by fraud. In PSW, managers seek outside funding for their projects and provide (potentially fraudulent) information on the prospects of these projects to potential investors who face adverse selection in deciding whether or not to provide funding. In their model, managers receive non-contractible control benefits from any (even negative NPV) investment and, as a result, they commit fraud in an effort to mislead investors into funding negative NPV projects so that they can obtain these benefits of control.

In both models, the cost of fraud derives from an over-investment problem similar to Myers and Majluf (1984).

In contrast to these models, fraud in our model stems from an agency problem between managers and shareholders, with managers seeking to manipulate prices upward in order to increase their equity-based compensation. As in Goldman and Slezak (2006), equity-based compensation is a “double-edged sword” in that it provides both the incentive for the managers to exert costly effort in improving the profitability of the firm and the incentive to commit fraud. Given this dual role, the possibility of fraud alters the incentive contract, which alters the equilibrium level of effort and the productivity of firms. Thus, in contrast to PSW and Wang (2006), which take the distribution of firm productivity as given and consider how fraud affects the allocation of resources among the fixed set of firms, our model endogenously determines the productivity of the set of firms via the incentive contract. In contrast to Goldman and Slezak (2006), which takes the investigation of the regulatory agent as given, our model considers the strategic interaction between the regulatory agent and the manager in the context of this agency problem. We show that this combination of elements generates new insights.

Another key difference between our model and the model in Wang (2006) is that we consider the behavior of a regulatory agency that is concerned with social welfare.⁶ In Wang (2006), the monitor chooses whether or not to investigate by trading off the investigation cost against the penalties the monitoring agency “earns” when fraud is detected. That is, the monitor in Wang (2006) seeks to maximize the expected profit from monitoring, with the penalties -- set exogenously -- representing revenue to the monitor. Thus, since the

⁶ In PSW, firms are not monitored by a regulatory agency such as the SEC. Rather, the potential investors decide whether or not to investigate the claims of firms further prior to investing. Although there is no regulatory agency in PSW, their potential investors’ decision to investigate depends upon trade-offs that are analogous to those considered by the RA in our model.

exogenously set penalty is not tied to the endogenously-determined cost of fraud, the behavior of the monitor in Wang (2006) is not motivated by social welfare. In contrast, our monitor's behavior is motivated by social welfare.⁷

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 discusses the equilibria. Section 4 discusses the empirical implications of the model. Section 5 concludes. All proofs, as well as a numerical example, are provided in the appendix.

2. The Model

The model consists of a large number of competitive firms (in a variety of different industries) owned by atomistic risk-neutral investors and managed by risk neutral agents. Every firm within a given industry has exactly the same characteristics. The sequence of events unfolds over four periods as following.

2.1. Period 1: The Contracting Stage

In period $t = 1$, an entrepreneur with an idea starts a firm consisting of real and intellectual assets whose initial value is normalized to 1. The entrepreneur has limited expertise at managing the on-going operations of the firm and, thus, hires a wealth-constrained professional manager from a competitive managerial labor market to manage the firm for her. Because the manager is wealth constrained, the first-best contracting solution, in which the entrepreneur sells the firm to the manager, is not feasible. Instead, the entrepreneur offers the manager a compensation contract (\bar{w}, α) , where \bar{w} is a nonnegative fixed wage (paid to the manager at $t = 1$) and α is the percentage of the firm's shares offered to the manager in the form of a stock option with a zero strike price; the option vests in period $t = 2$. The manager either accepts or rejects the contract,

⁷ Both the investors in PSW and the regulatory agency in our model trade off the benefit of reduced over-investment against the investigation cost (which includes the cost of investigating truthful firms). In both models, the deadweight loss associated with fraud-induced resource misallocation and the costs incurred to limit fraud are minimized.

based on a comparison of the manager's expected utility under the contract and his reservation utility, which for simplicity (and without a loss of generality) is assumed to be zero. Once a manager has been hired and the terms of the contract are set, the entrepreneur sells her ownership stake in the firm at an initial public offering. The risk-neutral entrepreneur chooses the contract in order to maximize her expected wealth, given that the value of the firm will depend, via rational expectations, on the incentives embodied in the contract and on other features of the market, especially the regulatory environment and the behavior of the regulatory agency (hereafter referred to as the RA).

After the manager is hired and the IPO is complete, the manager exerts an unobservable amount of costly effort e , which affects the return on the firm's assets realized in period $t = 2$ (described further in the next section). The manager chooses the amount of effort to exert given the trade off between its beneficial effect on his compensation (via its effect on firm value) and its detrimental effect on his utility via a disutility of effort given by $\frac{\delta}{2}e^2$, where δ is a positive constant. That is, the manager's objective function is $U(.) = E[W] - \frac{\delta}{2}e^2$, where $E[W]$ is his expected wealth conditional on the contract and the economic/regulatory environment.

2.2. Period 2: The Reporting and Investigating Stage

In period $t = 2$, the return on the firm's initial assets is realized and privately observed by the manager. For simplicity, we assume that the gross return of the firm is either a_H or $a_L < a_H$.⁸ The probability that the gross return is a_H is $P(e) = \varphi e$, where φ (i.e., the marginal

⁸ In the real world, the returns of firms are likely to be continuous. However, all that is required for our results is that the return support be bounded above. If the return distribution is bounded from above, then there is a limit to the amount that higher-type managers can exaggerate their return. As a result, since the higher-type managers will not always report returns exceeding those of lower-type managers (at some point their claims cease to be feasible), there will exist situations in which lower-type managers will pool (with some positive probability) with higher-type managers.

productivity of effort) is a positive constant and $e \in [0, 1/\phi]$; while the probability that the gross return is a_L is $1 - P(e)$.

Once the manager privately observes the realized return on assets, he must make an earnings report (denoted r) to the market. Since the realized return is not directly observable by anyone other than the manager, the manager can choose to either report truthfully or fraudulently. We assume the manager can either truthfully disclose or inflate his earning. Specifically, a manager with a realized return a_H reports earning truthfully thus $r(a_H) = a_H$. However, a manager with a realized return of a_L may report truthfully (i.e., $r(a_L) = a_L$) or fraudulently (i.e., $r(a_L) = a_H$).⁹ Given the equilibrium information content of the manager's equilibrium reporting strategy, market investors rationally value the firm conditional on the firm's reported earnings. The manager then exercises his stock option and sells all his vested shares to the market.¹⁰

The RA, which seeks to protect the interests of investors (including the entrepreneur), is responsible for investigating and detecting fraud. For simplicity, we assume that if the RA chooses to investigate fraud, it will always detect fraud when it exists and will never "detect" fraud when it does not exist. That is, the RA does not make ex-post Type I or Type II errors. In order

⁹ Since there are only two return values possible, the manager will report either $r = a_H$ or $r = a_L$. Any other reports will not be credible as other values of r are not feasible.

¹⁰ Here we assume the manager has a short horizon. We do this for two reasons. First, there is some evidence that this is consistent with the situations in many real-world fraud cases. (Bergstresser and Philippon (2006) present evidence that CEOs exercise unusually large amount of options during periods of high accruals (which indicate intensive earning manipulation).) Second, it markedly simplifies the analysis without affecting the nature of the results. Even if the manager is given a multi-period contract, as long as he receives some compensation based on intermediate value of the firm, he will still have an (albeit mitigated) incentive to manipulate earnings reports to raise the intermediate price of the firm. In a setting in which the manager receives part of his compensation based on long-term value, the weight placed on long-term value will reduce the incentive to commit fraud and will complement the incentives created by penalties for fraud. Since our model has both incentives (EBC) and disincentives (penalties) to commit fraud, adding an additional disincentive (by placing weight on the terminal value) will not change the nature of our results. See Wang (2006) for an analysis of the case where the manager has a long horizon.

to detect fraud, however, the RA must choose to investigate fraud. Thus, a manager who has committed fraud can “get away with it” if the RA chooses not to investigate the manager’s firm. We assume that it costs $C_I > 0$ to investigate fraud and that the RA decides whether to investigate fraud to minimize the deadweight loss associated with fraudulent reporting, taking into consideration both the expected benefits (i.e., the deterrent effect) and costs associated with its investigation policy. If the RA detects fraud, the manager is assessed a penalty $fa_D \equiv f(a_H - a_L)$ proportional to the extent of the misreporting, where f is the constant of proportionality.¹¹

2.3. Period 3: The Investment Stage

At $t = 3$, new investment opportunities may become available. The probability that a new investment opportunity arrives is λ . For simplicity, all new investment opportunities require an additional investment of capital equal to I , which is raised by issuing equity. If a new investment opportunity arrives, its gross return will be $\mu + \varepsilon$, where μ is the mean of the gross return and ε is a white-noise term following $N(0, \sigma^2)$. We assume that the manager privately observes the realized value of mean μ , but that, with respect to the investors information set, μ is a random variable distributed as $Uniform(\underline{\mu}, \bar{\mu})$ with $0 < \underline{\mu} < 1 < \bar{\mu}$ and $E(\mu) \leq 1$.¹² The white noise error ε is realized at the end of the economy at $t = 4$. The distributions of μ and ε are common knowledge. The manager makes an optimal investment decision to maximize his own payoff; if he is indifferent between investing and forgoing the project, he will invest in any positive

¹¹ As we will see below, the only type of fraud committed is when the manager reports a_H when in fact a_L has occurred. Thus, the “extent” of the fraud is $a_D \equiv (a_H - a_L)$ and f is the marginal penalty. For simplicity, we do not model the optimal choice of f by the RA.

¹² We assume $E(\mu) \leq 1$ to reflect the fact that it is not easy to find positive NPV projects in the real world.

NPV project to maximize the ultimate shareholders' value.¹³

2.4. Period 4: The Liquidation Stage

At $t = 4$, the firm is liquidated and the gross returns from old and (if existing) new projects are distributed to shareholders. Once the firm is liquidated, the truth in past reports may be revealed. For example, consider the situation in which there is undetected fraud (i.e., the gross return was a_L , the manager reported a_H but the RA did not investigate at $t = 2$). If there is no new investment at $t = 3$, the realized terminal cash flow of the firm, a_L , will make it apparent that the manager committed fraud when he reported a_H . The RA has no discretion and has to investigate such cases. If, however, there is new investment at $t = 3$, the terminal cash flow will be $a_L + \mu + \varepsilon$. As a result, if the new investment is taken, there will be no direct evidence of fraud since the support of $a_L + \mu + \varepsilon$ overlaps with the support of $a_L + \mu + \varepsilon$. We assume that if the cash flow is sufficiently low, such that the probability that the $t = 2$ gross return was a_L is sufficiently high, then the RA will investigate.¹⁴ That is, there is a set critical value K such that if the terminal cash flow is at or below this critical value, the RA will investigate fraud. Since the realized cash flow of a non-fraudulent firm may also fall below K , non-fraudulent firms may be investigated. However, again we assume that the RA does not make any ex-post Type I or Type II errors once it decides to investigate. That is, if it investigates a fraudulent firm, the manager is caught and assessed the penalty; if it investigates a non-fraudulent firm, the manager is exonerated and no penalty is assessed.

¹³ These assumptions are employed to abstract away from the Myers-Majluf type underinvestment problem associated with high-return firms. These assumptions simplify our model and are consistent with the manager receiving some performance-based compensation based on the overall terminal value of the firm. If we instead assume the manager acts to maximize old shareholder value as is in Myers and Majluf (1984), the fraudulent firm manager will have additional incentive to overinvest (i.e., to exploit the overvaluation of his firm's stock price). Thus, the nature of our results will be the same.

¹⁴ Wang (2006) provides a detailed justification for this behavior by showing how mingling cash flows from multiple projects negatively affects the information content of realized returns with respect to fraud. We adopt a simple abstraction of her model to capture this feature.

Given that the RA investigates at $t = 4$ whenever the reported gross return from new investment is lower than K , if prior undetected fraud exists and there is new investment at $t = 3$, the firm will be investigated if its gross return from the new investment is lower than $\frac{a_D}{I} + K$. The first term is the difference in the return that the new investment must make up for the final return to be consistent with the manager's prior earnings report. The value of K is an addition term that requires the reported return from new investment to be sufficiently unusual to warrant investigation. We assume that K is small ($K \ll 1$) and its value is set prior to $t = 1$.^{15,16} The investigation cost of the RA is again C_I per case.

3. Equilibrium

The model is solved by backward induction. Section 3.1 determines the optimal investment rule of the manager at $t = 3$ as a function of the exogenous parameters as well as the endogenous variables determined prior to $t = 3$ (i.e., the compensation contract (\bar{w}, α) and the manager's reporting strategy). Section 3.2 derives the optimal reporting and auditing strategies, anticipating the investment strategy derived in section 3.1 and taking as given the compensation contract (\bar{w}, α) . Section 3.3 characterizes the optimal compensation contract (\bar{w}, α) offered to

¹⁵ Since the objective of the RA is to minimize deadweight loss, if the RA chooses K at $t = 4$, it will set K equal to zero. This is true because (as can be seen in the next section) the only benefit of fraud detection in our model is to prevent the cost associated with over-investment, which, at $t = 4$, is sunk. Thus, there is no benefit to investigating at $t = 4$ and, given the positive investigation cost, the optimal amount to investigate is zero.

¹⁶ We assume that K is small and exogenously set (rather than endogenously determined) in the model. We do this because there are likely to be many factors outside the bounds of this model that will affect the setting of K in the real world. For example, in the model, for simplicity we assume that the RA makes no ex-post type I and type II errors, and that the investigation cost is fixed per case. However, in the real world, corporate executives are insiders and understand the complicated businesses of their firms much better than outsiders. When a firm is big and there exist multiple investment projects, it is very difficult for outsiders to distinguish cash flows from different projects of the firm. The investigation and litigation process will become very lengthy and very costly, and ex-post type I and type II errors, which are very costly to stakeholders of the firm, are also possible to be made. Thus, although ideally an optimal K should be set through trading off the expected benefits of deterring fraud and the investigation costs, there will be other important factors outside the boundary of this model that will affect the optimal K , which justifies K to be set exogenously and be small.

managers at $t = 1$, anticipating all of the optimal strategies in the subsequent sub-games. To fully characterize the equilibrium, Section 3.4 describes the conditions under which a specific type (e.g., separating, pooling, or mixed) of equilibrium obtains.

The equilibrium concept we adopt is the rational expectation perfect Bayesian equilibrium (PBE) characterized by: (a) common belief of the RA and market investors (regarding the investment behavior of the manager at $t = 3$, the probability that a firm that reports a high return is truly an a_H -type firm at $t = 2$, and the managerial effort choice at $t = 1$) is reasonable (derived from the manager's effort choice and reporting and investing strategies with rational expectation and using Bayes' rule whenever possible); and (b) given the reasonable common belief, the effort choice and reporting strategy of the manager, the evaluation strategy of market investors and the detecting strategy of the RA are sequentially rational.¹⁷

3.1. The Investment Decision of the Manager at $t = 3$

In the liquidation process (i.e., $t = 4$) prior undetected fraud (if any) will be investigated and the fraudulent-reporting (a_L -type) manager will be subsequently penalized if either new investment does not occur at $t = 3$, or the realized return from new investment is too low (i.e.,

$$\mu + \varepsilon < \frac{a_D}{I} + K).$$

At $t = 3$, if there is no prior undetected fraud, the manager will invest in the expansion opportunity when $\mu \geq 1$. That is, a non-fraudulent manager only invests in opportunities with an expected NPV greater than or equal to zero. If there is prior undetected fraud, however, the manager will rationally invest in any project (even a negative expected NPV project) that arrives.

¹⁷ The PBE concept here should be equivalent to the sequential equilibrium concept of Kreps and Wilson (1982) since there are only two types of firms in the model – the sequential equilibrium concept will be stronger if the types of firms are more than two. See Fudenberg and Tirole (1991) for a proof.

Thus, fraud causes an overinvestment problem.¹⁸ The reason for this overinvestment is straightforward. If the manager does not invest in the expansion opportunity, evidence of his prior fraud will be exposed in liquidation and he will be penalized with probability 1. If, however, he invests in the project, the probability of his getting caught for prior fraud will be

$$\Pr ob(\varepsilon < \frac{a_D}{I} + K - \mu) = \Phi\left(\frac{\frac{a_D}{I} + K - \mu}{\sigma}\right) < 1. \quad \text{Since this probability is less than } 1 \text{ for all projects,}$$

the manager with undetected fraud will rationally invest in any available project in order to reduce his probability of being penalized at $t = 4$. Consequently, the ex ante probability of a manager with new investment (and undetected fraud) being prosecuted at $t = 4$ is

$$H \equiv \int_{\underline{\mu}}^{\bar{\mu}} \frac{1}{\mu - \underline{\mu}} \Phi\left(\frac{\frac{a_D}{I} + K - \mu}{\sigma}\right) d\mu. \quad (1)$$

Moreover, at $t = 4$ the RA is also likely to investigate a non-fraudulent a_H -type firm with new investment at $t = 3$, if the firm's gross return from new investment happens to be lower than K . The ex-ante probability of this (ex-ante) type I error is

$$L \equiv \int_1^{\bar{\mu}} \frac{1}{\mu - \underline{\mu}} \Phi\left(\frac{K - \mu}{\sigma}\right) d\mu, \quad (2)$$

which is less than H .

The expected value of future investments (including the possibility λ that a new project will arrive) from non-fraudulent managers is given by

¹⁸ One way to solve the overinvestment problem is to decouple the investment decision at $t=3$ with the (fraudulent) reporting decision at $t=2$ so that the person making the investment decision is not interested in covering up fraudulent reports. One way to do this is to always require the manager at $t=2$ be replaced. But, this policy is likely to be very costly (especially in non-fraud situations) as continuity in management/leadership is important and the expertise of the initial manager is likely to be useful as old projects come to fruition.

$$G \equiv \frac{\lambda(\bar{\mu} - 1)^2}{2(\bar{\mu} - \underline{\mu})} > 0. \quad (3)$$

While the expected value of future investments from fraudulent manager is simply

$$B \equiv \frac{\lambda(\bar{\mu} + \underline{\mu} - 2)}{2} \leq 0. \quad (4)$$

The difference between the expected value of optimal future investment (G) and sub-optimal future investment associated with fraud (B) is the expected over-investment (deadweight) loss associated with fraud:

$$J \equiv G - B \equiv \frac{\lambda(1 - \underline{\mu})^2}{2(\bar{\mu} - \underline{\mu})}.^{19} \quad (5)$$

3.2. The Reporting and Auditing Strategies at $t = 2$

The following proposition describes the optimal reporting strategy of the manager and the auditing strategy of the RA as a function of the contract α and the parameters of the model. Intuition is provided following the proposition. First, we define:

$$P_{RA}^{Crit} \equiv \frac{J - C_I \lambda(1 - H)}{J - C_I \lambda(1 - H) + C_I(1 - \lambda L)}, \quad (6)$$

$$P_{Manager}^{Crit} \equiv \frac{(1 - \lambda + \lambda H) f a_D / \alpha + J}{a_D + J}, \quad (7)$$

and

$$\lambda^{Crit} \equiv \frac{1 - \alpha / f}{1 - H}. \quad (8)$$

In what follows, we assume that $f \geq 1$ (i.e., penalties such as loss of reputation and a prison term

¹⁹ So that the problem is non-trivial, we assume that the expected overinvestment loss J is larger than $\lambda(1 - H)C_I$; otherwise the RA would never audit at $t = 2$.

are substantial) so that $\alpha / f \leq 1$. Further define P^* to be the probability of realizing the high return a_H .

Proposition 1: For given values for α and P^* (and the exogenous parameters):

a. If $\lambda \leq \lambda^{Crit}$, all managers truthfully report and the RA does not audit (i.e., separating).

b. If $\lambda > \lambda^{Crit}$, the exact reporting and audit strategies depend on the following conditions.

- i. When $P^* \geq \text{Max}[P_{RA}^{Crit}, P_{Manager}^{Crit}]$, the optimal reporting strategy is pooling (i.e., $r(a_L) = a_H$ and $r(a_H) = a_H$) and the RA does not audit.
- ii. When $P^* < \text{Max}[P_{RA}^{Crit}, P_{Manager}^{Crit}]$, and $P_{RA}^{Crit} \geq P_{Manager}^{Crit}$ (or, equivalently, $\alpha \geq f(1 - \lambda + \lambda H)a_D / V_D^M$), the optimal reporting strategy is mixed. Specifically, a_H managers always truthfully report, but a_L managers fraudulently report a_H with probability m and truthfully report a_L with probability $1 - m$, where

$$m = \frac{C_I P^* (1 - \lambda L)}{(1 - P^*) [J - C_I \lambda (1 - H)]}. \quad (9)$$

In this case, the RA investigates any claimed a_H -type firm with probability n :

$$n = 1 - \frac{1}{\lambda(1 - H)} \left(1 - \frac{\alpha V_D^M}{f a_D}\right), \quad (10)$$

where

$$V_D^M = a_D - \frac{C_I (1 - \lambda L)(a_D + J)}{J - C_I \lambda (1 - H) + C_I (1 - \lambda L)}. \quad (11)$$

- iii. When $P^* < \text{Max}[P_{RA}^{Crit}, P_{Manager}^{Crit}]$, and $P_{RA}^{Crit} < P_{Manager}^{Crit}$ (or, equivalently, $\alpha < f(1 - \lambda + \lambda H)a_D / V_D^M$), the optimal reporting strategy is mixed, with a_H managers always truthfully reporting, but a_L managers fraudulently reporting a_H with probability m and truthfully reporting a_L with probability $1 - m$, where

$$m = \frac{P^* a_D [1 - (1 - \lambda + \lambda H) f / \alpha]}{(1 - P^*) [J + (1 - \lambda + \lambda H) f a_D / \alpha]}. \quad (12)$$

In this case, however, the RA never investigates.

Proof. See the appendix.

Henceforth we refer to situations in which the condition specified in part a holds as either the truthful or separating equilibrium. Similarly, situations in which the condition in part b holds will be referred to as fraudulent equilibria; within the set of fraudulent equilibria, we will refer to pooling (as in part b.i), fully mixed (as in part b.ii), or partially mixed (as in part b.iii) equilibrium.

When the firm's growth potential λ is sufficiently low (i.e., $\lambda \leq \lambda^{Crit}$ as in part a.), the probability that the manager will be able to mask his fraud via new investment will be low enough to prevent fraudulent reporting. Furthermore, since there is no fraudulent reporting, the RA does not need to audit at $t = 2$. The proposition also shows that the higher the EBC, the lower the threshold growth potential. This is true because the higher the EBC, the greater the manager's gain from committing fraud and selling his shares at the fraudulently inflated price. Thus, for a given distribution of potential growth levels within an industry, the larger the value of α , the larger is the set of firms in that industry that will commit fraud.²⁰

When the industry's growth potential is sufficiently high (i.e., $\lambda > \lambda^{Crit}$), low-return firms may misreport their earnings, knowing that it is very likely that they will receive a new investment opportunity at $t = 3$ that will allow them to "hide" their prior fraud. When P^* is sufficiently high (as in part b.i), the market will believe that any firm reporting a high earning is very likely telling the truth, and thus will give it a value close to that given to a true high earning type. In this case, an a_L manager has much to gain by reporting a_H and selling his shares at the relatively high pooled price. Thus, in addition to the higher likelihood (due to high λ) of being able to obscure

²⁰ This is consistent with recent empirical studies. In particular, Johnson, Ryan and Tian (2003) find that their sample of exposed fraud firms uses a significantly higher level of EBC than their (size- and industry-matched) control sample. Peng and Röell (2004) find that incentive pay in the form of vested options increases the probability of securities class action litigation.

past fraud with new investment (which lowers the expected penalty), the expected benefit to misreporting is also greater due to the higher pooled price. In addition, the RA will not investigate any claimed high earning firm (even if all low return firms misreport their earnings) since, in this case, the deadweight cost C_I of investigating potential fraud is large relative to the low expected gain from preventing the infrequently occurring a_L firms from committing fraud and over-investing. In such a “favorable” environment, all low-earnings firms will naturally misreport their earnings to pool with the high-return firms.

When P^* is small (as in parts *b.ii* and *b.iii*), the optimal reporting strategies are mixed. To understand this result, suppose that all low-earning firms chose to pool with the high earning firms by reporting $r(a_L) = a_H$. The RA would rationally investigate any firm claiming to be a a_H type, since the expected gain from preventing the deadweight overinvestment loss of a potential fraud firm will outweigh the investigation cost. It then would not be profitable for the a_L firm managers to misreport earnings. However, if a_L firms misreport earnings with a certain equilibrium probability (less than 1), the RA will choose not to investigate all claimed high earnings firms. The proposition shows that there exists a mixed fraudulent reporting probability that makes the RA indifferent between investigating any claimed high-earnings firm and not investigating such a firm. Similarly, there exists a mixed strategy investigation probability that makes low-earnings managers indifferent between truthful disclosure and fraudulent reporting. Hence, the proposition characterizes the mixed strategy probabilities m and n such that the mixed strategies of low-earnings managers and the RA are rational reactions to each other. When the parameters are such that $P_{RA}^{Crit} < P_{Manager}^{Crit}$, the manager optimally mixes even though the RA never investigates at $t = 2$ because the mixed fraudulent reporting probability m is low enough

(due to low α) such that the RA cannot justify bearing the investigation costs at $t = 2$.

Corollary 1 is self evident given Proposition 1.

Corollary 1. *If the equity-based executive compensation α is small enough such that $\alpha \leq fH$, that is, $\lambda^{crit} \geq 1$, it will be impossible for fraud to exist in any equilibrium; if α is large enough such that $\alpha \geq f$, that is, $\lambda^{crit} \leq 0$, fraud will exist in any equilibrium.*

It is clear from Corollary 1 that if the probability of a fraudulent-reporting manager with new investment being prosecuted at $t = 4$, H , is big, then it will be difficult for fraud to exist in equilibrium. However, if σ is small, K is small and I is relatively large compared with a_D such that $a_D / I + K \leq \underline{\mu}$, then $H \rightarrow 0$, which we assume in the rest of the paper to ensure the existence of equilibrium fraud.²¹

3.3. The Contracting Problem at $t = 1$

In this section, we examine the contracting problem faced by the entrepreneur who anticipates the equilibrium strategies that will occur in the subsequent sub-games. For each of the potential equilibria, we solve for the managerial compensation contract that maximizes the IPO price, which reflects the effect of the contract on managerial effort and the assumed subsequent strategies. In Section 3.4, we then refine the set of potential equilibria by including only those for which the assumed reporting/auditing strategies are optimal given the optimal contract under those assumed reporting/auditing strategies.

The next proposition specifies the optimal contracts for each of the potential equilibria.

Proposition 2: *Let Σ denote the set of all possible collections of the exogenous parameters. Let $\Omega^T \in \Sigma$ denote the set of parameters for which the truthful equilibrium obtains at $t = 2$. Similarly denote $\Omega^P \in \Sigma$, $\Omega^M \in \Sigma$, and $\Omega^N \in \Sigma$ as the sets of parameters for which, respectively,*

²¹ Corollary 2 presented in the appendix provides partial comparative static results on how the mixing probabilities in the fully mixed case (characterized in b.ii of Proposition 1) vary with growth potential and marginal fraud penalty given values for α and P^* . These partial comparative static results are useful in the proofs of subsequent propositions and corollaries.

the pooling, the fully mixed, and the partially mixed equilibrium (in which “No” monitoring occurs) obtain. For each type of $t=2$ equilibrium, the optimal compensation contract (\bar{w}^*, α^*) and the managerial effort e^* induced by that optimal contract are as follows:

a. **Separating:** For any $\Omega \in \Omega^T$,

$$\bar{w}_T^* = 0, \alpha_T^* = \frac{1}{2} \left[1 - \frac{\delta(a_L + G)}{(a_D \phi)^2} \right]; \quad (13)$$

$$e_T^* = \alpha_T^* \frac{\phi a_D}{\delta}. \quad (14)$$

b. **Pooling Equilibrium:** For any $\Omega \in \Omega^P$,

$$\bar{w}_P^* = 0, \alpha_P^* = \frac{1}{2} \left[1 - \frac{(1 - \lambda + \lambda H) f a_D}{a_D + J} - \frac{\delta(a_L + B)}{\phi^2 (a_D + J)^2} \right]; \quad (15)$$

$$e_P^* = \alpha_P^* \frac{\phi(a_D + J)}{\delta} + \frac{\phi(1 - \lambda + \lambda H) f a_D}{\delta}. \quad (16)$$

c. **Fully Mixed:** For any $\Omega \in \Omega^M$,

$$\bar{w}_M^* = 0, \alpha_M^* = \frac{1}{2} \left[1 - \frac{\delta(a_L + G)}{\phi^2 \phi V_D^M} \right]; \quad (17)$$

$$e_M^* = \alpha_M^* \frac{\phi V_D^M}{\delta}, \quad (18)$$

where

$$\phi \equiv V_D^M \left[1 + \frac{C_I(1 - \lambda L)}{J - C_I \lambda(1 - H)} \right] = a_D - \frac{C_I(1 - \lambda L)J}{J - C_I \lambda(1 - H)} > V_D^M,$$

and V_D^M is as defined in Proposition 1.

d. **Partially Mixed:** For any $\Omega \in \Omega^N$,

Optimal contract does not exist;

$$e_N^* = \frac{\phi(1 - \lambda + \lambda H) f a_D}{\delta}. \quad (19)$$

Proof. See the appendix.

Since no optimal compensation contract exists when agents anticipate the partially mixed strategy equilibrium at $t = 2$, the partially mixed strategy equilibrium does not exist for the overall game. Thus, this case is not analyzed further below.

Corollary 3: *The following comparative static relationships hold.*

a. **Separating Equilibrium:** *for any Ω strictly inside Ω^T :*

$$i. \quad \frac{d\alpha_T^*}{d\lambda} < 0, \quad \frac{d\alpha_T^*}{df} = 0, \quad \frac{d\alpha_T^*}{d\varphi} > 0, \quad \frac{d\alpha_T^*}{d\delta} < 0;$$

$$ii. \quad \frac{de_T^*}{d\lambda} < 0, \quad \frac{de_T^*}{df} = 0, \quad \frac{de_T^*}{d\varphi} > 0, \quad \frac{de_T^*}{d\delta} < 0;$$

$$iii. \quad \frac{dP_T^*}{d\lambda} < 0, \quad \frac{dP_T^*}{df} = 0, \quad \frac{dP_T^*}{d\varphi} > 0, \quad \frac{dP_T^*}{d\delta} < 0.$$

b. **Pooling Equilibrium:** *for any Ω strictly inside Ω^P :*

$$i. \quad \frac{d\alpha_P^*}{d\lambda} > 0, \quad \frac{d\alpha_P^*}{df} < 0, \quad \frac{d\alpha_P^*}{d\varphi} > 0, \quad \frac{d\alpha_P^*}{d\delta} < 0;$$

$$ii. \quad \frac{de_P^*}{d\lambda} < 0 \text{ if } f \text{ is sufficiently large, } \frac{de_P^*}{df} > 0, \quad \frac{de_P^*}{d\varphi} > 0, \quad \frac{de_P^*}{d\delta} < 0;$$

$$iii. \quad \frac{dP_P^*}{d\lambda} < 0 \text{ if } f \text{ is sufficiently large, } \frac{dP_P^*}{df} > 0, \quad \frac{dP_P^*}{d\varphi} > 0, \quad \frac{dP_P^*}{d\delta} < 0.$$

c. **Fully Mixed Equilibrium:** *For any Ω strictly inside Ω^M :*²²

$$i. \quad \frac{d\alpha_M^*}{d\lambda} \text{ is ambiguous, } \frac{d\alpha_M^*}{df} = 0, \quad \frac{d\alpha_M^*}{d\varphi} > 0, \quad \frac{d\alpha_M^*}{d\delta} < 0;$$

$$ii. \quad \frac{de_M^*}{d\lambda} \text{ is ambiguous, } \frac{de_M^*}{df} = 0, \quad \frac{de_M^*}{d\varphi} > 0, \quad \frac{de_M^*}{d\delta} < 0;$$

$$iii. \quad \frac{dP_M^*}{d\lambda} \text{ is ambiguous, } \frac{dP_M^*}{df} = 0, \quad \frac{dP_M^*}{d\varphi} > 0, \quad \frac{dP_M^*}{d\delta} < 0.$$

Proof: These results can be easily verified using Proposition 2.²³

3.4. The Equilibrium of the Overall Game

²² If we take into consideration the fact that market investors can rationally infer the probability n that the RA adopts in investigating fraud (i.e., market investors can rationally expect the RA to correct the overinvestment problem of a portion n of the fraudulent reporting firms) at $t = 2$, V_H^M thus V_D^M will be greater, and the changes in V_D^M and n will always reinforce each other. Since $\partial n / \partial f < 0$, we will have $\partial V_D^M / \partial f < 0$, and, as a result $d\alpha_M^* / df < 0$, $de_M^* / df < 0$, and $dP_M^* / df < 0$. See also footnote 28.

²³ Since the proofs are fairly tedious but straightforward, we do not provide proofs in the appendix for brevity. However, they are available upon request from the authors.

In this section we show that for each of the three types of equilibrium (separating, pooling, and fully mixed) there exists a non-empty set of parameters for which a particular type of equilibrium obtains. Proposition 3 is obvious given Propositions 1 and 2:

Proposition 3: *For λ sufficiently small, the equilibrium to the overall game is such that the truthful separating equilibrium obtains in the $t = 2$ reporting/auditing sub-game. Thus, the set Ω^T is non-empty and includes firms/industries that have low growth opportunities.*

Proof: *See the Appendix.*

In the event that there are high growth options, Proposition 4 states that, depending upon the collection of exogenous parameters, either the pooling or the fully mixed equilibrium obtains.

Proposition 4: *For λ sufficiently high, the equilibrium to the overall game is such that either the pooling or the fully mixed equilibrium obtains in the $t = 2$ reporting/auditing sub-game. Thus, the sets Ω^P and Ω^M are non-empty and include firms/industries that have high growth opportunities.*

Proof: *See the appendix.*

Figure 1 illustrates how the equilibrium depends upon the parameters. The figure plots α_T^* , α_P^* , and α_M^* as a function of λ . Corollary 3 and the proof of Proposition 4 justify the ranking and the shape of each curve. In addition, figure 1 also plots $\lambda^{Crit} = \frac{1-\alpha/f}{1-H}$ as a function of α . That is, the line labeled λ^{Crit} denotes, for a given α , the specific value of λ (i.e., the value on the λ^{Crit} line associated with that α) that is such that if λ is smaller (greater) than that value, the separating (a fraudulent) equilibrium obtains. The line $\hat{\lambda}^{Crit}$ denotes, for a given α , the boundary where $P_{RA}^{Crit} = P_{Manager}^{Crit}$ (or equivalently $\alpha = f(1-\lambda + \lambda H)a_D / V_D^M$); any λ strictly to the right of this line corresponds to a value such that $\alpha \geq f(1-\lambda + \lambda H)a_D / V_D^M$, which is the condition for the fully mixed equilibrium.

As implied by Proposition 1, in order to obtain a separating equilibrium, α and λ must

be below (to the left of) the λ^{Crit} line. Similarly, in order for a pooling equilibrium to obtain, α and λ must be above (to the right of) the λ^{Crit} line. A fully mixed equilibrium obtains if α and λ are above (to the right of) the $\hat{\lambda}^{Crit}$ line. For both the pooling and fully mixed potential equilibria, Proposition 1 imposes an additional condition on the equilibrium level of P^* which will further refine which equilibrium obtains. Finally, if, for a specific λ , all of the corresponding values of α are such that none of the above rankings is satisfied, then no equilibrium exists. We next consider some specific ranges.

First consider the equilibrium for any value of $\lambda \leq \lambda_1$. Specifically, consider $\lambda = \lambda_0$. For λ_0 , the optimal α under the assumption that the separating equilibrium obtains is $\alpha_T^*(\lambda_0)$ (i.e., the value of α on the α_T^* line that corresponds to λ_0). For this combination of α and λ , the conditions required for the separating equilibrium are satisfied: $\lambda_0 < \lambda^{Crit}(\alpha_T^*(\lambda_0))$. None of the other potential equilibria obtain for $\lambda < \lambda_1$ because the optimal α for that λ under other potential equilibria do not satisfy the conditions required for those equilibria (since $\lambda_0 < \lambda^{Crit}(\alpha_p^*(\lambda_0))$ and $\lambda_0 < \lambda^{Crit}(\alpha_M^*(\lambda_0))$).

Next consider any value of $\lambda > \lambda_3$. In particular, consider $\lambda = \lambda_4$. For λ_4 , the optimal α under the various potential equilibria are $\alpha_T^*(\lambda_4)$, $\alpha_p^*(\lambda_4)$, and $\alpha_M^*(\lambda_4)$. For each combination of α and λ , only the conditions required for the pooling and fully mixed equilibria are satisfied. The separating equilibrium does not obtain in this case since $\lambda_4 > \lambda^{Crit}(\alpha_T^*(\lambda_4))$. Let $P_M^*(\alpha_M^*)$ denote the probability of the high return given the optimal α and the optimal managerial effort under fully mixed and $P_p^*(\alpha_p^*)$ denote that probability under pooling. If $P_M^*(\alpha_M^*) < \text{Max}[P_{RA}^{Crit}(\alpha_M^*), P_{Manager}^{Crit}(\alpha_M^*)]$ and $P_p^*(\alpha_p^*) < \text{Max}[P_{RA}^{Crit}(\alpha_p^*), P_{Manager}^{Crit}(\alpha_p^*)]$, then the only equilibrium at $\lambda = \lambda_4$ is the fully mixed equilibrium. If $P_M^*(\alpha_M^*) > \text{Max}[P_{RA}^{Crit}(\alpha_M^*), P_{Manager}^{Crit}(\alpha_M^*)]$

and $P_p^*(\alpha_p^*) > \text{Max}[P_{RA}^{Crit}(\alpha_p^*), P_{Manager}^{Crit}(\alpha_p^*)]$, then the only equilibrium at $\lambda = \lambda_4$ is the pooling equilibrium. If $P_M^*(\alpha_M^*) < \text{Max}[P_{RA}^{Crit}(\alpha_M^*), P_{Manager}^{Crit}(\alpha_M^*)]$ and $P_p^*(\alpha_p^*) > \text{Max}[P_{RA}^{Crit}(\alpha_p^*), P_{Manager}^{Crit}(\alpha_p^*)]$, then both pooling and mixed equilibria are possible, and which one occurs depends on the entrepreneurial utility in these two equilibria (as the entrepreneur is the Stackelberg leader in the overall game and can pick the equilibrium by setting α). A similar analysis implies that for values of λ such that $\lambda_2 < \lambda < \lambda_3$, the only possible equilibrium is pooling while for values of λ such that $\lambda_1 < \lambda < \lambda_2$, no equilibrium exists.

Figure 2 is similar to Figure 1 and isolates the effect of f on the equilibrium. Specifically, it shows how α and f are related for the separating (T), pooling (P), and mixed (M) equilibria. The line labeled λ^{Crit} indicates the boundary between separation and fraudulent equilibria, with values of f below (i.e., to the right) such that separation obtains (i.e., $\lambda > \lambda^{Crit}$). The line labeled $\hat{\lambda}^{Crit}$ denotes the boundary between fully mixed and partially mixed, with values of f to the left implying fully mixed (i.e., $\alpha \geq f(1 - \lambda + \lambda H)a_D / V_D^M$). As can be seen, for f greater than f_3 , only separating equilibria occur. For f between f_2 and f_3 , either pooling or separating occur (depending on whether the condition on P is satisfied for pooling or not and on the entrepreneur's utility). Between f_1 and f_2 only pooling is possible. And for $f < f_1$, either pooling or fully mixed occurs (again depending on the conditions on P and on the entrepreneurial utility).

4. Empirical Implications

In this section we examine the implications of the model with respect to (1) the commission, detection, and observation of fraud, (2) the impact of fraud on equity based executive compensation, and (3) public policy and economic performance.

4.1. The Incidence of Fraud

With respect to the incidence of fraud, the first empirical implication of the model is that fraudulent reporting will be concentrated in “new economy” industries for which growth opportunities are high (Propositions 1, 3, and 4). Conversely, fraud will not occur in ‘old economy’ industries for which the expected arrival of new projects is not sufficient to provide enough potential masking of fraud to create the incentive to commit fraud in the first place.

The second empirical implication of the model is that while fraud incentive is strongest in good times (i.e., when the marginal productivity of effort and the probability of realizing good earnings are high, and the pooling PBE occurs), a significant amount of detected fraud is more likely to occur when the ‘new economy’ industries fall into downturns (i.e., when the marginal productivity of effort and the probability of realizing good earnings are low, and the fully mixed PBE occurs). To understand this point, consider a situation where a high growth industry is in the pooling PBE. If there is a decrease in the marginal productivity of effort (which usually is a result of industry downturns), the equilibrium probability of realizing good returns, P_p^* , will decrease. However, the threshold probability for pooling, $Max[P_{RA}^{Crit}, P_{Manager}^{Crit}(\alpha_p^*)]$, will weakly increase due to a reduction in α_p^* caused by the decrease in φ . If this decrease in φ is big enough, it can result in $P_p^* < Max[P_{RA}^{Crit}, P_{Manager}^{Crit}(\alpha_p^*)]$. The pooling PBE is no longer feasible and a drop from pooling to fully mixed occurs (the proof of Proposition 4 shows that the fully mixed PBE is the only feasible equilibrium in this case since $P_M^* < P_p^* < Max[P_{RA}^{Crit}, P_{Manager}^{Crit}(\alpha_p^*)] \leq Max[P_{RA}^{Crit}, P_{Manager}^{Crit}(\alpha_M^*)]$). Thus we will observe substantially worse economic performance and a significant amount of detected fraud in the industry.

The third main empirical implication of the model concerns the relationship between the commission of fraud and the ex-post observation of fraud in the fully mixed strategy equilibrium.

Combining the results of Corollary 2 (in the Appendix) and Corollary 3, we have the following comparative statics regarding the probability of committing fraud (i.e., $(1 - P_M^*)m$), the probability of detecting fraud (i.e., n), and the probability of observing firms being caught for fraud (i.e., $(1 - P_M^*)mn$).

Corollary 4: *If the set of parameter values are such that the fully mixed strategy equilibrium obtains, the following comparative statics results hold for the probability of committing fraud, the probability of detecting fraud, and the probability of observing firms being caught for fraud:*

- 1) $\frac{d(1 - P_M^*)m}{d\lambda} < 0$, $\frac{dn}{d\lambda} > 0$, $\frac{d(1 - P_M^*)mn}{d\lambda} > 0$ if $\frac{f}{\alpha_M^*} \geq 2$;
- 2) $\frac{d(1 - P_M^*)m}{df} = 0$, $\frac{dn}{df} < 0$, $\frac{d(1 - P_M^*)mn}{df} < 0$;²⁴
- 3) $\frac{d(1 - P_M^*)m}{d\phi} > 0$, $\frac{dn}{d\phi} > 0$, $\frac{d(1 - P_M^*)mn}{d\phi} > 0$;
- 4) $\frac{d(1 - P_M^*)m}{d\delta} < 0$, $\frac{dn}{d\delta} < 0$, $\frac{d(1 - P_M^*)mn}{d\delta} < 0$.

Proof: Parts 2), 3), and 4) can be easily verified. Part 1) can only be verified numerically. We examine a wide range of parameter values and find the results of Part 1) hold.

Part 1 shows that an increase in an industry's growth potential has opposite effects on the probability of committing fraud and the probability of detecting fraud in the fully mixed equilibrium, with the increase in the detection probability dominating the decrease in the commission probability, resulting in a net increase in the amount of frauds that are exposed. A numerical illustration is provided in Panel D of Table 1. Thus, if the number of exposed frauds is taken as a (proportional) proxy for the amount of fraud being committed, cross-sectional

²⁴ If we take into consideration the fact that market investors can rationally infer the probability n that the RA adopts in investigating fraud (i.e., market investors can rationally expect the RA to correct the overinvestment problem of a portion n of the fraudulent reporting firms) at $t = 2$, we should also have $\frac{d(1 - P_M^*)m}{df} < 0$. See also footnotes 22 and 28.

comparisons of exposed frauds in industries differing only in their growth potential would inappropriately conclude that the industries with fewer exposed frauds have less fraud being committed.

The basic intuition is as follows. When the industry's growth potential increases, the expected overinvestment loss of a potential fraud firm will increase and it will be more difficult for fraud to get exposed in future liquidation. Thus, the RA will choose to detect fraud at $t = 2$. Low-earnings managers hence need to commit fraud with lower probability in order to keep the RA indifferent between investigating and not investing a claimed high earning firm in equilibrium. When the industry's growth potential increases, however, fraudulently-reporting managers are less likely to suffer a penalty (from being caught) in the future (since fraud is now less likely to get exposed in liquidation), and the difference in market value between a claimed high earning firm and a claimed low earning firm (i.e., V_D^M) is larger (since the low earning firms now misreport their earnings with a lower probability). Hence, earning misreporting becomes more attractive to the low-earnings managers. To keep the low-earnings managers indifferent between truthful disclosure and misreporting, the probability of detecting fraud at $t = 2$ must increase. When the fraud penalty is substantial, the effect of a reduction in personal cost to fraudulent reporting managers due to the increase in growth potential (thus less likelihood of future fraud exposure) will also be quite significant. Therefore, the probability of detecting fraud has to substantially increase in order to balance this reduction in personal cost to fraudulent reporting managers, resulted in (ex post) higher probability of any firm in the industry being caught for committing fraud.

Since a marginal change in fraud penalty does not affect the ($t = 2$ or $t = 4$) payoff to the RA, it should not directly cause the probability of committing fraud to change, as this probability

should keep the RA indifferent between investigating and not investigating a claimed high earning firm.²⁵ However, as fraud penalty increases, it will be more costly for the low-earnings managers to commit fraud. Thus, the probability of detecting fraud at $t = 2$ need to be reduced so that the low earning managers can still be indifferent between fraudulent reporting and truthful disclosure in equilibrium, resulted in lower probability of observing firms being caught for fraud.

Similar to Goldman and Slezak (2006), our model also predicts that EBC is a double-edged sword. Higher EBC induces more managerial effort but also makes fraud easier to occur in the reporting/auditing sub-game, and increases both the probability of committing fraud and the probability of detecting fraud in the fully mixed equilibrium. Given a marginal increase in EBC (caused by either an increase in marginal productivity of effort φ or a decrease in managerial disutility of effort δ , or other reasons not incorporated in our simple model), ceteris paribus, in the fully mixed equilibrium the optimal managerial effort will increase, which in turn will increase the probability of realizing high earnings thus make the RA choose not to investigate a claimed high earning firm at $t = 2$. Consequently, the equilibrium probability of (any low-earnings firm) committing fraud will increase (so that the RA can still be indifferent between investigating and not investigating a claimed high earning firm). Furthermore, when EBC increases, earning misreporting will become more attractive to the low return manager as his personal benefit from misreporting increases while his personal cost remains unchanged. Therefore, in order to keep the manager still indifferent between truthful disclosure and misreporting in equilibrium, the probability of detecting fraud has to increase. Hence, this increase in both the probability of committing fraud and that of detecting fraud results in higher probability of observing firms being caught for fraud in the industry.

²⁵ However, as pointed out in footnote 22, a marginal increase in f reduces P_M^* , thus it also reduces the probability of fraud commission.

4.2. Incentive Compensation, Effort, and Fraud

The model also generates empirical implications regarding the effects of growth options and fraud penalties on the equilibrium level of α and effort (thus economic performance). Corollary 3 specifies the effects. In general, growth opportunities and fraud penalties will have different marginal effects on α and effort, depending on the type of equilibrium that obtains for those parameters changes.

As the growth potential increases, the equilibrium α under separation decreases. When λ increases, the firm's growth value (thus the firm value) also increases. However, keeping the compensation contract constant, the manager's chosen effort levels remain unchanged given an increase in λ . The entrepreneur would pay the manager too much for his effort if she did not adjust the original compensation level accordingly – the original α would be too costly to the entrepreneur given the higher firm value associated with a higher λ . Therefore, the entrepreneur will lower the manager's α in response to an increase in λ . Since changes in the severity of fraud penalty f affect neither the manager's effort choice nor the firm value in the truthful disclosure separating equilibrium, the optimal α will not vary with f .

As the growth potential increases, the equilibrium α under pooling generally increases. In the pooling PBE, a marginal increase in λ will make fraudulent reporting less likely to be exposed in the future. If f is big enough, this reduced likelihood of future penalty will be quite substantial to the manager. The manager will optimally respond by reducing his costly effort. Since he can fraudulently report return as a_H if the firm actually realizes a_L , and he is now less likely to suffer penalty in the liquidation stage, he needs not exert the same level of costly effort as before. This reduction in managerial effort hurts the entrepreneur (as the firm now has more chance to realize low return and suffer overinvestment loss due to misreporting). Moreover,

when λ increases, the firm's overinvestment loss (i.e., J) will also increase. Therefore, it is meaningful for the entrepreneur to respond by optimally increasing α in order to induce more managerial effort. On the contrary, a marginal increase in f will make the manager work harder in order to avoid becoming an a_L type and suffer the intensified expected future penalty. This intensified penalty can partially substitute for the effort incentive provided by costly executive compensation. The entrepreneur will then respond by optimally reducing α .

As the growth potential increases, the change in equilibrium α under fully mixed is ambiguous. In the mixed strategy equilibrium, keeping the compensation contract constant and given a marginal increase in λ , the manager will increase his effort. The reason is that a marginal increase in λ will enlarge the difference in market value between the claimed a_H and a_L types (i.e., V_D^M) through reducing the a_L type's probability of misreporting; thus the manager will work harder to increase his probability of being an a_H type. Thus, an increase in λ can partially substitute for the effort incentive provided by α , which becomes increasingly costly to the entrepreneur given the increase in λ as the firm's growth value is now higher. This effect will cause the entrepreneur to reduce the α . However, since the increase in λ enlarges the difference in market value between the claimed a_H - and claimed a_L -type firms, it will increase the difference in personal benefits between the a_H -type and a_L -type managers. Therefore, α will become more effective in providing managerial effort incentive, i.e., a marginal increase in α will spur more incremental managerial effort. Furthermore, given the now increased market value difference between the claimed a_H - and claimed a_L -type firms, managerial effort becomes more important and valuable to the entrepreneur. Therefore, it makes sense for the entrepreneur to respond by increasing α . The simultaneous working of these two

effects results in the overall effect of the increase in λ on α being ambiguous. As pointed out by footnote 22, since a marginal increase in f reduces the detection probability n thus reduces V_D^M (under fully mixed), it also reduces α .

In the US, the ‘old economy’ industries (e.g., the manufacturing industry) have low growth potential. Thus, they are usually in the truthful disclosure PBE. Our model then implies a negative effect of a marginal change in λ on equilibrium α and no effect of a marginal change in f on α for such industries. On the contrary, the ‘new economy’ industries (e.g., the high-tech industry) have high growth potential. Thus they are usually in the fraudulent equilibria. Our model implies a positive effect of a marginal change in λ on α especially when the marginal productivity of effort is high thus these industries are booming (i.e., under pooling PBE), and a negative effect of f on α for such industries.^{26,27}

4.3. Growth Opportunities, Fraud, and Economic Performance

This section examines a situation in which a marginal increase in growth potential can cause a regime switch from a high-performance, no-exposed-fraud equilibrium (i.e., the high EBC and high effort pooling equilibrium) to a low-performance, exposed-fraud equilibrium (i.e., the lower EBC, lower effort fully mixed PBE). Thus, paradoxically, an increase in growth potential, which typically implies increased future prosperity, can, via its impact on fraud and EBC, result in worse economic performance and a significant increase in the amount of fraud exposed. That is, there is a dark side to innovation. For example, following innovations in financial instruments

²⁶ A numerical illustration of the situation where the marginal productivity of effort is high and the pooling PBE dominates the fully mixed PBE (in terms of entrepreneurial utility) for the ‘new economy’ industries can be obtained from the authors.

²⁷ This implication can potentially explain the empirical finding of Murphy (2003), Itner, Lambert and Larcker (2003), and Anderson, Banker and Ravindran (2000). These papers document that the high growth ‘new economy’ sector substantially more and more rely on EBC to provide managerial incentive, while the phenomenon is not as pronounced in the low growth ‘old economy’ sector.

designed to increase efficiency in risk sharing and capital allocation (for example, from the financial innovation during the roaring twenties to the recent wide-spread use of mortgage-backed securities), there have been episodes of significant declines in economic performance (the great depression following the twenties and the credit/liquidity crisis following the mortgage “meltdown” in 2007) combined with the revelation of significant amounts of fraudulent behavior (widespread abuse during the twenties and over dozens of SEC investigations of corporate fraud in connection with the sub-prime mortgage meltdown). Similarly, deregulation which implies increased growth opportunities can also lead to increased exposure of fraud. For example, deregulation of energy markets lead to increased growth opportunities associated with efficiency gains from production and demand smoothing – but is also associated with the Enron scandal.

Figure 3 illustrates this regime shift. The figure plots P_p^* and P_M^* as a function of λ . It also plots P_{RA}^{Crit} , $P_{Manager}^{Crit}(\alpha_p^*)$ and $P_{Manager}^{Crit}(\alpha_M^*)$ as a function of λ . Equations 6 and 7, Corollary 3 and the proof of Proposition 4 justify the ranking and the shape of each curve. Since P_p^* is decreasing in λ (due to the reduction in managerial effort) and P_{RA}^{Crit} is increasing in λ (due to the strategic reaction of the RA to higher deadweight overinvestment loss and lower likelihood of fraud exposure at $t = 4$), the pooling equilibrium is not feasible beyond λ_1 (where $P_p^* = P_{RA}^{Crit}$). Consider an increase in λ from λ_0 to λ_2 . If the high growth industry is in the pooling PBE at λ_0 , this increase in growth potential alone will cause the industry to drop from the pooling PBE to the fully mixed PBE (the fully mixed PBE is the only feasible equilibrium at $\lambda = \lambda_2$ since $P_M^* < P_p^* < P_{RA}^{Crit}$). We will then observe significantly worse economic performance and a significant amount of detected fraud in the industry. A numerical illustration of such a fall is presented in Panel B and Panel C of Table 1. When the growth potential λ

increases from 0.9 to 0.93, the pooling PBE is no longer feasible and a drop from pooling to fully mixed PBE occurs. The economic performance of the industry substantially deteriorates (the probability of realizing good earnings falls from 0.819 to 0.242) and the amount of exposed fraud cases in the industry significantly increases (from 0 percent to 0.52 percent of the firms in the industry).

4.4. Public Policy, Fraud, and Economic Performance

Next we examine how an increase in the penalty for fraud will affect the extent of fraud and the economic performance of the firm/industry. Proposition 1 shows that, holding fixed the compensation contract, an increase in f will lead to less fraud being committed in the economy as a whole since an increase in f raises λ^{Crit} ; as λ^{Crit} increases, more firms will fall into the parameter space in which separation obtains, thus reducing fraudulent activity. Thus effect is presumably the intended effect of increased penalties.

Proposition 2, however, implies that as penalties increase, the compensation contract may change. Firms already in Ω^T are obviously unaffected by an increase in f . There is no impact on the compensation contract, and, as a result, no change in the manager's effort (e_T^*) or productivity (P_T^*). Firms that are in Ω^P , however, will respond to the increase in f by lowering α_p^* . This happens because the fraud penalties act as substitutes for α under pooling. Holding fixed α , the manager will exert more effort when f increases so as to raise P^* , which results in a lower probability of realizing the low return state in which the manager knows he will commit fraud. An increase in the penalties make the manager want to avoid the low return state more and, thus, raises his effort level. Given the increased effort, the entrepreneur can scale back α , thus allowing shareholders to retain more of the firm's cash flow, which raises the entrepreneur's wealth since shareholders are willing to pay more for the firm at the IPO. Proposition 2 shows

that an increase in f on net increases e and P^* , with the increase in effort due to an increase in f dominating the drop in effort due to the resulting fall in α . Thus, for the pooling case, an increase in f results in both a drop in fraud (as intended) and the spillover effect of an increase in economic performance as the probability of realizing the high return P increases.

On the other hand, if the firm is in Ω^M , penalties generate mixed results. Although an increase in f will be successful at reducing the amount of fraud committed and observed (see Corollary 4 and footnote 24), an unfortunate side-effect of the increase in f is that both e and P^* fall (footnote 22), resulting in declining economic performance. While there are clearly other forces at work, this result indicates that the increased penalties imposed by Sarbanes-Oxley may have contributed to the weak performance of the U.S. economy following its enactment.

5. Conclusion

In contrast to existing models of fraud, this paper considers an agency model in which the auditing/investigation strategy of the regulatory agency is strategically determined by considering the equilibrium fraud commission strategy of managers and investigation costs. We show that the strategic interaction of the manager's and the regulator's strategies under an agency framework generates a rich set of implications on (1) the commission, detection, and observation of fraud, (2) the impact of growth opportunities and fraud penalties on managerial equity based compensation, (3) the surprising effect of growth opportunities on short-run economic performance and exposed fraud, and (4) the relationship between public policy with respect to fraud and economic performance. For example, we show that in some cases, an increase in growth potential alone can result in worse economic performance and a significant amount of detected fraud for a high growth industry. We also show that when growth opportunities increase, the amount of fraud being committed may fall, but that the amount of fraud observed (i.e., committed fraud that is detected)

will always increase. Thus, cross-sectional and time-series comparisons in the amount of fraud observed may not be indicative of differences in the amount of fraud being committed. The model highlights the existence of an equilibrium link between the level of EBC, economic performance, and the extent of fraud committed and observed.

Table 1: A Simple Numerical Illustration of the Model

We assume the following parameter values: the high gross return from the firm's assets in place is $a_H=2$, and the low return is $a_L=0.5$; the scale of new investment is $I=5$, its mean gross return is $\mu \sim \text{Uniform}(0.7, 1.2)$, and its white noise return error is $\varepsilon \sim N(0, 0.05^2)$; the critical reported level from new investment below which the RA will have to investigate fraud at $t = 4$ is $K=0.3$; the marginal fraud penalty is $f=1.2$; the manager's effort disutility coefficient is $\delta=0.45$; the RA's investigation cost is $C_I=0.08$ per case; the marginal productivity of effort $\varphi=0.62$, thus the probability of realizing a high return is $P(e)=0.62e$. Based on these parameter values, it can be calculated that the ex ante probability of a manager with prior undetected fraud and new investment being prosecuted at $t = 4$ is $H=8.48 \times 10^{-4}$ and the ex ante probability of the RA investigating a non-fraudulent firm (with new investment) in period 4 is virtually zero, i.e., $L=0$.

Panel A: Separating (Truthful Disclosure) Equilibrium

λ	$f(1-\lambda+\lambda H)$	G	α_T^*	e_T^*	$P(e_T^*)$	Principal's Utility
0.1	1.080	0.020	0.365	0.754	0.467	0.776
0.2	0.960	0.040	0.360	0.743	0.461	0.788
0.3	0.840	0.060	0.354	0.732	0.454	0.801
0.4	0.720	0.080	0.349	0.722	0.447	0.814
0.5	0.601	0.100	0.344	0.711	0.441	0.827
0.6	0.481	0.120	0.339	0.700	0.434	0.841
0.7	0.361	0.140	0.334	0.689	0.427	0.854

Panel B: Pooling Equilibrium

λ	$f(1-\lambda+\lambda H)$	J	P_{RA}^{Crit}	$P_{Manager}^{Crit}$	G	B	α_p^*	e_p^*	$P(e_p^*)$	Principal's Utility
0.8	0.241	0.360	0.787	0.745	0.160	(0.200)	0.352	1.400	0.868	1.240
0.85	0.181	0.383	0.797	0.582	0.170	(0.213)	0.380	1.361	0.844	1.162
0.9	0.121	0.405	0.806	0.446	0.180	(0.225)	0.408	1.321	0.819	1.086
0.93	0.085	0.419	0.811	0.375	0.186	(0.233)	0.424	1.297	0.804	1.042
0.95	0.061	0.428	0.815	0.331	0.190	(0.238)	0.435	1.281	0.794	1.013
0.97	0.037	0.437	0.818	0.290	0.194	(0.243)	0.445	1.265	0.784	0.985
0.99	0.013	0.446	0.821	0.251	0.198	(0.248)	0.456	1.249	0.774	0.957
1	0.001	0.450	0.822	0.232	0.200	(0.250)	0.461	1.241	0.769	0.943

Table 1 (Continued)

Panel C: Mixed Strategy Equilibrium

λ	$f(1-\lambda+\lambda H)$	P_{RA}^{Crit}	$P_{Manager}^{Crit}$	$f(1-\lambda+\lambda H) * a_D / V_D^M$	V_D^M	ϕ	α_M^*	e_M^*	$P(e_M^*)$	Principal's Utility
0.85	0.181	0.797	0.780	0.243	1.118	1.403	0.250	0.385	0.239	1.005
0.9	0.121	0.806	0.595	0.160	1.131	1.403	0.249	0.388	0.241	1.018
0.93	0.085	0.811	0.485	0.112	1.138	1.403	0.248	0.390	0.242	1.025
0.95	0.061	0.815	0.413	0.080	1.143	1.403	0.248	0.390	0.242	1.030
0.97	0.037	0.818	0.341	0.048	1.147	1.403	0.248	0.391	0.243	1.034
0.99	0.013	0.821	0.270	0.017	1.151	1.403	0.247	0.392	0.243	1.039
1	0.001	0.822	0.234	0.001	1.153	1.403	0.247	0.392	0.243	1.041

Panel D: Probabilities of Fraud Commission and Detection in Mixed Strategy Equilibrium

λ	$P(e_M^*)$	m	n	$(1 - P(e_M^*)) * m$	$(1 - P(e_M^*)) * m * n$
0.85	0.2388	0.0798	0.0054	0.0607	0.0003
0.9	0.2407	0.0761	0.0620	0.0578	0.0036
0.93	0.2416	0.0740	0.0929	0.0562	0.0052
0.95	0.2421	0.0727	0.1124	0.0551	0.0062
0.97	0.2426	0.0714	0.1310	0.0541	0.0071
0.99	0.2429	0.0701	0.1488	0.0530	0.0079
1	0.2431	0.0694	0.1574	0.0526	0.0083

Figure 1: Equilibrium in EBC-Growth Potential Space

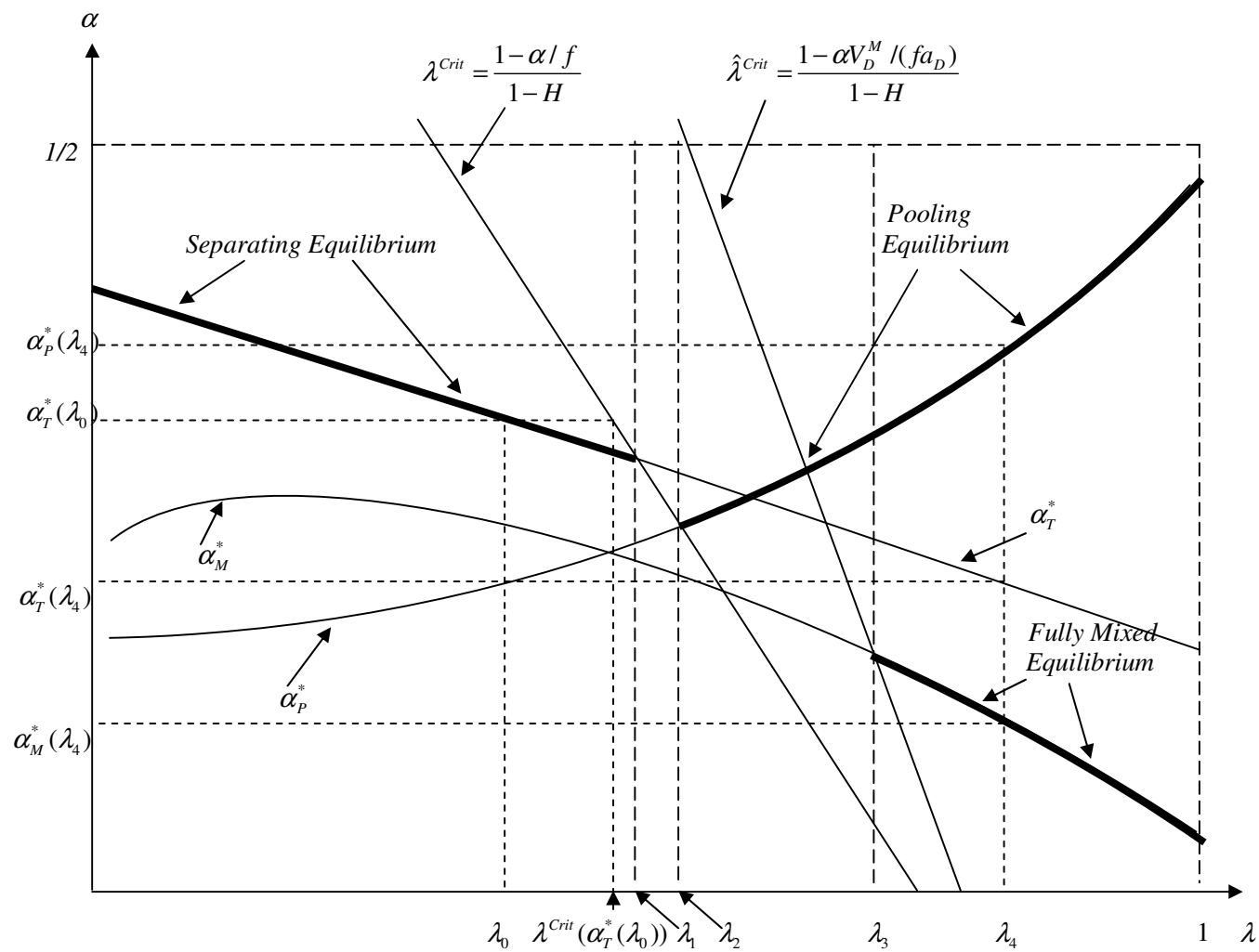


Figure 2: Equilibrium in EBC-Fraud Penalty Space

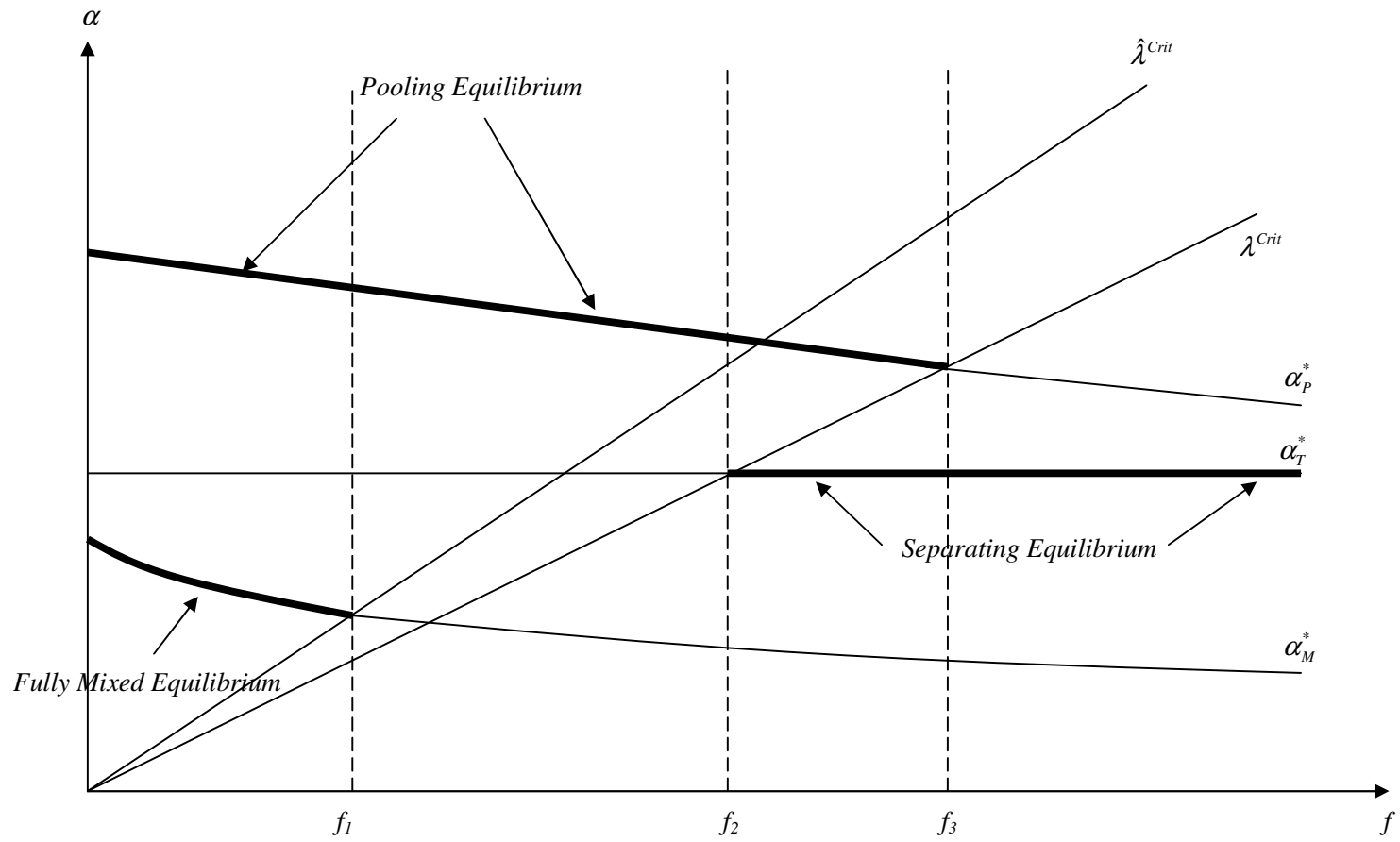
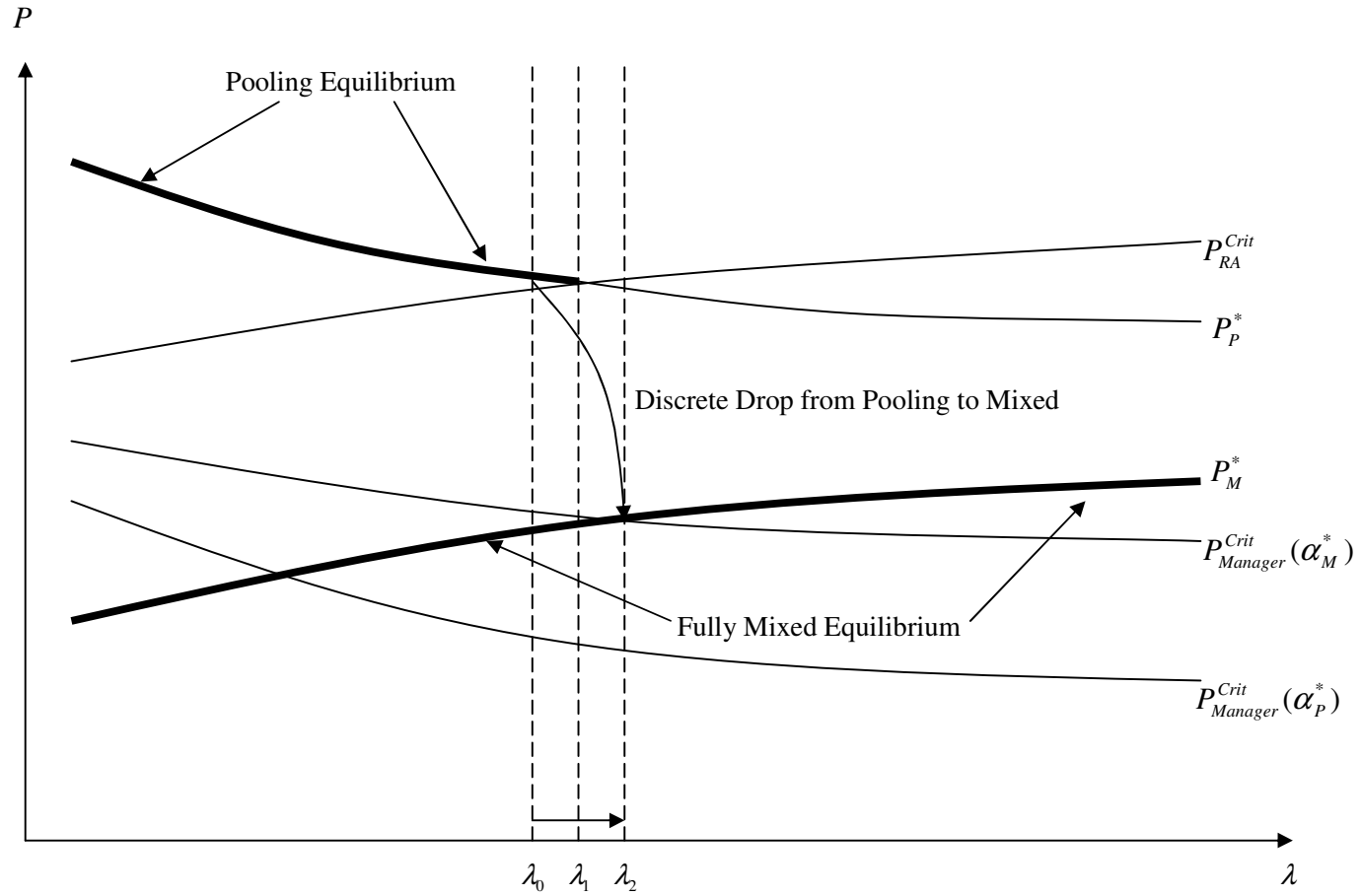


Figure 3: Discrete Drop from Pooling to Mixed Given an Increase in Growth Potential



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Appendix

Proof of Proposition 1, Part a: As is discussed in Section 3.1, truthful managers will only invest in positive NPV projects. As a result, in a truthful revelation separating PBE to the reporting/auditing sub-game (in which all managers are truthful at $t = 2$), market investors will rationally value firms reporting $r = a_H$ as $V_H = a_H + G$ and firms reporting $r = a_L$ as $V_L = a_L + G$. Furthermore, since there is no auditing of claims made at $t = 2$ in the proposed separating equilibrium, a fraudulent manager only faces the penalty fa_D if he is caught at $t = 4$, which occurs with probability $(1 - \lambda) + \lambda H$. Thus, a separating PBE for the reporting sub-game exists if every a_L -type manager prefers the expected utility under truthful reporting $r = a_L$ to fraudulent reporting $r = a_H$: $\alpha a_D \leq [(1 - \lambda) + \lambda H]fa_D$, where the left-hand side is the incremental benefit (in terms of extra compensation) from misreporting and the right hand side is the expected penalty associated with fraud. (Obviously, the a_H -type manager never has an incentive to misreport when α is non-negative.) This condition can be rearranged as follows:

$$\lambda \leq \frac{1 - \alpha/f}{1 - H}. \quad (\text{A1})$$

If condition (A1) holds, then the a_L -type manager has no incentive to mimic the a_H type, and the conjectured market expectations (conditional on the reports) are rational and the conjectured auditing strategy of the RA is also optimal (there is no need to audit if all agents truthfully report even when no auditing occurs), thus the separating PBE in which both types truthfully disclose earnings and the RA never audits at $t = 2$ exists. Note that this equilibrium is also unique. When

inequality (A1) is strictly satisfied, the a_L -type manager will find reporting a_H unprofitable, even when market investors believe that the firm is a real a_H type and his fraudulent reporting will never be detected by the RA at $t = 2$. When (A1) holds only weakly and the a_L -type manager is indifferent between reporting a_L or a_H , if any a_L -type managers commit fraud in (a different candidate) equilibrium, the benefit of committing fraud will fall (as the market rationally discounts firms with a_H -reported returns), then the a_L type managers will strictly prefer truthfully disclosing their returns. Thus, only the pure strategy separating equilibrium exists when condition (A1) holds. **Q.E.D.**

Proof of Proposition 1, Part b.i: In the pooling PBE to the reporting/auditing sub-game with both types reporting returns as a_H and RA never detecting any firm at $t = 2$, market investors will rationally value a firm reporting a_H as

$$V_{pooling} = P^*(a_H + G) + (1 - P^*)(a_L + B),$$

where P^* denotes the probability of realizing the high return (anticipated by investors given the expected optimal effort level induced in the equilibrium) in the pooling equilibrium. When all firms claim returns as a_H , the payoff to RA if it chooses to investigate a firm at $t = 2$ will be $(1 - P^*)J - C_I$, where the first term is the expected overinvestment loss prevented by the investigation, and the second term is the investigation cost. If RA chooses not to investigate any firm at $t = 2$, its payoff will be $-[(1 - P^*)(1 - \lambda + \lambda H) + P^* \lambda L]C_I$. Thus, for such a pooling equilibrium to exist, we must have

$$(1 - P^*)J - C_I \leq -[(1 - P^*)(1 - \lambda + \lambda H) + P^* \lambda L]C_I,$$

which implies

$$P^* \geq \frac{J - C_I \lambda (1 - H)}{J - C_I \lambda (1 - H) + C_I (1 - \lambda L)} \equiv P_{RA}^{Crit} < 1. \quad (A2)$$

Otherwise, RA would optimally choose to investigate any firm reporting return as a_H at $t = 2$ (thus the a_L -type manager would never want to pool with the a_H type). For an a_L -type manager to be willing to fraudulently report return as a_H , his payoff from doing so must be no less than that from truthfully disclose a_L . Thus, we must have $\alpha V_L \leq \alpha V_{pooling} - (1 - \lambda + \lambda H) f a_D$, which is $\alpha(a_L + G) \leq \alpha[P^*(a_H + G) + (1 - P^*)(a_L + B)] - (1 - \lambda + \lambda H) f a_D$. This inequality can be simplified to $(1 - \lambda + \lambda H) f a_D / \alpha + J \leq P^*(a_D + J)$. We then have

$$P^* \geq \frac{(1 - \lambda + \lambda H) f a_D / \alpha + J}{(a_D + J)} \equiv P_{Manager}^{Crit} \quad (A3)$$

Therefore, given $\lambda > \frac{1 - \alpha/f}{1 - H}$ thus $\alpha > f(1 - \lambda + \lambda H)$, the necessary condition for the pooling PBE to exist is

$$P^* \geq \text{Max}[P_{RA}^{Crit}, P_{Manager}^{Crit}] \quad (A4)$$

This proves the ‘only if’ part. Now let us prove the ‘if’ part and the claim that the pooling outcome is the only possible outcome simultaneously. When inequality (A4) is strict, the RA will never investigate any firm in the industry at $t = 2$, and the a_L -type manager will strictly prefer reporting return as a_H (i.e., no a_L -type manager would want to mix), therefore the pooling PBE exists and is the only PBE to the reporting/auditing sub-game. Now let us consider the case where (A4) is weakly satisfied, that is $P^* = \text{Max}[P_{RA}^{Crit}, P_{Manager}^{Crit}]$. If $P^* = P_{Manager}^{Crit} > P_{RA}^{Crit}$, then again RA will never audit at $t = 2$. Suppose in equilibrium some a_L type chooses to truthfully disclose

earning. Market value of firms reporting a_H will be higher, then the a_L type will strictly prefer fraudulently reporting a_H . Again, the pooling PBE is the only PBE in this case. Finally, if $P^* = P_{RA}^{Crit} \geq P_{Manager}^{Crit}$, for the same reason the a_L type cannot adopt mixed strategy – when the a_L type mixes, the RA will never investigate at $t = 2$ and market value of firms reporting a_H will be higher, thus all a_L -type firms will strictly prefer reporting returns as a_H . So the pooling PBE (in which RA never investigates fraud at $t = 2$ and all a_L -type firms pool with the a_H type) again exists and the pooling outcome is the only possible outcome. **Q.E.D.**

Proof of Proposition 1, Part b.ii: Given the conditions Part b.ii, we know from prior proofs of Part a and Part b.i that the only possible PBE to the reporting/auditing sub-game should be a mixed strategy PBE in which the a_L type mixes between reporting a_L and a_H . If the RA believes that an a_L -type manager fraudulently reports a_H with probability m , then the RA will be indifferent between investigating a claimed a_H -type firm and not investigating such a firm if

$$\frac{m(1-P^*)}{P^* + m(1-P^*)} J - C_I = -\left[\frac{m(1-P^*)}{P^* + m(1-P^*)} (1 - \lambda + \lambda H) + \frac{P^*}{P^* + m(1-P^*)} \lambda L \right] C_I,$$

where P^* denotes the probability of realizing the high return (anticipated by investors given the expected optimal effort level induced in the equilibrium) in the mixed strategy equilibrium. The left-hand-side is the payoff to the RA if it chooses to investigate any claimed a_H -type firm, and the right-hand-side is the (future) payoff to the RA if it chooses not to investigate such a firm. Rearrange the equation, we get

$$m = \frac{C_I P^* (1 - \lambda L)}{(1 - P^*) [J - C_I \lambda (1 - H)]}. \quad (A5)$$

Probability m equals 1 when $P^* = P_{RA}^{Crit}$. Since we have $0 < P^* < P_{RA}^{Crit}$, we should also have

$0 < m < 1$. For such a mixed strategy equilibrium to exist, the a_L -type manager has to feel indifferent between reporting a_L and a_H . If the RA investigates a claimed a_H -type firm with probability n , and market investors evaluate a claimed a_L -type firm as V_L^M and a claimed a_H -type firm as V_H^M in at $t = 2$, then we should have

$$\alpha V_L^M = n(\alpha V_H^M - fa_D) + (1-n)[\alpha V_H^M - (1-\lambda + \lambda H)fa_D].$$

Rearrange the above equation, we get

$$n = 1 - \frac{1}{\lambda(1-H)} \left(1 - \frac{\alpha V_D^M}{fa_D}\right), \text{ with } V_D^M \equiv V_H^M - V_L^M. \quad (\text{A6})$$

A claimed a_H -type firm should be valued by the market as²⁸

$$V_H^M = \frac{P^*}{P^* + m(1-P^*)} (a_H + G) + \frac{m(1-P^*)}{P^* + m(1-P^*)} (a_L + B).$$

From equation (A5), we have

$$\frac{m(1-P^*)}{P^* + m(1-P^*)} = \frac{C_I(1-\lambda L)}{J - C_I\lambda(1-H) + C_I(1-\lambda L)}, \text{ and}$$

$$\frac{P^*}{P^* + m(1-P^*)} = \frac{J - C_I\lambda(1-H)}{J - C_I\lambda(1-H) + C_I(1-\lambda L)}.$$

Therefore, we get

$$V_H^M = \frac{J - C_I\lambda(1-H)}{J - C_I\lambda(1-H) + C_I(1-\lambda L)} (a_H + G) + \frac{C_I(1-\lambda L)}{J - C_I\lambda(1-H) + C_I(1-\lambda L)} (a_L + B).$$

²⁸ In the current version of the model, when calculating market value of the claimed a_H -type firm at $t = 2$, we do not take into consideration the fact that market investors can rationally infer the probability n that the RA adopts in investigating fraud (i.e., market investors can rationally expect the RA to correct the overinvestment problem of a portion n of the fraudulent reporting firms). In another version of the model, we take this rational expectation of market investors into consideration – all main results remain the same (if not stronger) as those in the current version, but the mathematical formulation is substantially more complicated. The reason why our results will not be changed, however, is quite straightforward. If we take this rational expectation of market investors into account, V_H^M thus V_D^M will be greater, and the changes in V_D^M and n will always reinforce each other. Thus, the effects in our results will actually be stronger. We adopt the current version for the ease of mathematical presentation.

A claimed a_L type firm should be valued as $V_L^M = a_L + G$. So we have

$$V_D^M \equiv V_H^M - V_L^M = \frac{J - C_I \lambda(1-H)}{J - C_I \lambda(1-H) + C_I(1-\lambda L)} a_D - \frac{C_I(1-\lambda L)}{J - C_I \lambda(1-H) + C_I(1-\lambda L)} J.$$

Rearrange the above equation, we get

$$V_D^M = a_D - \frac{C_I(1-\lambda L)(a_D + J)}{J - C_I \lambda(1-H) + C_I(1-\lambda L)}. \quad (\text{A7})$$

From equation (A7) we know that $V_D^M < a_D$, thus equation (A6) gives us $n < 1$. To complete the proof, we still need to make sure that $n \geq 0$, which is equivalent to require that

$$V_D^M \geq f a_D (1 - \lambda + \lambda H) / \alpha. \quad (\text{A8})$$

Substitute the expression of V_D^M into inequality (A8), we get

$$\frac{C_I(1-\lambda L)(a_D + J)}{J - C_I \lambda(1-H) + C_I(1-\lambda L)} \leq a_D [1 - f(1 - \lambda + \lambda H) / \alpha]. \quad (\text{A9})$$

But from the conditions of the proposition we already have

$$\frac{(1 - \lambda + \lambda H) f a_D / \alpha + J}{a_D + J} \leq \frac{J - C_I \lambda(1-H)}{J - C_I \lambda(1-H) + C_I(1-\lambda L)}.$$

Therefore,

$$\frac{a_D - (1 - \lambda + \lambda H) f a_D / \alpha}{a_D + J} \geq \frac{C_I(1-\lambda L)}{J - C_I \lambda(1-H) + C_I(1-\lambda L)}.$$

Rearrange the above inequality and we can get inequality (A9). Therefore, we are sure that $n \geq 0$.

Up to this point, we complete the proof of the existence of the mixed strategy PBE to the reporting/auditing sub-game. This proof, combined with prior proofs of Part a and Part b.i, also show that the mixed strategy PBE is the only possible PBE given the conditions of Part b.ii.

Q.E.D.

Proof of Proposition 1, Part b.iii: Given the conditions of Part b.iii, we first prove the existence

of such a mixed strategy equilibrium. If the a_L -type fraudulently reports a_H with probability m and the RA never detects any claimed a_H -type firm, we must have

$$\alpha V_L = \alpha V_H - (1 - \lambda + \lambda H) f a_D,$$

Where

$$V_L = a_L + G, \text{ and}$$

$$V_H = \frac{m(1 - P^*)}{P^* + m(1 - P^*)} (a_L + B) + \frac{P^*}{P^* + m(1 - P^*)} (a_H + G).$$

We then have

$$(1 - \lambda + \lambda H) f a_D / \alpha = V_H - V_L = \frac{P^*}{P^* + m(1 - P^*)} a_D - \frac{m(1 - P^*)}{P^* + m(1 - P^*)} J$$

Simplify the above equation, we get

$$P^* [a_D - (1 - \lambda + \lambda H) f a_D / \alpha] = m(1 - P^*) [J + (1 - \lambda + \lambda H) f a_D / \alpha], \text{ and}$$

$$m = \frac{P^* a_D [1 - (1 - \lambda + \lambda H) f / \alpha]}{(1 - P^*) [J + (1 - \lambda + \lambda H) f a_D / \alpha]}.$$

The RA will never investigate any claimed a_H -type firm if

$$\frac{m(1 - P^*)}{P^* + m(1 - P^*)} J - C_I < -\left[\frac{m(1 - P^*)}{P^* + m(1 - P^*)} (1 - \lambda + \lambda H) + \frac{P^*}{P^* + m(1 - P^*)} \lambda L \right] C_I.$$

The above inequality is equivalent to

$$m(1 - P^*) [J - C_I \lambda (1 - H)] < C_I P^* (1 - \lambda L).$$

Thus, for such a mixed strategy equilibrium to exist, we must have

$$m = \frac{P^* a_D [1 - (1 - \lambda + \lambda H) f / \alpha]}{(1 - P^*) [J + (1 - \lambda + \lambda H) f a_D / \alpha]} < \frac{P^* C_I (1 - \lambda L)}{(1 - P^*) [J - C_I \lambda (1 - H)]}.$$

However, from the condition of Part b.iii we have

$$\frac{(1-\lambda+\lambda H)fa_D/\alpha+J}{a_D+J} > \frac{J-C_I\lambda(1-H)}{J-C_I\lambda(1-H)+C_I(1-\lambda L)}, \text{ thus}$$

$$\frac{a_D-(1-\lambda+\lambda H)fa_D/\alpha}{a_D+J} < \frac{C_I(1-\lambda L)}{J-C_I\lambda(1-H)+C_I(1-\lambda L)}, \text{ which is equivalent to}$$

$$\frac{a_D+J}{a_D-(1-\lambda+\lambda H)fa_D/\alpha} > 1 + \frac{J-C_I\lambda(1-H)}{C_I(1-\lambda L)}.$$

Thus,

$$\frac{a_D+J}{a_D-(1-\lambda+\lambda H)fa_D/\alpha} - 1 = \frac{J+(1-\lambda+\lambda H)fa_D/\alpha}{a_D-(1-\lambda+\lambda H)fa_D/\alpha} > \frac{J-C_I\lambda(1-H)}{C_I(1-\lambda L)}.$$

Then we have

$$\frac{a_D[1-(1-\lambda+\lambda H)f/\alpha]}{J+(1-\lambda+\lambda H)fa_D/\alpha} < \frac{C_I(1-\lambda L)}{J-C_I\lambda(1-H)}.$$

Therefore, inequality $m < \frac{P^*C_I(1-\lambda L)}{(1-P^*)[J-C_I\lambda(1-H)]}$ is satisfied.

To complete the proof of the existence of such a mixed strategy equilibrium, we still need to show

that $0 < m < 1$. $m > 0$ is obvious given $\lambda > \frac{1-\alpha/f}{1-H}$ (thus $f(1-\lambda+\lambda H)/\alpha < 1$). We still need

to show that $m = \frac{P^*a_D[1-(1-\lambda+\lambda H)f/\alpha]}{(1-P^*)[J+(1-\lambda+\lambda H)fa_D/\alpha]} < 1$, which is equivalent to require that

$$P^* \left\{ 1 + \frac{a_D[1-(1-\lambda+\lambda H)f/\alpha]}{J+(1-\lambda+\lambda H)fa_D/\alpha} \right\} = P^* \frac{a_D+J}{J+(1-\lambda+\lambda H)fa_D/\alpha} < 1. \text{ Thus we need to have}$$

$$P^* < \frac{(1-\lambda+\lambda H)fa_D/\alpha+J}{a_D+J}, \text{ which is true given the conditions of Part b.iii. We complete the}$$

proof of the existence of the specified mixed strategy PBE. Given the conditions of the Part b.iii,

we know (from Part a, Part b.i and Part b.ii) that none of the truthful disclosure separating

equilibrium, the pooling equilibrium or the mixed strategy equilibrium with both the a_L type and the RA adopt mixed strategies can exist, thus the specified mixed strategy equilibrium is the only PBE to the reporting/auditing sub-game in this case. **Q.E.D.**

Corollary 2. *Given values for α and P^* (and the exogenous parameters), in the mixed strategy case characterized in b.ii of Proposition 1, the following (partial) comparative statics results with respect to growth potential and marginal fraud penalty hold:*

$$\frac{\partial m}{\partial \lambda} < 0, \quad \frac{\partial n}{\partial \lambda} > 0, \quad \text{and} \quad \frac{\partial(mn)}{\partial \lambda} > 0 \quad \text{if} \quad \frac{f}{\alpha} \geq 2;$$

$$\frac{\partial m}{\partial f} = 0, \quad \frac{\partial n}{\partial f} < 0, \quad \text{and} \quad \frac{\partial(mn)}{\partial f} < 0.$$

Proof. $\frac{\partial m}{\partial \lambda} < 0$ is straightforward from equation (A5). To prove $\frac{\partial n}{\partial \lambda} > 0$, we need to prove

$\frac{\partial V_D^M}{\partial \lambda} > 0$ first. It is not difficult to show that

$$\frac{\partial V_D^M}{\partial \lambda} = \frac{J[a_D - C_I(1 - \lambda L)]/\lambda - C_I(1 - H)a_D + LJ[J - C_I\lambda(1 - H) + C_I(1 - \lambda L)]}{[J - C_I\lambda(1 - H) + C_I(1 - \lambda L)]^2}.$$

From equation (A9), we have

$$\frac{C_I(1 - \lambda L)(a_D + J)}{J - C_I\lambda(1 - H) + C_I(1 - \lambda L)} \leq a_D[1 - f(1 - \lambda + \lambda H)/\alpha] < a_D.$$

The above inequality gives us

$$J[a_D - C_I(1 - \lambda L)] > C_I\lambda(1 - H)a_D, \quad \text{thus}$$

$$J[a_D - C_I(1 - \lambda L)]/\lambda > C_I(1 - H)a_D.$$

Therefore, we have $\frac{\partial V_D^M}{\partial \lambda} > 0$. Since

$$\frac{\partial n}{\partial \lambda} = \frac{1}{\lambda^2(1-H)} + \frac{\alpha}{fa_D(1-H)} \frac{\partial(V_D^M / \lambda)}{\partial \lambda} = \frac{1}{\lambda^2(1-H)} \left(1 - \frac{\alpha V_D^M}{fa_D}\right) + \frac{\alpha}{fa_D \lambda(1-H)} \frac{\partial V_D^M}{\partial \lambda},$$

$\frac{\partial n}{\partial \lambda} > 0$ is obvious due to the fact that $(1 - \frac{\alpha V_D^M}{fa_D}) > 0$ and $\frac{\partial V_D^M}{\partial \lambda} > 0$.

Now let us prove that $\frac{\partial(mn)}{\partial \lambda} > 0$ given $\frac{f}{\alpha} \geq 2$.

$$mn = \frac{C_I P^* (1 - \lambda L)}{(1 - P^*) [J - C_I \lambda (1 - H)]} - \frac{C_I P^* (1 - \lambda L)}{(1 - P^*) [J - C_I \lambda (1 - H)] \lambda (1 - H)} \left(1 - \frac{\alpha V_D^M}{fa_D}\right).$$

It can be shown that

$$\frac{\partial(mn)}{\partial \lambda} = \frac{C_I P^* [(2 - \lambda L)(1 - \frac{\alpha V_D^M}{fa_D}) - \lambda(1 - H)]}{(1 - P^*) [J - C_I \lambda (1 - H)] \lambda^2 (1 - H)} + \frac{C_I P^* (1 - \lambda L) \alpha}{(1 - P^*) [J - C_I \lambda (1 - H)] \lambda (1 - H) fa_D} \frac{\partial V_D^M}{\partial \lambda}. \text{ The}$$

refore, a sufficient condition for $\frac{\partial(mn)}{\partial \lambda} > 0$ is

$$(2 - \lambda L)(1 - \frac{\alpha V_D^M}{fa_D}) > \lambda(1 - H). \quad (\text{A10})$$

Given $\frac{f}{\alpha} \geq 2$, we have $2(1 - \frac{\alpha V_D^M}{fa_D}) > 2(1 - \frac{\alpha}{f}) \geq 1 > \lambda\{1 - [H - L(1 - \frac{\alpha V_D^M}{fa_D})]\}$, thus inequality

(A10) is satisfied. So we have $\frac{\partial(mn)}{\partial \lambda} > 0$ given $\frac{f}{\alpha} \geq 2$.

$\frac{\partial m}{\partial f} = 0$ and $\frac{\partial n}{\partial f} < 0$ are self-evident from equations (A5) and (A6) respectively. Therefore, we

should have $\frac{\partial(mn)}{\partial f} < 0$. **Q.E.D.**

Proof of Proposition 2: In our risk-neutral principal-agent setting, giving a positive fixed wage to the agent (i.e., the manager) can be shown to be suboptimal for the principal (i.e., the entrepreneur), thus we should have $\bar{w}^* = 0$ in any equilibrium. The entrepreneur will then solve the following

simplified optimal contracting problem if she anticipates the separating PBE in the subsequent sub-games.

$$(1) \quad \underset{0 \leq \alpha \leq f(1-\lambda+\lambda H)}{\text{Max}} \quad (1-\alpha)\{P(e_T^*)[a_H + G] + (1-P(e_T^*))[a_L + G]\}$$

$$\text{s.t.} \quad \alpha\{P(e_T^*)[a_H + G] + (1-P(e_T^*))[a_L + G]\} - \frac{1}{2}\delta e_T^{*2} \geq 0$$

$$e_T^* = \arg \max_e \alpha\{P(e)[a_H + G] + (1-P(e))[a_L + G]\} - \frac{1}{2}\delta e^2$$

Solving the optimal effort choice problem of the manager (i.e., the second constraint) given constant the compensation contract α , we have $e_T^* = \alpha \frac{\varphi a_D}{\delta}$. Further notice that the first-order derivative of the left-hand-side of the first constraint (i.e., the manager's individual rationality constraint) with regard to α is always positive. Therefore, the first constraint will be binding if

$$\alpha = 0 \quad \text{thus} \quad e_T^* = \alpha \frac{\varphi a_D}{\delta} = 0. \quad \text{Substitute} \quad e_T^* = \alpha \frac{\varphi a_D}{\delta} \quad \text{into the objective function of the}$$

entrepreneur, then the first-order condition being zero will give us $\alpha_T^* = \frac{1}{2}\left[1 - \frac{\delta(a_L + G)}{(a_D\varphi)^2}\right]$. It is

easy to see that there always exists some set of parameter values (e.g., the marginal productivity of effort, φ , being sufficiently large and/or managerial effort disutility, δ , being sufficiently small)

such that $\alpha_T^* > 0$, i.e., it is worthwhile for the entrepreneur to hire the manager and give him some portion of the firm's shares. Since $\alpha_T^* < 1/2$, given that λ is sufficiently small,

$$\alpha_T^* \leq f(1-\lambda+\lambda H) \text{ can always be satisfied.}$$

The entrepreneur will solve the following simplified optimal contracting problem if she anticipates the pooling PBE in the subsequent sub-games.

$$(2) \quad \underset{\alpha > f(1-\lambda+\lambda H)}{\text{Max}} \quad (1-\alpha)[P(e_p^*)(a_H + G) + (1-P(e_p^*))(a_L + B)]$$

$$\text{s.t. } \alpha V_{pooling} - (1 - P(e_p^*)) (1 - \lambda + \lambda H) f a_D - \frac{1}{2} \delta e_p^{*2} \geq 0$$

$$e_p^* = \arg \max_e \alpha V_{pooling} - (1 - P(e)) (1 - \lambda + \lambda H) f a_D - \frac{1}{2} \delta e^2$$

Given constant the compensation contract α and solving the optimal effort choice problem of the manager, we have

$$e_p^* = \alpha \frac{\varphi(a_D + J)}{\delta} + \frac{\varphi(1 - \lambda + \lambda H) f a_D}{\delta}.$$

Substitute the expression of e_p^* into the objective function of the principal, then the first-order condition being zero will give us

$$\alpha_p^* = \frac{1}{2} \left[1 - \frac{(1 - \lambda + \lambda H) f a_D}{a_D + J} - \frac{\delta(a_L + B)}{\varphi^2(a_D + J)^2} \right].$$

There always exists some set of parameter values (e.g., φ being sufficiently large and/or δ being sufficiently small, and λ being sufficiently high) such that $\alpha_p^* > f(1 - \lambda + \lambda H) > 0$ (recall that $H \rightarrow 0$). Notice that the manager's individual rationality constraint will also be satisfied (the first-order derivative of the left-hand-side of the constraint with regard to α is always positive; given that λ is sufficiently high, the individual rationality constraint will bind when α is close to zero).

The entrepreneur will solve the following simplified optimal contracting problem if she anticipates the first mixed strategy PBE (in which the RA adopts mixed strategy at $t = 2$) in the subsequent sub-games.

$$(3) \quad \underset{\alpha \geq f(1 - \lambda + \lambda H) a_D / V_D^M}{\text{Max}} \quad (1 - \alpha) \{ [P(e_M^*) + (1 - P(e_M^*)) m] V_H^M + (1 - P(e_M^*)) (1 - m) V_L^M \}$$

$$\text{s.t. } \alpha [P(e_M^*) V_H^M + (1 - P(e_M^*)) V_L^M] - \frac{1}{2} \delta e_M^{*2} \geq 0$$

$$e_M^* = \arg \max_e \alpha [P(e)V_H^M + (1-P(e))V_L^M] - \frac{1}{2} \delta e^2$$

Solving the optimal effort choice problem of the manager given α , we get $e_M^* = \alpha \frac{\phi V_D^M}{\delta}$.

Substitute the expressions of e_M^* and m (from Part b.ii of Proposition 1) into the objective function of the principal, and the first-order condition being zero will give us

$$\alpha_M^* = \frac{1}{2} \left[1 - \frac{\delta(a_L + G)}{\phi^2 \phi V_D^M} \right], \quad \text{where} \quad \phi \equiv V_D^M \left[1 + \frac{C_I(1-\lambda L)}{J - C_I \lambda(1-H)} \right] = a_D - \frac{C_I(1-\lambda L)J}{J - C_I \lambda(1-H)} > V_D^M.$$

Again, there always exists some set of parameter values (e.g., ϕ being sufficiently large and/or δ being sufficiently small) such that $\alpha_M^* > 0$. For the first mixed strategy PBE to occur, we need $\alpha_M^* \geq f(1-\lambda + \lambda H)a_D / V_D^M$, which will be satisfied given that λ is high enough. The manager's individual rationality constraint will also be satisfied (since the first-order derivative of the left-hand-side of the constraint with regard to α is always positive, the individual rationality constraint will bind when $\alpha = 0$).

The entrepreneur will solve the following simplified optimal contracting problem if she anticipates the second mixed strategy PBE (in which the RA never audits at $t = 2$) in the subsequent sub-games.

$$(4) \quad \text{Max}_{f(1-\lambda+\lambda H)a_D < \alpha < f(1-\lambda+\lambda H)a_D / V_D^M} (1-\alpha) \{ [P(e_N^*) + (1-P(e_N^*))m]V_H + (1-P(e_N^*))(1-m)V_L \}$$

$$\text{s.t.} \quad \alpha [P(e_N^*)V_H + (1-P(e_N^*))V_L] - \frac{1}{2} \delta e_N^{*2} \geq 0$$

$$e_N^* = \arg \max_e \alpha [P(e)V_H + (1-P(e))V_L] - \frac{1}{2} \delta e^2$$

Solving the optimal effort choice problem of the manager given α and utilizing the fact that $V_H - V_L = (1-\lambda + \lambda H)fa_D / \alpha$ (from the proof of Part b.iii of Proposition 1), we get

$e_N^* = \frac{\varphi(1-\lambda+\lambda H)fa_D}{\delta}$, which is independent of α . Thus, $P(e_N^*)$ is also independent of α .

Substitute the expressions of m (from Part b.iii of Proposition 1) into the objective function of the principal, and it can be shown that the first-order derivative of the objective function with regards to α is always negative. Thus the principal will want to minimize α in this case (this is intuitive given that the managerial effort level is independent of α). It can also be verified that the left-hand-side of the individual rationality constraint of the manager can be simplified to

$\alpha V_L + \frac{[\varphi(1-\lambda+\lambda H)fa_D]^2}{2\delta}$, which is always greater than zero. Therefore, the entrepreneur will

pick α to be as close to $f(1-\lambda+\lambda H)$ as possible as, but can never set $\alpha_N^* = f(1-\lambda+\lambda H)$ (recall that we must have $\alpha_N^* > f(1-\lambda+\lambda H)$, otherwise, the separating equilibrium will occur in the subsequent sub-games). The optimal α_N^* hence does not exist. **Q.E.D.**

Proof of Proposition 3: For a given set of exogenous parameters Ω , let

$\lambda_T^{Crit}(\Omega) = \lambda^{Crit}(\alpha_T^*(\Omega), \Omega)$, where $\lambda^{Crit}(\alpha_T^*(\Omega), \Omega)$ is the function defined in equation (8)

evaluated at $\alpha = \alpha_T^*(\Omega)$, conditional on all the relevant parameters in Ω . From equation (13) it

is clear that $\alpha_T^*(\Omega) < 1/2$. From equation (8) we then have $\lambda_T^{Crit}(\Omega) > 0$, thus there will always

exist sufficiently low value of λ such that $\lambda < \lambda_T^{Crit}(\Omega)$ and the separating equilibrium occurs.

Q.E.D.

Proof of Proposition 4: There always exists some subset of Ω such that $\alpha_T^* > 0$ (e.g., given

that the coefficient of disutility of effort, δ , is small and/or marginal productivity of effort, φ , is

big), thus (since $\alpha_T^* > fH$) we will have $\lambda_T^{Crit}(\Omega) < 1$. Therefore, for sufficiently high value of

λ such that $\lambda > \lambda_T^{Crit}(\Omega)$, the separating equilibrium cannot exist, thus fraud must exist in any

alternative equilibrium. In that case, depending on different parameter values, either the pooling equilibrium or mixed strategy equilibrium obtains, or no equilibrium exists. Similarly, there always exist some subset of Ω such that $\alpha_p^* > 0$ (e.g., δ is small and/or ϕ is big and λ is high), thus (since $\alpha_p^* > fH$) we will have $\lambda_p^{Crit}(\Omega) < 1$. Therefore, for sufficiently high value of λ such that $\lambda > \lambda_p^{Crit}(\Omega)$, if $P^*(\Omega) \geq \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\Omega)]$, the pooling equilibrium obtains. Also, there always exist some subset of Ω such that $\alpha_M^* > 0$ (e.g., δ is small and/or ϕ is big), thus (given that $\alpha_M^* > (\frac{a_D}{V_D^M})fH$) we will have $\frac{1 - \alpha_M^* V_D^M / (fa_D)}{1 - H} < 1$. Thus, for sufficiently high value of λ such that $\lambda \geq \frac{1 - \alpha_M^* V_D^M / (fa_D)}{1 - H}$, if $P^*(\Omega) < \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\Omega)]$, the fully mixed equilibrium obtains.

From Proposition 2, it is easy to verify that $\alpha_T^* > \alpha_M^*$. For sufficiently high value of λ , we also have $\alpha_p^* > \alpha_T^* > \alpha_M^*$, thus,

$$\text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_M^*(\Omega), \Omega)] \geq \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_p^*(\Omega), \Omega)].$$

It is also easy to verify that

$$P_p^*(\Omega) = \frac{\phi^2}{2\delta} [a_D + J + (1 - \lambda + \lambda H)fa_D] - \frac{a_L + B}{2(a_D + J)} > P_M^*(\Omega) = \frac{\phi^2}{2\delta} V_D^M - \frac{a_L + G}{2\phi}.$$

Therefore, for some subset of Ω such that $\lambda_p^{Crit}(\Omega) < 1$ and $\frac{1 - \alpha_M^* V_D^M / (fa_D)}{1 - H} < 1$ (i.e., $\alpha_p^* > 0$ and $\alpha_M^* > 0$; recall that $H \rightarrow 0$), for sufficiently high value of λ such that

$$\lambda \geq \frac{1 - \alpha_M^* V_D^M / (fa_D)}{1 - H} > \frac{1 - \alpha_p^* / f}{1 - H},$$

we should have either

$$\text{i) } P_p^*(\Omega) > P_M^*(\Omega) > \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_M^*(\Omega), \Omega)] \geq \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_p^*(\Omega), \Omega)],$$

or

$$\text{ii) } \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_M^*(\Omega), \Omega)] \geq \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_P^*(\Omega), \Omega)] > P_P^*(\Omega) > P_M^*(\Omega),$$

or

$$\text{iii) } P_P^*(\Omega) > \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_M^*(\Omega), \Omega)] \geq \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_P^*(\Omega), \Omega)] > P_M^*(\Omega),$$

or

$$\text{iv) } \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_M^*(\Omega), \Omega)] > P_P^*(\Omega) > \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_P^*(\Omega), \Omega)] > P_M^*(\Omega),$$

or

$$\text{v) } P_P^*(\Omega) > \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_M^*(\Omega), \Omega)] > P_M^*(\Omega) > \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_P^*(\Omega), \Omega)],$$

or

$$\text{vi) } \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_M^*(\Omega), \Omega)] > P_P^*(\Omega) > P_M^*(\Omega) > \text{Max}[P_{RA}^{Crit}(\Omega), P_{Manager}^{Crit}(\alpha_P^*(\Omega), \Omega)].$$

It is clear that pooling is the only possible equilibrium under i) and fully mixed is the only possible equilibrium under ii). Under iii) to vi), both pooling and fully mixed are possible. Which one obtains then depends on the entrepreneurial utility (since the entrepreneur is the Stackelberg leader in the overall game). Thus, when λ is sufficiently high, either the pooling equilibrium or the mixed strategy equilibrium obtains. **Q.E.D.**