Competition, Human Capital and Innovation Incentives

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Abstract

This paper sheds light on the changing nature of the firm and offers an explanation for why innovative, human capital intensive firms tend to operate in competitive environments, as discussed in Zingales (*JF*, 2000). We develop a model in which employees' incentives to acquire human capital, a necessary input for innovation, depend on the number of firms competing for employee human capital. A key insight of our paper is that firms operating in the same competitive industry have a higher degree of relatedness, since they invest in similar and more compatible technologies. This implies that employees in more competitive industries develop human capital that can be transferred more easily from one firm to another and, thus, can extract more rents (i.e., greater wages) from their firms. Anticipating their higher rent extraction ability, employees invest more in human capital and innovate more. Thus, competition in the market for innovation leads to competition for employee human capital and promotes innovation. We show that, under certain conditions, firms prefer to operate in a competitive product market in order to become more innovative. In this way, greater competition emerges endogenously and leads to more innovation. Hence, our analysis obtains endogenously the emergence of today's human capital intensive firms which are highly innovative in spite of operating under greater competition than ever. We also show that horizontal mergers may be detrimental to innovation generation, and that firms may prefer to preserve a competitive environment in order to provide better incentives for their employees. This finding implies that, while *internal* competition within a firm may be detrimental for innovation (as in Rotemberg and Saloner, 1994), *external* competition may promote effort and foster innovation. Finally, our paper has implications for the choice of industry standards, such as *open source* technologies, and the adoption of contractual measures that hinder employee mobility, such as *no-compete* clauses.

1 Introduction

The last two decades have witnessed three major changes on the nature of firms, as discussed in Zingales (2000). The first is that ownership of physical assets no longer represents a source of comparative advantage for firms since developments in financial markets and easier access to capital markets significantly increased their ability to invest in physical assets. As a result, firms' value generating ability today depends much more on the human capital they employ rather than on the physical capital they own. Thus, recruiting, retaining and motivating employees becomes a critical step for value creation. The second change is that shortened product life cycles and increased worldwide competition have made it more vital for firms to continuosly innovate. This trend has raised even further the importance and the price of human capital, the most critical and expensive input for innovation. The third major change is that employees now have a higher ability to move from one firm to another as a result of increased competition. The presence of a greater number of firms competing in similar product markets has increased employee mobility, and hence the price of human capital. Interestingly, in spite of being exposed to greater competition than ever, today's firms are remarkably successful in generating new products and technologies at an unprecedented pace.

Our paper ties these trends together by establishing a new, positive link between competition and innovation incentives in human capital intensive firms. We argue that human capital intensive firms may benefit from competition in the product market because it creates competition for employee human capital, increasing the price of human capital and thus enhancing employees' incentives to invest in their own human capital. Hence, our model helps to explain the emergence of today's human capital intensive firms as highly innovative ventures, despite being subject to intense competition. Our paper also helps to explain why firms may benefit from employee mobility, and why human capital intensive firms may choose not to impose on their employees "no-compete" clauses that would restrict employee mobility across firms in the same industry. Finally, our paper examines the effect of merger activity on innovation incentives. We show that, in order to preserve strong innovation incentives, human capital intensive firms may prefer to forego horizontal mergers, even when these mergers increase their market power.

The intuition of our model is as follows. In our set-up, innovation arises as an outcome of costly investment in human capital by employees. Human capital originates from an investment in the knowledge and skills necessary to generate an innovation. In our model, firms can operate in a protected environment, facing no competition from other firms, or operating in a competitive environment in the presence of rival firms. A key insight of our paper is that firms competing in similar product markets have a high degree of relatedness, since they invest in similar technologies. This implies that employees in more competitive industries will develop human capital that can be transferred more easily from one firm to another. Due to their ability to transfer their human capital to competing firms, employees can extract higher rents from their employers and internalize a greater part of the return from their human capital investment. Thus, anticipating their higher rent extraction ability, employees invest more in human capital and innovate more. In this way, our paper establishes a link between product market competition and the rate of innovation through the beneficial effect of competition on the accumulation of human capital.

An interesting implication of our paper is that not only employees, but firms as well can benefit from product market competition in spite of the fact that competition allows employees to extract higher rents and reduces a firm's market share. We show that competition presents firms with the following trade-off. On the one hand, it improves employee incentives to innovate by creating an inter-firm market in which firms compete for employees. Since firms profit from the innovations generated by their employees, they will also benefit from the competitive environment that encourages innovation. On the other hand, competition hinders innovation incentives and firm profitability by reducing a firm's market share and its ability to profit from innovation. In our paper, we show that the positive effect of competition on incentives may outweigh its negative effect, and that firms may ultimately benefit from competition. We show that the benefit of competition is stronger when innovation incentives are low, which happens when either employees have low bargaining power or when the cost of exerting innovation effort is large. We also suggest that a competitive environment is more likely to emerge in sectors characterized by greater human capital intensity. Projects with greater human capital intensity involve more costly human capital investments and therefore benefit most from the additional incentives provided by a competitive environment. Thus, our model helps to explain why today's human capital intensive firms tend to operate in highly competitive environments (as discussed in Zingales, 2000).

Economists have long discussed whether innovation flourishes better in an open and competitive environment (where innovators threaten incumbents with new products), or in a protected environment (where innovators harvest the fruit from their effort while shielded from potential imitators). For example, Schumpeter himself evolved from his earlier entrepreneurial theory of innovation (see Schumpeter, 1911) to his later view that "a market structure involving large firms with a considerable degree of market power is the price that society must pay for rapid technological progress."[1] Most of the theoretical work in Industrial Organization has stressed the later Shumpeterian view that competition, by decreasing the potential rents available to innovators, is detrimental to innovation (see, for example, Salop, 1977, Spence, 1984, Aghion and Howitt, 1992). This predicted negative relationship between competition and innovation incentives, however, has been challenged by recent empirical work, such as Nickell (1996), Blundell, Griffith, and Van Reenen (1999), and Knott and Posen (2003), which document a positive relationship between product market competition and innovation.

A remarkable exception to this literature is provided by Aghion, Bloom, Blundell, Griffith, and Howitt (2002), which develops a model predicting an inverted u-shaped relationship between innovation and competition. In this model, moderate competition is beneficial for innovation because, by reducing *pre*-innovation profits, it increases the incremental benefits that an innovator gains from leapfrogging its rivals. A positive effect of competition on managerial incentives (and thus on innovation) is also predicted by some agency-based models. For example, in Hart (1983) more competition in the product market allows a firm's owners to write better incentive contracts for managers, strengthening managerial incentives and thus inducing more effort. In Schmidt (1997) and Aghion, Dewatripont and Rey (1999) the threat of bankruptcy induces managers to exert more effort as a way to avoid liquidation. More recently, in a model with endogenous market entry, Raith (2003) shows that the impact of competition on effort depends on the sources of variation in the degree of competition. The paper finds that greater competition increases effort if it is due to greater product substitutability, while it decreases effort if it is due to lower entry costs.

Our paper presents a new explanation for the relationship between innovation and competition. In our model, competition and innovation are *both* endogenous and interact in essential ways. Our analysis shows that, under certain conditions, firms prefer to compete in the product market in order to become more innovative. Thus, greater competition emerges endogenously in the industry and leads to more innovation: the desire to innovate spurs competition and competition leads to more innovation.

An additional implication of our model is that human capital-intensive firms may benefit from promoting their employees' mobility. For example, firms may benefit from operating in a legal environment that does not put any restrictions on the mobility of employees. This implication is consistent with the idea that the presence of legal restrictions to inter-firm mobility, such as the inclusion of *no-compete* clauses in

[1] Nelson and Winter (1985), page 278. See, also, the discussion in Schumpeter (1942), pages 88-90.

employment contracts, may hinder innovation. In an extension of our model, we show that in equilibrium firms choose not to impose such clauses on their employees in order to enhance employee incentives. Thus, our model supports the view discussed in Gilson (2004) suggesting that an important element accounting for the better relative performance of Silicon Valley with respect to Boston's Route 128 can be traced to the differences in their legal environments (since California does not enforce *no-compete* clauses, while Massachusetts does).

Our mode also sheds new insights on the choice of industry standards and on the location of firms. Since employee mobility among firms can be promoted by the adoption of compatible technologies, our model shows that firms may benefit from the creation of homogeneous industry standards that facilitate the transferability of employee human capital from one firm to another. In a similar way, firms may benefit from creating industry clusters or forming "industrial hubs." By locating in the same geographical area, firms increase employee mobility from one firm to another and facilitate the creation of an intra-industry market for employee human capital, with a positive spillover on employee incentives and innovation. In this way, our paper is related to a recent paper by Almazan, De Motta and Titman (2004), which studies the link between human capital investment and firm location. Since geographical proximity promotes the development of a competitive labor market, this paper shows that firms prefer to cluster when employees pay for their own training, while they locate apart when firms pay for their employees' human capital development. Our paper differs from theirs in an important way. Almazan, de Motta and Titman (2004) focuses on the interaction between location and the development of human capital, abstracting from the interaction between location and product market competition. In our paper we focus precisely on the link between product market competition and incentives to accumulate human capital. In our model competition in the product market for innovation leads to competition for employee human capital, promoting human capital formation and ultimately fostering innovation. Our model implies that firms may be willing to cluster even if this choice results in greater employee wages and competition in the product market.

In our analysis, the beneficial effect of competition on incentives is particularly appealing for firms when employees have low bargaining power in wage negotiations. When firms have a high level of bargaining power, they can appropriate a large share of the value from the innovation generated by their employees, reducing employee incentives to innovate. The creation of an inter-firm labor market through competition in the product market allows employees to retain a greater share of the value of the innovation they create, promotes innovation and ultimately benefits firms. In this way, our work is related to the earlier literature, started by Fama (1980) and Holmstrom (1982), which emphasizes the positive role of the inter-firm managerial labor market as an incentive device.

Finally, our paper examines the effects of horizontal mergers on employee incentives to develop human capital, and shows that horizontal mergers may be detrimental for innovation incentives. Horizontal mergers affect incentives in three ways. The first one is that, by consolidating competing firms, horizontal mergers reduces competition for human capital which affects their incentives adversely. The second way is that a horizontal merger introduces internal competition between the employees in the merged firm, again impacting employee incentives negatively. This reduction in innovation incentives (due to the internal competition in the merged firm) is similar to that identified in Rotemberg and Saloner (1994). The third effect of a merger on incentives is positive, since the merger eliminates competition between firms in the product market and increases its market power. Our analysis shows that, under certain conditions, firms prefer to forego the increased market power provided by a horizontal merger in order to preserve the beneficial effect of a competitive environment on their employees' incentives. This implies that, while *internal* competition may be detrimental for innovation, *external* competition may promote it. Thus, firms will benefit from belonging to a competitive industry, even if this comes at the expense of competition in the product market.

The paper is organized as follows. In section 2, we take the market structure under which the two firms operate as given and we present the basic results of the paper. In section 3, we endogenize the choice of the market structure, that is, whether the firms choose to compete by entering into the same market or to have monopoly power by entering into different markets. Section 4 analyzes firms' incentives to invest in innovation under competition and monopoly. In section 5 we examine firms' incentives to limit or enhance the ability of their employees to leverage their human capital outside the firm. Section 6 studies the impact of a horizontal merger on innovation incentives. Section 7 concludes. All proofs are in the Appendix.

2 The Basic Model

We consider an economy endowed with two firms and two employees. All agents are risk-neutral and there is no discounting. We assume that at the beginning of the game each firm is already matched with one of the two employees. The two firms are human capital intensive in the sense that they create value by implementing their employees' innovations. The innovation cycle involves two stages of a project. The first stage of the project is performed by the employee and, if successful, generates an innovation.[2] The second stage involves the development and commercialization of the innovation and is performed by the firm with the collaboration of the employee. We assume that the active participation and effort of the employee who initially generated the innovation is necessary also in the second stage for its development into a final product.[3]

The success probability in the first stage of the project depends on an unobservable effort exerted by the employee, which is denoted by e_i, $i = 1, 2$. Employee effort determines the success probability of the project: $p_i(e_i) = e_i \in [0, 1]$. Exerting effort is costly: we assume that effort costs are convex and given by $\frac{k}{2}e_i^2$. The parameter k measures the cost of exerting such effort. We interpret employee effort broadly, as representing the costly investment made by the employee to acquire the knowledge and human capital necessary for the success of the project.[4]

The human capital acquired by an employee in the first stage of the project is essential for both the generation and the development of an innovation. The key feature of our model is that employee incentives to acquire human capital depend on the market structure in which the two firms operate. If a firm operates as a monopolist, there are no rival firms in the product market and employee innovations can only be developed and commercialized within the original firm. This is because the human capital developed by the employee is firm-specific and is valuable only if deployed within the original firm. If the two firms operate in the same product market, they will have technologies that are related to each other. In this case, it will be easier for employees with an innovation to transfer (albeit imperfectly) their innovation and human capital from one firm to the other. This implies that the presence of other firms in the same product market enables employees to develop human capital that can be valued outside the original firm, allowing employee mobility. Thus, competition in the product market facilitates the transfer of employee skills and promotes the creation of an external market for employee human capital. As we

[2]Innovation can be broadly interpreted as any new idea, process or product which improves firm profitability.

[3]This assumption implies that if an employee with a successful innovation leaves his firm at the end of the first stage, the firm cannot implement the innovation without the original employee. Similarly, we assume that if the employee leaves the firm, he cannot implement the innovation by himself but he must join another firm with the resources and capabilities necessary to implement the innovation. We assume also that the employee needs the firm's resources during both stages of the production process, which implies that he can generate an innovation only if he has joined a firm at the beginning of the game. In section 4, we model firms' investment in innovation explicitly.

[4]As such, parameter k can be also interpreted as measuring the degree of human capital intensity of the project. This may be seen by noting that projects characterized by greater human capital intensity require more knowledge and effort and therefore are inherently more costly, per unit of success probability e_i, than projects with lower human capital intensity.

will discuss below, the presence of an external market in which firms compete for human capital allows employees to extract more rents (i.e., higher wages) from their firms. In this way, competition increases employee incentives to invest in human capital and promotes innovation.

We assume that employee effort is not observable, exposing firms to moral hazard. Following Stole and Zwiebel (1996a and 1996b), and in the spirit of Grossman and Hart (1986) and Hart and Moore (1990), we also assume that firms and employees cannot write binding contracts on the development of successful innovations and that they can withdraw their participation from the project before the development phase. This implies that if an employee generates an innovation the allocation of the surplus from the innovation is determined, as in Stole and Zwiebel (1996a and 1996b), by intra-firm bargaining between the firm and the employee at the interim date before the second stage of the project is performed. This also implies that contracts written ex-ante between a firm and its employee on how to share the final surplus, such as equity contracts (or options, as in Noldeke and Schmidt, 1998), are ineffective since both the firm and the employee can (unilaterally) withdraw their essential assets and participation (human capital, in the case of the employee) from the development phase of the innovation.[5]

The outcome of bargaining process between the employee and the firm (that is, the division of the project's payoff between them) depends on their relative bargaining power and on each party's outside option. We assume that each firm's outside option while bargaining with its employee is limited by the fact that the firm cannot replace its current employee with a new one from the general labor market population, but it can only hire an employee from a rival firm in the same product market. This assumption captures the notion that it is impossible for the firm to continue production by replacing the original employee with a new one from the generic (unskilled) labor market pool. This assumption is easy to justify if employees need a training in the first period to produce in the second period. The presence of an outside option for the employee depends on the employee's ability to transfer his human capital from one firm to the other. This will be possible only if the employee can move from his original firm to a rival firm in the same product market, that is, if a rival firm exists in the same product market. In this case, competition between firms in the product market, by creating competition for employee human capital, affects the allocation of the surplus between the firm and the employee. Thus, product market competition influences employee effort and the overall efficiency of the innovation generation process.

[5]For a further discussion on the role of employment *at will* and renegotiation on surplus allocation, see Stole and Zwiebel (1996a) and (1996b).

The market structure (that is, whether the firms operate under monopoly or competition) affects project payoffs as follows. If a firm is a monopolist in its own market, and the first stage of the project has been successful, then the project generates a payoff M.[6] If the two firms operate in the same product market, then the payoff from the project will depend on whether one or both firms have been successful in the first stage of their projects. If both firms have been successful (that is, if employees at both firms obtain an innovation) the two firms will compete in the commercialization of the innovation. We assume that in this case the two firms will engage in Bertrand competition, and each project's payoff will be equal to 0.[7] If, instead, one of the two firms fails, then the successful firm will be a monopolist, and the payoff from the project will be M_C. Note that the two market structures may have product markets of different size, with the competitive structure having a greater (smaller) market size than the monopolistic market structure if $M_C > (<) M$. Finally, if an employee fails to obtain an innovation, the payoff from the project is zero.

In this section we take the market structure under which the two firms operate as given, and we compare the innovation incentives under monopoly and competition.[8] The basic game unfolds as follows. At $t = 0$, after observing whether his firm is operating under monopoly or competition, each employee exerts effort, which determines the success probability of his project. Effort choices are made simultaneously by the two employees.

At $t = 1$, the outcome of the first stage of the project is known. If the first stage is successful, then each employee bargains with his firm over the division of the surplus from the commercialization of the innovation. The share of the surplus earned by the employee may be interpreted as the wage (or bonus) that the employee receives for his contribution necessary for the further development and commercialization of the innovation. When bargaining with his firm, the employee captures a share $\beta \in [0, 1]$ of the net joint surplus that depends on his bargaining power. Thus, we will refer to the parameter β as measuring the employee "bargaining power."

The payoffs from bargaining depend on the employee outside options, and on whether the two firms

[6]We assume in this section that the size of the product market, which determines the project payoff M, is given. This assumption is relaxed in Section 4, where we allow firms to make at the outset of the game a costly investment that affects the size of the product market.

[7]We make this assumption for analytical tractability. The main results of our paper can be extended to include different forms of product market competition between the two firms.

[8]This assumption is relaxed in Section 3, where we allow firms to endogenously choose the market structure in which they operate.

operate in the same product market, and therefore compete, or they operate in different product markets, and therefore are monopolists in their respective markets. If the two firms are monopolists in their own market, the employees cannot transfer their firm-specific skills to the other firm. Thus, both employees and firms have zero outside options while bargaining. If the two firms operate in the same product market, employee human capital can be redeployed at the rival firm. The possibility of redeploying human capital at the competing firm gives the employee an outside option when bargaining with his own firm. We assume that an employee can transfer his innovation to the competing firm and commercialize it with a potential payoff $\delta \leq M_C$. We interpret the parameter δ as measuring the degree of transferability of employee human capital.[9]

At $t = 3$, the payoff is realized and the cash flow is distributed.

We first consider the case in which the firms do not compete, that is, each firm operates in its own market and employee human capital is not transferable from one firm to another. We consider next the case in which the two firms compete in the same product market, allowing employees to transfer their human capital from one firm to another.

2.1 Innovation Incentives under Monopoly

If firms operate in a monopolistic environment, employees develop firm-specific human capital and innovation that they cannot transfer from one firm to another. This implies that both employees and firms have zero outside options when bargaining at the interim date over the division of the project's payoff, M.

We model the bargaining game between an employee and his firm as a standard alternating-offers game in which the two parties make alternating offers under the threat that bargaining breaks down with a certain exogenous probability (see, for example, Binmore, Rubinstein, and Wolinski, 1986). If bargaining breaks down, employees and firms obtain their outside options, which, in the case of monopoly, are equal to zero for both parties. One can show that, as the probability that the bargaining process breaks down tends to zero, the outcome of the subgame perfect equilibrium of the bargaining game is such that the employee and the firm receive a fraction β and $1-\beta$, respectively, of the surplus that they jointly generate. As a result, employees obtain a payoff equal to βM, while firms capture the remainder, which is equal to

[9]We assume initially that a firm cannot prevent its employee from joining the rival firm. We relax this assumption in Section 5, where we assume that each firm can require its employee to sign a "no-compete" agreement, effectively preventing the employee from joining the rival firm.

$(1-\beta)M$.[10]

In anticipation of his payoff from bargaining, employee i chooses his effort level e_i^M by maximizing his expected profits, $\pi_{E_i}^M$, given by

$$\max_{e_i} \ \pi_{E_i}^M \equiv \beta M e_i^M - \frac{k}{2}(e_i^M)^2; \ i = 1, 2. \tag{1}$$

Correspondingly, firm i's expected profits, $\pi_{F_i}^M$, are given by

$$\pi_F^M \equiv (1-\beta)M e_i^M; \ i = 1, 2. \tag{2}$$

The optimal level of effort under monopoly, e^{M*}, and the corresponding level of expected profits for the employee, $\pi_{E_i}^{M*}$, and the firm, $\pi_{F_i}^{M*}$, and total profits, $\pi_{T_i}^{M*}$, are characterized in the following lemma.[11]

Lemma 1 *The optimal level of effort under monopoly is*

$$e_i^{M*} = e_j^{M*} = e^{M*} \equiv \frac{\beta M}{k}. \tag{3}$$

The corresponding expected profits for employees and firms are

$$\pi_{E_i}^{M*} = \frac{\beta^2 M^2}{2k}; \qquad \pi_{F_i}^{M*} = \frac{(1-\beta)\beta M^2}{k}; \qquad \pi_{T_i}^{M*} \equiv \pi_{E_i}^{M*} + \pi_{F_i}^{M*} = \frac{(2-\beta)\beta M^2}{2k}, \ i = 1, 2. \tag{4}$$

It is immediate to see that the optimal level of effort, e^{M*}, and profit levels are increasing functions of the monopoly payoff, M, and decreasing functions of the cost of exerting effort, k. This implies that, all else equal, the probability of obtaining an innovation is increasing in the project's market size, and declining in the cost of exerting effort. The effect of employee bargaining power β, in contrast, is ambiguous, and is characterized in the following lemma.

Lemma 2 *The optimal level of effort under monopoly (3) is an increasing function of employee bargaining power, β. Furthermore, the effect of employee bargaining power, β, on the expected profits is as follows:*

$$\frac{\partial \pi_{E_i}^{M*}}{\partial \beta} > 0; \qquad \frac{\partial \pi_{F_i}^{M*}}{\partial \beta} \geq 0 \quad for \ \beta \leq \frac{1}{2}; \qquad \frac{\partial \pi_{T_i}^{M*}}{\partial \beta} \geq 0, \ i = 1, 2. \tag{5}$$

A greater level of bargaining power, β, allows the employee to extract more surplus from his firm and leads to a greater innovation probability, e^{M*}, and employee expected profits, $\pi_{E_i}^{M*}$. The effect of

[10] Note that this division of the surplus corresponds to the Nash-bargaining solution in which the employee's and the firm's bargaining powers are, respectively, β and $1-\beta$, and neither party has an outside option. See Binmore, Rubinstein, and Wolinski (1986).

[11] To ensure interior solutions we assume that $k \geq \beta M$.

employee bargaining power on firm's expected profits, $\pi_{F_i}^{M*}$, is ambiguous and depends on the trade-off of two opposing effects. On the one hand, greater employee bargaining power leads to a higher level of effort and innovation probability, which always benefits the firm under monopoly. On the other hand, a greater bargaining power allows the employee to extract more rents, which is costly for the firm. The firm benefits from higher employee bargaining power when the positive incentive effect dominates the negative rent extraction effect. This happens when the employee bargaining power, and therefore effort, is small, that is, when $\beta \leq \frac{1}{2}$. Firms' expected profits are maximized when the incentive and rent extraction effects balance each other out exactly, which in our model occurs when the firm and the employee have the same bargaining power, that is, when $\beta = \frac{1}{2}$. Finally, the total expected profits, $\pi_{T_i}^{M*}$, are always an increasing function of employee bargaining power. This is an implication of the fact that the rent extracted by the employee is just a transfer of surplus between the two bargaining parties. Hence, the only effect of an increase in employee bargaining power on total expected profits is its positive effect on employee incentives.

2.2 Innovation Incentives under Competition

If firms operate in a competitive environment employees develop human capital and innovation that can be transferred from one firm to another, which gives them an outside option while bargaining with their firms. The outcome of bargaining between firms and employees, and thus the allocation of the surplus, depends on whether only one, or both, employees generate an innovation.

If only one employee, say employee i, is successful in generating an innovation, he will bargain with his firm over the division of the payoff, M_C. Note that this case differs from the monopoly case in that now employee i has the option to transfer his innovation and human capital to the competing firm j, with a payoff of δ where $\delta \leq M_C$.[12] We model the bargaining game between employee i and firm i again as one in which the two parties make alternating offers under the threat that the bargaining process breaks down with a certain exogenous probability. If bargaining with firm i breaks down, employee i has the option to start a new round of bargaining with firm j. Thus, the payoff from bargaining with firm j represents employee i's outside option when bargaining with firm i. One can show that, as the probability that the bargaining process breaks down tends to zero, the outcome of the subgame perfect equilibrium of the

[12]Note that in equilibrium employees will never transfer their human capital and innovation to the rival firm since $\delta \leq M_C$. It is straightforward to extend our model to the case where (with some exogenous probability) employees may be more valuable when they redeploy their human capital outside their original firm. This case originates, for example, when $\delta > M_C$, and therefore employees move from one firm to another in equilibrium.

bargaining game between firm i and employee i is such that the employee and the firm receive the value of their outside options (the value that they can obtain in the case of a breakdown in bargaining), plus the fractions β and $1-\beta$, respectively, of the surplus that they jointly generate net of the sum of their outside options.[13]

We can determine the payoffs from bargaining between firm i and employee i by proceeding backwards. If bargaining between employee i and firm i breaks down, employee i has the opportunity to bargain with firm j. In this second bargaining game, both employee i and firm j have zero outside options. The employee and the firm will therefore divide the joint surplus, δ, according to their bargaining power obtaining $b\delta$ and $(1-b)\delta$ respectively, where b represents employee i's bargaining power when bargaining with firm j.[14] The payoff that employee i obtains when bargaining with firm j, which we denote as $\Delta \equiv b\delta$, with $0 \leq \Delta \leq M_C$, represents the value of employee i's outside option while bargaining with firm i. Since employee j has failed to obtain an innovation, firm i has an outside option with value zero. This implies that employee i's payoff from bargaining with firm i is equal to $\Delta + \beta(M_C - \Delta) = \beta M_C + (1-\beta)\Delta$. Correspondingly, firm i's payoff is given by $(1-\beta)(M_C - \Delta)$.

If both employees have been successful, the firms compete in the product market and both firms and employees earn zero profits.

In anticipation of his payoff from bargaining, employee i chooses his effort level, e_i^C, by maximizing his expected profits, $\pi_{E_i}^C$, given by

$$\max_{e_i} \; \pi_{E_i}^C \equiv e_i^C(1-e_j^C)(\beta M_C + (1-\beta)\Delta) - \frac{k}{2}(e_i^C)^2; \; i=1,2; \; i \neq j. \tag{6}$$

Correspondingly, firm i's expected profits, $\pi_{F_i}^C$, are given by

$$\pi_{F_i}^C \equiv e_i^C(1-e_j^C)(1-\beta)(M_C - \Delta); \; i=1,2; \; i \neq j. \tag{7}$$

The first-order condition of (6) provides employee i's optimal response, given employee j's choice of effort, and is as follows:

$$e_i^C(e_j^C) = \frac{(1-e_j^C)(\beta M_C + (1-\beta)\Delta)}{k} = \frac{\beta M_C - e_j^C \beta M_C + (1-e_j^C)(1-\beta)\Delta}{k}; \; i=1,2; \; i \neq j. \tag{8}$$

[13] Note that this division of the surplus corresponds to the Nash-bargaining solution with outside options, in which the employee's and the firm's bargaining powers are, respectively, β and $1-\beta$. See again Binmore, Rubinstein, and Wolinski (1986).

[14] Note that this specification allows employee i's bargaining power while bargaining with firm j, given by b, to differ from his bargaining power while bargaining with firm i, given by β.

From the first-order condition (8), it can immediately be seen that employee i's effort is a decreasing function of employee j's effort, which implies that effort choices by the two employees are strategic substitutes, due to competition. Furthermore, employee i's effort is an increasing function of project payoff, M_C, and of the value of his outside option, Δ, and a decreasing function of the cost of exerting effort, k.

A comparison of the first-order conditions under monopoly (3) and under competition (8) allows us to identify the impact of competition on employee incentives to exert effort. Competition affects employee incentives in three ways. The first effect, measured in the RHS of (8) by the term βM_C, is of ambiguous sign and depends on the size of the competitive market relative to the monopolistic market (that is, whether $M_C > M$ or $M_C < M$). The second effect, measured by the term $-e_j^C \beta M_C$, is negative and reflects the reduction in profits due to competition in the product market. If the rival firm successfully obtains an innovation, which occurs with probability e_j^C, competition between the two firms reduces employee payoff to zero (from βM_C) with a negative impact on employee effort. The third effect, measured by the term $(1 - e_j^C)(1 - \beta)\Delta$, is positive and originates from the beneficial impact of competition on employee incentives to acquire human capital. If firms compete in the same product market, employee i's human capital can be redeployed at the competing firm (although this will never happen in equilibrium). The presence of this outside option enables the employee to extract more rents from his firm, enhancing his incentives to exert effort. Note that this effect arises only when the employee of the rival firm has failed, which happens with probability $1 - e_j^C$.

We now characterize the Nash-equilibrium of the effort subgame.

Lemma 3 *The unique (symmetric) Nash-equilibrium of the effort subgame is given by:*

$$e_i^{C*} = e_j^{C*} = e^{C*} \equiv \frac{\beta M_C + (1-\beta)\Delta}{k + \beta M_C + (1-\beta)\Delta}. \tag{9}$$

The corresponding expected profits for firms, employees and total profits are

$$\pi_{E_i}^{C*} = \frac{k\left(\beta M_C + (1-\beta)\Delta\right)^2}{2\left(k + \beta M_C + (1-\beta)\Delta\right)^2};\ i = 1, 2, \tag{10}$$

$$\pi_{F_i}^{C*} = \frac{k(1-\beta)\left(\beta M_C + (1-\beta)\Delta\right)\left(M_C - \Delta\right)}{\left(k + \beta M_C + (1-\beta)\Delta\right)^2};\ i = 1, 2, \tag{11}$$

$$\pi_{T_i}^{C*} \equiv \pi_{E_i}^{M*} + \pi_{F_i}^{M*} = \frac{k\left(\beta M_C + (1-\beta)\Delta\right)\left(M_C + (1-\beta)(M_C - \Delta)\right)}{2\left(k + \beta M_C + (1-\beta)\Delta\right)^2};\ i = 1, 2. \tag{12}$$

We now establish some preliminary results that will be useful in our subsequent analysis. Note first that an important difference between the monopoly case and the competitive case is that a greater (equilibrium) level of effort is not always desirable for the two firms when they compete in the product market while it always benefits them under monopoly. This property reflects the fact that under competition a greater level of effort by *both* employees increases the probability that both firms successfully develop the innovation and thus compete in the product market, obtaining a zero payoff. The net effect of employee effort on firms' expected profits will then depend on the balance between the benefit of a higher probability of innovation and the reduction in profitability due to competition. This is established in the following lemma.

Lemma 4 *The expected profits of the firms as a function of the Nash-equilibrium effort level, e^{C*}, have the following property:*

$$\frac{\partial \pi_{F_i}^{C*}}{\partial e^{C*}} \geq 0 \quad \textit{iff} \quad e^{C*} \leq \frac{1}{2}; \tag{13}$$

or, from (9), if and only if $k \geq \beta M_C + (1-\beta)\Delta$.

The following lemmas establish the properties of the Nash-equilibrium level of effort and expected profits under competition.

Lemma 5 *The Nash-equilibrium level of effort of under competition, e_C^*, is an increasing function of the project payoff, M_C, of the degree of transferability of human capital, δ (and, thus, the value of the outside option, Δ), and of the employee's bargaining power, β, and is a decreasing function of the cost of exerting effort, k.*

Lemma 6 *Expected profits of firms and employees under competition are increasing functions of the project payoff, M_C.*

The effect of the employee outside option, Δ, bargaining power, β, and the cost of exerting effort, k, on individual profits are, in contrast, ambiguous and are examined in the following lemmas.

Lemma 7 *Employee expected profits are increasing in the value of their outside option, Δ. Firm expected profits and total expected profits are increasing in Δ only if employees have low bargaining power:*

$$\frac{\partial \pi_{E_i}^{C*}}{\partial \Delta} > 0; \; i = 1, 2, \tag{14}$$

$$\frac{\partial \pi_{F_i}^{C*}}{\partial \Delta} \geq 0 \quad \textit{iff} \quad \beta \leq \overline{\beta}_F \equiv \frac{kM_C - \Delta(M_C + 2k)}{(M_C + 2k)(M_C - \Delta)}; \; i = 1, 2, \tag{15}$$

$$\frac{\partial \pi_{T_i}^{C*}}{\partial \Delta} \geq 0 \quad \textit{iff} \quad \beta \leq \overline{\beta}_T \equiv \frac{kM_C - \Delta(M_C + k)}{(M_C + k)(M_C - \Delta)}; \; i = 1, 2, \tag{16}$$

with $\overline{\beta}_F < \overline{\beta}_T$.

Lemma 8 *Employee expected profits are increasing in the value of their bargaining power, β. Firm expected profits and total expected profits are increasing in β only if the value of the employee outside option Δ is low:*

$$\frac{\partial \pi_{E_i}^{C*}}{\partial \beta} > 0;\ i = 1, 2, \tag{17}$$

$$\frac{\partial \pi_{F_i}^{C*}}{\partial \beta} \geq 0 \text{ iff } \Delta \leq \overline{\Delta}_F \equiv \frac{M_C \left(k(1-2\beta) - \beta M_C\right)}{(1-\beta)(2k + M_C)};\ i = 1, 2, \tag{18}$$

$$\frac{\partial \pi_{T_i}^{C*}}{\partial \beta} \geq 0 \text{ iff } \Delta \leq \overline{\Delta}_T \equiv \frac{M_C \left(k(1-\beta) - \beta M_C\right)}{(1-\beta)(k + M_C)};\ i = 1, 2, \tag{19}$$

with $\overline{\Delta}_F < \overline{\Delta}_T$.

The presence of the employee outside option is, as expected, always beneficial for the employee. As the value of his outside option, Δ, increases, the employee can extract more rents from the firm. This always increases the level of employee effort and employee expected profits. The impact of the outside option on the firm's expected profits, in contrast, is ambiguous and depends on the balance of two opposing effects. On the one hand, the availability of the outside option allows the employee to extract more rents from the firm, which, all else equal, has a negative effect on the firm's expected profits. On the other hand, a greater value of Δ increases the probability of an innovation by inducing more effort. The positive effect on employee incentives dominates the negative effect on rents when employee effort is low, which happens when the employee has a low bargaining power (i.e., when $\beta \leq \overline{\beta}_F$).[15] By a similar argument, total expected profits are increasing in the value of the outside option when effort is sufficiently low, which again happens when the employee has limited bargaining power (i.e., when $\beta \leq \overline{\beta}_T$).[16]

The effect of employee bargaining power on expected profits mirrors the effect of the outside option. As expected, employees always benefit from having greater bargaining power when negotiating with their firms. The impact of employee bargaining power on the firm's expected profits, is again ambiguous, reflecting the dual role of the bargaining power discussed in Lemma 2. On the one hand, greater bargaining power allows the employee to extract more rents, which is always detrimental to the firm at the negotiating stage. On the other hand, greater bargaining power gives the employee better incentives and leads to a greater probability of innovation. The positive incentive effect dominates the negative rent extraction effect

[15] From the definition of $\overline{\beta}_F$ in (15), note also that $\overline{\beta}_F > 0$ if and only if $k > \frac{\Delta M_C}{M_C - 2\Delta}$, that is, when the cost of exerting effort is high (and, therefore, the level of equilibrium effort is low).

[16] Note again that, from (16), $\overline{\beta}_T > 0$ if and only if $k > \frac{\Delta M_C}{M_C - \Delta}$, that is, when the cost of exerting effort is high and therefore the level of equilibrium effort is low.

when employee effort is sufficiently low. This happens when the value of the employee outside option is limited, i.e., when $\Delta \leq \overline{\Delta}_F$. In a similar way, total expected profits, $\pi_{T_i}^{C*}$, are an increasing function of employee bargaining power when the value of the outside option is low, i.e., when $\Delta \leq \overline{\Delta}_T$.[17]

Lemma 9 *Employee and firm expected profits under competition are decreasing in the cost of exerting effort, k, if and only if $k > \beta M_C + (1-\beta)\Delta$.*

An increase in k affects employee and firm expected profits in two different ways: first, it makes employee effort more costly, which leads the employee to reduce his own effort level; second, it reduces employee effort at the rival firm. By the envelope theorem, the first effect is always negative on employee profits, while the second is always positive, since each employee is better off when the rival employee exerts lower effort. Lemma 9 implies that the first (negative) effect dominates the second (positive) effect only when the equilibrium level of effort is low, that is, when the cost of exerting effort is sufficiently high, giving $k > \beta M_C + (1-\beta)\Delta$. Similarly, firms suffer when their own employee decreases effort and benefit when the rival firm's employee lowers effort. Lemma 9 shows that the first (negative) effect dominates the second (positive) effect again when the equilibrium level of effort is low, that is, when $k > \beta M_C + (1-\beta)\Delta$.[18]

2.3 Competition and Innovation

We now present the first two main results of our paper. First, we show that the probability of obtaining an innovation can be greater under competition than under monopoly. Next, we show that firm profits can be greater under competition than monopoly, which implies that firms themselves may benefit from operating in a competitive rather than in a monopolistic environment.

Proposition 1 *The probability of obtaining an innovation under competition (9) is strictly greater than that under monopoly (3) if and only if*

$$\left(\frac{k}{\beta M} - 1\right)(1-\beta)\Delta - \beta M_C + k\left(\frac{M_C}{M} - 1\right) > 0. \tag{20}$$

Competition influences employee effort and the probability of innovation through three separate channels, which are represented by the three terms in (20). The first channel is the positive impact of the

[17] Note that $\overline{\Delta}_F$ and $\overline{\beta}_F$, and $\overline{\Delta}_T$ and $\overline{\beta}_T$ are related to each other as follows: $\overline{\Delta}_F = \frac{M_C\left(k(1-2\overline{\beta}_F)-\beta M_C\right)}{(1-\overline{\beta}_F)(2k+M_C)}$ and $\overline{\Delta}_T = \frac{M_C\left(k(1-\overline{\beta}_T)-\beta M_C\right)}{(1-\overline{\beta}_T)(k+M_C)}$.

[18] Note that this is result is an implication of Lemma 4, which states that firms suffer from a decrease in the level of effort only when equilibrium effort is low (and, therefore, when the cost of effort k is large).

competition on employee incentives to acquire human capital, and is captured by the first term in (20). As discussed in the previous section, by enabling the employees to develop human capital that is transferable across firms, competition creates an outside option for the employees when they bargain with their firms. The presence of this outside option allows the employees to extract $(1-\beta)\Delta$ additional surplus from their firms, which always has a beneficial effect on employee incentives to exert effort. The second channel, represented by the second term in (20), is given by the reduction in expected profits due to competition when both employees obtain an innovation. This effect always has a negative impact on employee effort, and is captured by the loss in the employee's share of the payoff, βM_C. The third channel arises from the potential difference in market size under competition and under monopoly and, is measured by the term $\frac{M_C}{M}-1$ in (20). This effect is of ambiguous sign: it is positive when the market size under competition is greater than that under monopoly, and negative otherwise.

Note that condition (20) is more likely to be satisfied when either employee bargaining power is low, that is, when β is low, and when the cost of exerting effort is large, that is, when k is high. In either case, the employee exerts too little effort under monopoly. A competitive environment, on the contrary, allows the employee to extract more surplus from the firm, with a beneficial effect on his incentives to exert effort. An interesting implication of Proposition 1 is that firms characterized by greater human capital intensity are more likely to benefit from competition. This property reflects the fact that projects with greater human capital intensity require more costly human capital investment (that is, a greater value of k) and hence they benefit most from the better incentives provided by a competitive environment.

Proposition 1 shows that employee effort, and thus the probability of generating an innovation, can be greater under competition than monopoly. Since firms create value by developing their employee innovations, they may be better off operating in a competitive environment, even if this exposes them to a possible loss in profits and to greater employee rents.

Proposition 2 *Let $M_C \in (M_0, M_1)$, with $M_0 < M = 1 < M_1$. Then, there is a threshold level $\widehat{\beta}_F(M_C,\Delta,k)$ (defined in appendix) such that firm expected profits under competition (11) are strictly greater than those (4) under monopoly if and only if $\beta \leq \widehat{\beta}_F(M_C,\Delta,k)$. Furthermore $\frac{\partial\widehat{\beta}_F(M_C,\Delta,k)}{\partial k} > 0$.*

Proposition 2 shows that when product markets have similar sizes under competition and monopoly (that is, when M_C is close to M), firm profits under competition can be greater than those under monopoly when employee bargaining power is sufficiently low, that is, when $\beta \leq \widehat{\beta}_F(M_C,\Delta,k)$. Competition is particularly desirable when employees exert too little effort under monopoly, which happens either when employee bargaining power is low (i.e., a low value of β), or when the cost of exerting effort is high (i.e., a high value of k). Under these conditions, competition in the product market facilitates competition for employee human capital, which promotes the formation of human capital and improves effort incentives. Similarly, an increase in the cost of effort, k, makes competition relatively more desirable for firms: An increase in the cost of effort reduces employee incentives under monopoly and makes competition relatively more desirable due to its positive impact on effort. In contrast, when employee bargaining power is sufficiently high (that is, when $\beta > \widehat{\beta}_F$), employees already have sufficient incentives under monopoly. In this case, from the firm's perspective, the additional incentives provided by competition do not justify the higher rents extracted by employees and the expected losses due to competition. Hence, for high values of β (and low values of k) firm profits are lower under competition than under monopoly.

3 Endogenous Competition

In this section we endogenized the market structure and we allow firms to choose the market structure under which they operate. Specifically, we extend our basic model by allowing the two firms to choose whether to compete with each other by entering into the same product market, or not to compete by entering into different product markets where each firm has monopoly power. We will show that, under certain conditions, the two firms prefer to forgo the benefits of monopoly and, rather, to engage in direct competition for its beneficial effect on innovation incentives. Thus, greater competition emerges endogenously and leads to more innovation: the desire to innovate spurs competition and competition leads to more innovation.

We modify our model as follows. Each firm can choose one of the two possible types of technologies. The first type of technology is firm-specific. If a firm chooses this technology, it produces a differentiated product which effectively insulates the firm from (the other firm's) competition in the product market. The second type of technology is competitive in that if both firms choose that technology, they produce similar products and therefore are exposed to competition in the product market. Note that the technology selection decision can also be interpreted as geographical location selection, as in the literature on firm clustering. If firms choose the competitive technology, this would be equivalent to clustering. If they choose a firm specific technology, this would be analogous to isolating from each other by locating at distinct geographical places. Alternatively, the choice of technology may be interpreted as the selection of market wide product specifications. In this context, the choice of the firm-specific technology would be equivalent to the selection of proprietary standards requiring the development of firm-specific human capital that cannot be transferred across firms. The choice of the competitive technology would be equivalent to the selection of common standards, such as open sourcing, that allows employees to develop human capital that can be easily transferred across firms.[19]

We model the choice of technology, and thus competition between the two firms, as follows. Firms make their technology selection decision in a sequential manner. Firm 1 moves first and chooses whether to protect itself from future competition by choosing its firm-specific technology or to be exposed to competition by choosing the competitive technology. After observing the choice made by firm 1, firm 2 moves next and decides whether to stay protected from competition by choosing its own firm-specific technology, or to challenge firm 1 by undertaking the competitive technology if firm 1 has chosen the competitive technology as well. If firm 1 has chosen its firm-specific technology, both firms will be monopolists in their respective product markets regardless of the technology chosen by firm 2.[20] After that, the game unfolds as before.

We now characterize the equilibrium of the game.

Proposition 3 *Let* $M_C \in (M_0, M_1)$*, with* $M_0 < M = 1 < M_1$*. Then, there is a threshold level* $\widehat{\beta}_F(M_C, \Delta, k)$ *(defined in appendix) such that*

i) if $\beta \leq \widehat{\beta}_F(M_C, \Delta, k)$*, then the firms choose the competitive technology;*

ii) if $\beta > \widehat{\beta}_F(M_C, \Delta, k)$*, then firm 1 chooses the competitive technology if* $M_C \geq 1$ *and it chooses its firm-specific technology if* $M_C < 1$*; firm 2 always chooses its firm-specific technology.*

The equilibrium choice for the two firms will be the competitive technology when the benefit of competition on employee effort outweighs its cost through higher employee rents (i.e., greater wages) and the potential losses in the product market. The competitive technology is particularly desirable when employees exert too little effort under monopoly. As discussed in Propositions 1 and 2, this happens either when employee bargaining power is low (i.e., a low value of β), or when the cost of exerting effort is high (i.e., a high value of k). Proposition 3 shows that the competitive technology is chosen when employee bargaining power is lower than a certain threshold $\widehat{\beta}_F$. In this case, the choice of the competitive technology, by

[19] We thank Jim Brander for the suggestion of this interpretation.

[20] Note that the sequential nature of the game allows us to avoid the possibility of mixed strategy equilibria and does not affect our analysis in any substantial way.

facilitating the marketability of employee human capital, promotes the formation of human capital and improves effort incentives. In contrast, when employee bargaining power is sufficiently high (or the cost of effort is low), that is, when $\beta > \widehat{\beta}_F$, the employees incentives under monopoly are already sufficiently strong, and the choice of the competitive technology is not justified. Therefore, in this case, firm 1 will exploit its first-mover advantage and choose the competitive technology if it has a larger market size than its firm-specific technology ($M_C \geq 1$), and choose its firm-specific technology otherwise. Firm 2 will always choose its firm-specific technology.

Proposition 3 has also two important implications. First, note that from Proposition 2 we know that $\frac{\partial \widehat{\beta}_F(M_C,\Delta,k)}{\partial k} > 0$. This implies that that an increase in the cost of effort, k, enlarges the region in which the two firms choose to compete by both adopting the competitive technology. This property reflects the fact that an increase in the cost of effort reduces employee incentives making the competitive technology relatively more desirable (due to its positive impact on effort). It also implies that a competitive environment is more likely to emerge in sectors with technologies characterized by greater human capital intensity (that is, a greater value of k). As discussed in Proposition 1, projects with greater human capital intensity involve more costly human capital investment, and therefore they benefit most from the better incentives provided by a competitive environment. Thus, our model helps to explain why human capital intensive firms tend to operate in highly competitive environments (as discussed in Zingales, 2000).

The second interesting implication of Proposition 3 is that both firms may adopt the competitive technology even if it has a smaller market size than their firm-specific technology (i.e., when $M_C < 1$). When employees have low bargaining power β or high cost of exerting effort k, the two firms prefer to forego the monopoly power and the larger market size associated with their firm-specific technology for the benefit of providing stronger incentives to their employees.

The threshold level $\widehat{\beta}_F(M_C, \Delta, k)$ depends on the size of the competitive market, M_C, and the value of employee outside option, Δ, and it is characterized in the following proposition.

Proposition 4 *The threshold level $\widehat{\beta}_F(M_C, \Delta, k)$ has the following properties:*

(i) $\quad \frac{\partial \widehat{\beta}_F(M_C,\Delta,k)}{\partial M_C} > 0$;

$(ii.a)$ $\quad \frac{\partial \widehat{\beta}_F(M_C,\Delta,k)}{\partial \Delta} > 0$, *if* $\Delta < \widehat{\Delta}_F$ *(where $\widehat{\Delta}_F$ is defined in the appendix),*

$(ii.b)$ $\quad \frac{\partial \widehat{\beta}_F(M_C,\Delta,k)}{\partial \Delta} < 0$, *if* $\Delta > \widehat{\Delta}_F$.

The threshold level $\widehat{\beta}_F(M_C, \Delta, k)$ is, as predictable, an increasing function of M_C, which means that a greater market size makes the competitive technology relatively more attractive for the two firms. The effect of the value of the outside option on technology choice is ambiguous. When the value of the outside option is low (that is, when $\Delta < \widehat{\Delta}_F$), the choice of the competitive technology has a positive but relatively small impact on employee effort. This implies that an increase in the value of the outside option increases the beneficial effect of the competitive technology on incentives, making it relatively more desirable for the firms, leading to $(ii.a)$. In contrast, when the value of the outside option is high (that is, when $\Delta > \widehat{\Delta}_F$) the choice of the competitive technology gives employees too powerful incentives, which will be detrimental for the firms due to competition in the product market.[21] Thus, an increase in the value of the outside option increases employee effort even further, making the competitive technology relatively less desirable, leading to $(ii.b)$

Proposition 4 characterizes the firms' technology selection decision based on maximizing only firm expected profits rather than the total expected profits.[22] Proposition 5 summarizes the choice between the competitive technology and firm-specific technology when the choice is based on maximizing the total expected profits rather than only firms' expected profits.

Proposition 5 *Let $M_C \in (M_0^T, M_1^T)$, with $M_0^T < 1 < M_1^T$. Then, there is a threshold level $\widehat{\beta}_T(M_C, \Delta, k)$ (defined in the appendix) such that*

i) if $\beta \leq \widehat{\beta}_T(M_C, \Delta, k)$, then both firms choose the competitive technology;

ii) if $\beta > \widehat{\beta}_T(M_C, \Delta, k)$, then firm 1 chooses the competitive technology if $M_C \geq 1$ and it chooses its firm-specific technology if $M_C < 1$; firm 2 always chooses its firm-specific technology.

Furthermore, $\widehat{\beta}_T(M_C, \Delta, k) > \widehat{\beta}_F(M_C, \Delta, k)$.

Proposition 5 states that, when total profits are maximized, the competitive technology is more desirable than the firm-specific technology even at higher levels of employee bargaining power, i.e., $\widehat{\beta}_T > \widehat{\beta}_F$. This result derives from the fact that when the technology selection decision is based on maximizing firm profits rather than the total profits, the higher rent extraction ability of employees under the competitive technology represents, from the firms' point of view, a cost that must be balanced against better effort incentives. In contrast, when technology choice is made to maximize total profits, employee profits are internalized, and rent extraction represents a transfer of surplus from firms to employees with no direct

[21] Remember that, under competition, when employee effort is very high the probability that the firms will compete in the product market and earn zero profits is also very high, reducing firm's expected profits. This, from Lemma 4, makes a high level of employee effort undesirable.

[22] If transfers from one party to another are not possible at the beginning of the game, the firms will choose the technology that maximizes their own profits, rather than their own profits plus their employee's profits.

impact on total profits, leading to the adoption of the competitive technology for even greater levels of employee bargaining power.

4 Firms' Innovation Incentives

Our analysis so far has examined the impact of competition on employee incentives to exert effort only and remained silent about firms' innovation incentives. In this section we extend our analysis by examining the effect of competition on firms' incentives to invest in innovation. We modify the basic model presented in Section 2 by assuming that now each firm makes an investment at $t = 0$ at a cost of I which determines the project payoff from employee innovations. For simplicity, we assume now that $M = M_C = f(I)$, where $f(.)$ is an increasing and concave function of I with $f(0) = 0$ and $f'(0) = \infty$. After observing the firms' investments and market structure, each employee selects his level of effort. The rest of the game unfolds as before.

Consider first the case of monopoly. Anticipating the payoff that each firm will obtain from bargaining with its employee, at $t = 0$ each firm determines the optimal amount of investment, I^{M*}, by maximizing its expected profits, that is (after substituting M with $f(I)$ in (1)) by

$$\max_I \frac{(1-\beta)\beta f(I)^2}{k} - I. \tag{21}$$

In a similar way, under competition each firm i determines the optimal amount of investment, I_i^{C*}, $i = 1, 2$, by maximizing its expected profits, that is

$$\max_{I_i} \frac{(1-\beta)\ \ k(k-\beta f(I_i)-\check{\Delta})(f(I_i)-\Delta)\check{\Delta}\left(k-\beta\left(f(I_i)+f(I_j)\right)-\check{\Delta}\right)+f(I_i)\beta\left(k-\beta f(I_j)\right)}{\left(\beta^2 f(I_i)f(I_j)-\left(k-\check{\Delta}\right)\left(k-\check{\Delta}\right)+\beta\check{\Delta}\left(f(I_i)+f(I_j)\right)\right)^2} - f(I_i). \tag{22}$$

where $\check{\Delta} \equiv (1-\beta)\,\Delta$. We obtain the following proposition.

Proposition 6 *There is a $\tilde{\beta} > 0$ (defined in the appendix) such that if $0 \leq \beta \leq \tilde{\beta}$ firm investment in innovation is greater under competition than under monopoly.*

The benefit of competition on employee incentives to innovate creates a positive spillover effect on firms' incentives as well. Note that the positive effect on firms' incentives arises from the complementarity between employee effort and firms' investment. Since firms can obtain a profit only if their employees exert effort, each firm incentives to invest in innovation are greater when its employee exerts greater effort. Proposition 6 shows that firms have greater incentives to invest in innovation under competition than monopoly when employee bargaining power is low, that is when $\beta \leq \tilde{\beta}$. This result may be seen as follows. When employees have low bargaining power, they can extract too little rents under monopoly, leading to low effort and low expected profits for both firms and employees. As a result of low employee effort and low expected profits, firm investment turns out to be low as well. Competition, on the other hand, by allowing employees to extract greater rents from their firms, leads to greater employee effort and better incentives for firms to invest in innovation.

5 Human capital mobility and innovation

In Section 3 we have shown that one of the advantages of the competitive technology is that, by allowing employees to transfer their human capital to competing firms, it provides better incentives to exert effort. Since employee effort in the two firms are strategic substitutes, technology selection and thus human capital mobility can be used by each firm to achieve a strategic advantage over its rival. Firms may restrict the mobility of their employees by requiring them to sign ex-ante agreements that prevent them from joining rival firms. Such agreements, usually referred to as *no-compete* agreements, affect employees' bargaining position when they negotiate wages. In this section, we endogenize each firm's choice of whether or not to allow its employee to transfer his human capital (and the innovation) to the rival firm.

We modify the basic model as follows. For simplicity, we assume that the two firms have access only to the competitive technology, and we set $M_C = k = 1$. We endogenize the availability of outside options to employees by assuming that each firm can require its employee to sign ex-ante a *no-compete* agreement that prevents him from joining the rival firm at a later date. The effect of such an agreement is to eliminate the employee's outside option when bargaining with its firm, effectively setting δ (and, therefore, Δ) equal to 0.

The game is now as follows. The firms can only invest in the competitive technology. At time 0, the firms simultaneously choose whether or not to require their employees to sign a *no-compete* agreement. We will refer to this game as the *agreement-choice* game. After that, the game unfolds as before. The only difference with the basic model is that now an employee, while bargaining with his firm at the interim date, will have an outside option only if he has not entered into a *no-compete* agreement with his firm at time 0.

The following proposition characterizes the Nash-equilibrium of the *agreement-choice* game.

Proposition 7 *There is a* $\Delta^{NC} \in (0,1)$ *(defined in the appendix) such that*

i) if $0 \leq \Delta < \Delta^{NC}$ *neither firm requires its employee to sign a no-compete agreement. Furthermore, there is a* $\Delta_F^P \in [0, \Delta^{NC})$ *(defined in the appendix) such that, if* $\Delta_F^P \leq \Delta < \Delta^{NC}$*, the equilibrium profits for both firms are (Pareto) dominated by the profits obtained when both firms can commit to impose a no-compete agreement on their employees.*

ii) If $\Delta^{NC} \leq \Delta \leq 1$*, the agreement-choice game has two (Nash) equilibria. In the first equilibrium, neither firm requires its employee to sign a no-compete agreement. In the second equilibrium, both firms require their employee to sign a no-compete agreement. Furthermore, firm profits in the equilibrium where firms do not impose the no-compete agreement are (Pareto) dominated by the profits obtained when both firms impose a no-compete agreement.*

The imposition of a *no-compete* agreement on an employee has three effects on firm profits. First, the presence of the agreement, by preventing employee transfer to the rival firm, reduces the surplus extracted by the employee, with a positive direct effect on firm profits. Second, the reduction in the employee's ability to extract surplus has a negative effect on his incentives and leads to lower effort, with a negative impact on firm profits (all else equal). Third, since employee effort levels are strategic substitutes, the reduction in employee effort due to the *no-compete* agreement puts the firm at a strategic disadvantage since it promotes employee effort at the rival firm.

When the value of the employee outside option is not too large, that is, when $0 \leq \Delta < \Delta^{NC}$, the negative incentive effect and the strategic effect of the agreement dominate the positive effect on firm rents, and neither firm finds it optimal to impose a *no-compete* agreement on its employee. By preserving its employee's outside option, each firm can strategically empower its employee with better incentives and thus reduce innovation effort at the rival firm. Note that when $\Delta_F^P \leq \Delta < \Delta^{NC}$ the firms reach an inefficient outcome in the sense that the equilibrium profits are lower than those that would be obtained if both firms could coordinate ex-ante and commit to impose a *no-compete* agreement on their employees. This inefficiency arises because the strategic effect induces each firm not to impose the agreement even if the negative rent extraction effect dominates the positive incentive effect (that is, when the firms, absent competition and the strategic effect, would choose to impose such agreements).[23]

[23]This property may be seen as follows. Without a strategic benefit (i.e., by keeping the effort level of the rival firm's

When the value of the employee outside option is sufficiently large, that is, when $\Delta^{NC} \leq \Delta \leq 1$, two (symmetric) equilibria exist: in the first equilibrium, neither firm imposes a *no-compete* agreement, while in the second one they both impose an agreement. Furthermore, the equilibrium where the firms impose a *no-compete* agreement Pareto-dominates the one in which they do not so. These intuition for these results can be seen as follows. When the value of the employee outside option is large, the absence of a *no-compete* agreement allows an employee to extract more surplus from his firm, with a powerful effect on his incentives. Thus, by not imposing an agreement, a firm gains a significant strategic advantage over its rival, but at the cost of paying greater rents to its employee. If one of the firms imposes an agreement, the rival firm has the incentive to impose one as well since the strategic gain from not imposing the agreement does not compensate the firm for conceding greater rents to its employee. In contrast, if one of the firms does not impose an agreement, the rival firm does not to impose one either since imposing an agreement unilaterally would put the firm to a strategic disadvantage with respect to its rival firm. Furthermore, firm profits in the equilibrium without a *no-compete* agreement are again (Pareto) dominated by the profits obtained when both firms impose such an agreement.

Proposition 7 establishes the equilibrium decision of whether or not to impose a *no-compete* agreement where firms maximize their own profits. It is interesting to contrast this case to the one in which the firms and the employees act as a coalition, and the decision of whether or not to impose an agreement is made to maximize the joint profits.

Proposition 8 *If the firms maximize total profits, the unique Nash-equilibrium of the agreement-choice game is such that neither firm requires its employee to sign a no-compete agreement. Furthermore, there is a* $\Delta_T^P \in [0,1]$ *such that, if* $\Delta_T^P \leq \Delta \leq 1$*, the equilibrium profits for both firms are (Pareto) dominated by the profits obtained when both firms can commit to impose a no-compete agreement.*

Contrasting Proposition 7 and Proposition 8 reveals that, when the choice of whether or not to impose a no-compete agreement is made to maximize total profits, such agreements are never imposed in equilibrium. When the total profits are maximized, the reallocation of rents between a firm and its employee represents

employee constant) a firm has an incentive to impose a no-compete agreement on its employee when $\Delta > \bar{\Delta} \equiv \frac{1-2\beta}{1-\beta}$, with $\bar{\Delta} < \Delta^{NC}$. The firm's decision on whether or not to impose the agreement is made by trading off the positive effect on employee incentives against the negative effects on its own rents. Thus, if $\Delta \leq \bar{\Delta}$, the positive incentive effect dominates, and a firm would not impose the *no-compete* agreement even in the absence of a strategic benefit of the agreement. In contrast, when $\bar{\Delta} < \Delta \leq \Delta_1^{NC}$, absent a strategic benefit, the firm would prefer to impose a *no-compete* agreement on its employee.

an internal transfer of surplus between the two parties and it has no direct impact on the total profits. Thus, in this case, the rent extraction effect of imposing a *no-compete* agreement disappears, leaving only the strategic and the incentive effect in play. Hence the two firms will always agree with their employees on not signing a *no-compete* agreement. Furthermore, when Δ is sufficiently large, that is, when $\Delta_T^P \leq \Delta \leq 1$, the equilibrium outcome is not Pareto-optimal. This result reflects the property that a large value of the outside option Δ results in a too-high equilibrium level of employee effort, and induces more aggressive competition between the two firms (by maximizing the likelihood of the state where both employees are successful). In this case, the two firms would be better off if they could coordinate ex-ante and commit not to hire each-other's employee.[24]

Note that the results of this section shed some light on certain observed industry practices. For example, Procter and Gamble (P&G) and Unilever, the two rival firms in the consumer goods industry, require their employees to sign an agreement forbidding the P&G employees to join Unilever or vice versa. This practice can be interpreted, in our model set-up, as one in which the two firms cooperatively choose, in equilibrium, to impose a *no-compete* agreement on their employees. In contrast, there is anecdotal evidence that investment banking and consulting industries (two human capital-intensive industries) are characterized by an absence of *no-compete* agreements and thus, a high degree of employee mobility.

6 Innovation and Horizontal Mergers

The previous sections established that, under certain conditions, competition fosters innovation in human capital-intensive firms by promoting the development of employee human capital. Specifically, we have shown that two firms may prefer to compete in the same product market rather than to isolate from each other and enjoy monopoly power in their niche markets by choosing their firm-specific technology. In this section we analyze the incentives of the two firms to merge and undertake the competitive technology as a single firm, employing the two employees within the merged firm.

We modify the basic model as follows. For simplicity, we assume again that the two firms have only the competitive technology available, and we set $M_C = 1 = k$. At the beginning of the game the two firms have the option to merge or to remain as separate entities. If the two firms merge, the combined firm will become a monopolist in the product market. If they choose to remain as two independent firms, they will

[24]This commitment may be obtained, for example, by creating an industry standard which requires that employees of all firms sign such agreements.

compete in the product market as discussed in the previous sections.

The game now unfolds as follows. At time 0, the two firms decide whether to merge or to remain as separate entities. If the two firms decide not to merge, they both undertake the competitive technology, and the game is the same as before. If the two firms merge, they will retain the two employees in the merged firm.[25] The employees exert effort and, as before, they may generate an innovation with probability e_i^{HM}. We assume that the innovations generated by the two employees are perfect substitutes and that the merged firm can implement only one employee innovation.[26]

A merger between the two firms alters the nature of the competition between the two employees. When the two firms are separate, the employees compete only indirectly, as they belong to two separate firms. If successful, their ideas are always implemented by their firms, leading the two firms to compete in the product market. When, in contrast, the two firms merge, the employees compete directly within the merged firm since, if they are both successful, only one employee will have his innovation implemented. Thus, the merger has the consequence of replacing indirect, external competition with direct, internal competition. As a result, the firm is able to extract a higher surplus from its employees when both employees generate an innovation. In this state, when the firm bargains with one employee it has the option to bargain with its second employee if the bargain with the first employee fails. This allows the merged firm to extract a greater surplus from its employees (i.e., pay lower wages) relative to the case in which each firm employs only one employee.[27]

The division of the surplus between the firm and its employees depends on whether only one or both employees are successful. If only one employee generates an innovation, he will bargain with the firm for his profit share for his participation in the development of the innovation. Since the merged firm is a monopolist, and only one employee has an innovation, both the firm and the employee have zero outside options when they bargain. Thus, the employee will obtain a share β of the surplus created, and the firm will retain the remainder, $1-\beta$ (remember that now $M_C = 1$).

[25]Note that the merged firm can also decide to employ only one of the two employees and to fire the other. It is possible to show that this strategy is never optimal when $k = 1$.

[26]An example of such an innovation would be a "process innovation," that is, the discovery of a new technology that is necessary to produce a product. This assumption can be also justified by assuming that the firm is capacity and resource constrained, and can implement at most one innovation. See Rotemberg and Saloner (1994) for a similar assumption.

[27]Note that the fact that the merged firm extracts more rents when it employs more than one employee is similar to the result in Stole and Zwiebel (1996a and 1996b), which shows that firms may overemploy in order to gain a bargaining advantage in the wage negotiations with their employees.

If both employees generate an innovation, the firm selects randomly and with equal probability one of the two employee innovations as the one to be developed and commercialized. The selected employee, say employee i, will then bargain with the firm for the division of the surplus. This case differs from the monopoly and competitive cases discussed earlier in that now, while bargaining with employee i, the firm has the option of developing the innovation generated by employee j, if bargaining between the firm and employee i breaks down. Hence, a merger between the two firms creates an outside option for the merged firm and it eliminates the outside option that the employees have under the competitive technology.

We model the bargaining game between employee i and the firm as before, and we assume that the two parties make alternating offers under the threat that the bargaining process breaks down with a certain exogenous probability. If bargaining with employee i breaks down, the firm has the option to start a new round of bargaining with employee j. Thus, the payoff from bargaining with employee j represents the firm's outside option when bargaining with employee i. As before, the payoff of the subgame perfect equilibrium of the bargaining game is such that the employee and the firm receive the value of their outside options (the value that they can obtain in case of a breakdown in the bargaining game) plus, respectively, the fractions β and $1-\beta$ of the surplus they jointly generate, net of the sum of their outside options.

We can determine the payoffs of the bargaining game between the firm and employee i by proceeding backwards. If bargaining with employee i breaks down, the firm bargains with employee j. In this second bargaining game, both employee j and the firm will have zero outside options. The employee and the firm will therefore divide the joint surplus according to their bargaining power, obtaining β and $1-\beta$, respectively. Let $\Delta_F \equiv 1-\beta$ represent the value of the firm's outside option while bargaining with employee i. Since there are no other firms to which the employee i can go, he has no outside option when bargaining with the firm. This implies that employee i's payoff from bargaining with the firm is now equal to $\beta(1-\Delta_F) = \beta^2$. Furthermore, since employee i's innovation is chosen with probability $\frac{1}{2}$, his expected payoff is $\frac{1}{2}\beta^2$. Correspondingly, the firm's payoff is given by $\Delta_F + (1-\beta)(1-\Delta_F) = 1-\beta^2$.

In anticipation of his payoff from bargaining with the firm, employee i chooses his effort level, e_i^{HM}, by maximizing his expected profits, $\pi_{E_i}^{HM}$:

$$\max_{e_i} \quad \pi_{E_i}^{HM} \equiv e_i^{HM} e_j^{HM} \frac{1}{2}\beta^2 + e_i^{HM}(1-e_j^{HM})\beta - \frac{1}{2}(e_i^{HM})^2; \; i,j=1,2; \; i \neq j. \tag{23}$$

The firm's expected profits is

$$\pi_F^{HM} \equiv e_i^{HM} e_j^{HM}(1-\beta^2) + e_i^{HM}(1-e_j^{HM})(1-\beta) + e_j^{HM}(1-e_i^{HM})(1-\beta); \; i,j=1,2; \; i \neq j. \tag{24}$$

The first-order condition of (23) provides employee i's optimal response, given employee j's effort choice, and is as follows:

$$e_i^{HM}(e_j^{HM}) = \frac{\beta(1 - e_j^{HM}(1 - \frac{\beta}{2}))}{2};\ i, j = 1, 2;\ i \neq j. \tag{25}$$

From the first order condition (25) it can be immediately seen that employee i's effort is a decreasing function of employee j's effort, which implies that effort choices by the two employees are strategic substitutes. Furthermore, employee i's effort is again an increasing function of his bargaining power, β. We now characterize the Nash-equilibrium of the effort subgame.

Lemma 10 *The Nash-equilibrium of the effort subgame is given as follows:*

$$e_i^{HM*} = e_j^{HM*} = e^{HM*} \equiv \frac{\beta}{1 + \beta(1 - \frac{\beta}{2})}. \tag{26}$$

The corresponding expected profits of the firm and the employees are

$$\pi_{E_i}^{HM*} = \frac{\beta^2}{\left(1 + \beta(1 - \frac{\beta}{2})\right)^2},\ i = 1, 2; \tag{27}$$

$$\pi_F^{HM*} = \frac{\beta(1 - \beta)(2 + \beta)}{\left(1 + \beta(1 - \frac{\beta}{2})\right)^2}. \tag{28}$$

The following lemma establishes the conditions under which the equilibrium level of effort in the merged company is higher or lower than that in the competitive case.

Lemma 11 *The level of effort under horizontal merger, e^{HM*}, is greater than the level of effort when the firms choose to compete, e^{C*}, if and only if*

$$\Delta \leq \Delta^{HM} \equiv \frac{\beta^3}{(2 - \beta^2)(1 - \beta)}. \tag{29}$$

The horizontal merger affects employee incentives in several different ways. First, the merger eliminates the product market competition between the two firms and provides monopoly payoff when at least one of the two employees is successful in generating an innovation. This benefits the employees, as they now can obtain a nonzero payoff also in the state where they are both successful. Second, when both employees are successful, each employee will have his innovation implemented only with probability $\frac{1}{2}$. This effect has an adverse impact on incentives, which is similar to the one identified in Rotemberg and Saloner (1994). Moreover, when both employees are successful, the firm extracts more rents from each employee since the merger between the two firms introduces an outside option for the firm in bargaining. Finally, the horizontal merger prevents the formation of the inter-firm labor market, and the employees, therefore, lose their ability to extract higher rents by having the option to join the competing firm in the state where the rival's firm employee fails to generate an innovation.

Lemma 11 implies that the horizontal merger has a positive impact on employee effort when the benefits of the competitive technology on employee incentives given by the outside option Δ are not too large, that is, when $\Delta \leq \Delta^{HM}$. Furthermore, it is easy to see that the threshold level Δ^{HM} is an increasing function of employee bargaining power, β. This implies that employee effort in the horizontal merger is more likely to be greater than that under competition when the employees have high bargaining power. If employees have high bargaining power, they already have powerful incentives in the merged firm and external competition will improve effort only when the value of the outside option is sufficiently large.

In Proposition 9, we compare firm profits under the competitive technology to half of the total profits of the merged firm to characterize the firms' merger decision.

Proposition 9 *There is a threshold* $\beta^{HM}(\Delta)$ *(defined in the appendix) such that:*

i) if $\beta < \beta^{HM}(\Delta)$, *the two firms choose not to merge and to compete;*

ii) if $\beta > \beta^{HM}(\Delta)$, *the two firms choose to merge.*

The firms' merger decision is made by trading off the three effects of the merger on firm profits. First, of course, the merger eliminates competition in the product market and increases firm profits. Second, the merger, by eliminating the employees' outside options and inducing internal competition between the employees, allows the merged firm to extract more surplus from the employees at the bargaining stage. Third, holding everything else constant, the reduction in the employees' rent extraction ability leads to lower effort and lower profits for the merged firm. Proposition 9 states that, when employee bargaining power is low, the two firms will be better off under competition. The horizontal merger consolidates the two competing firms and restricts employees outside opportunities. Also, the merger replaces external, indirect competition between employees with direct, internal competition. Hence, it reduces the employees' ability to extract rents from their firms and affects their incentives adversely, hindering the rate of innovation. Thus, when the employees have low bargaining power, the two firms prefer to remain as separate, competing entities in order to preserve an active inter-firm market for their employees. The presence of an external market for employees promotes their effort in accumulating human capital and benefits firms through a higher likelihood of innovations. Thus, while internal competition within a firm may be detrimental for innovation (as in Rotemberg and Saloner, 1994), external competition may promote effort and induce innovation. When, in contrast, employee bargaining power is sufficiently high, internal incentives within the merged firm to exert effort are already sufficiently strong, and elimination of product market competition through the horizontal merger becomes more desirable.

7 Conclusions

This paper offers an explanation for the emergence of today's human capital intensive firms, which are subject to greater competition than before and yet extraordinarily innovative, by establishing a new link between product market competition and employee incentives to innovate. We argue that competition in the product market leads firms to compete for employee human capital and, thus, allows employees to extract more rents (i.e, greater wages). In turn, greater rents lead employees to invest more in human capital and to innovate more. Interestingly, not only employees, but also firms can benefit from competition in the product market. Although competition reduces firms' market shares and increases employee rents, its positive impact on employee incentives to innovate may have a positive impact on firm profits.

We also study the role of *no-compete* agreements on innovation incentives. We identify the conditions under which imposing a *no-compete* agreement on employees, which restricts employee mobility and eliminates their ability to join rival firms, will be detrimental for a firm. Our analysis shows that, under certain conditions, imposing such agreements on employees affects innovation incentives adversely and reduces firm profits. Moreover, a firm can gain a strategic advantage over its rival firm by not imposing such agreements, since this enhances its own employee's innovation effort and depresses innovation incentives at the rival firm. Hence, our analysis highlights a strategic motive for the use of no-compete agreements in competitive industries.

Finally we investigate the impact of horizontal mergers between rival firms on employee incentives to innovate. We show that, under certain conditions, a horizontal merger between two competing firms eliminates competition for employee human capital and affects employee incentives adversely. Hence, firms prefer not to merge and bear competition in the product market to maintain stronger employee incentives.

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Appendix

Proof of Lemma 1

The first order condition of (1) with respect to e_i^M gives the optimal value e_i^{M*} in (3). Direct substitution of (3) into (1) and (2) gives (4).

Proof of Lemma 2

From (3), it is easy to see that the optimal level of effort under monopoly, e^{M*}, is an increasing function of β. From (4), it is also easy to see that $\frac{\partial \pi_{E_i}^{M*}}{\partial \beta} > 0$. From (4), we obtain that $\frac{\partial \pi_{F_i}^{M*}}{\partial \beta} = M^2 \frac{1-2\beta}{k}$, which is positive for $\beta < \frac{1}{2}$. Finally, again from (4), we obtain that $\frac{\partial \pi_{T_i}^{M*}}{\partial \beta} = (1-\beta)\frac{M^2}{k}$, which is positive for all $0 < \beta < 1$.

Proof of Lemma 3

From the reaction function (8), the Nash-equilibrium effort level, e^{C*}, is obtained by setting $e^C = \frac{(1-e^C)(\beta M_C + (1-\beta)\Delta)}{k}$, and solving for e^C. Note that $\beta M_C < 1$ implies that the symmetric Nash-equilibrium is unique. The corresponding level of profits in (10), (11) and (12) are obtained by direct substitution of (9) into (6) and (7).

Proof of Lemma 4

The Nash-equilibrium firm profits are given by $e^{C*}(1-e^{C*})(1-\beta)(M_C-\Delta)$. Differentiating with respect to e^{C*}, we obtain

$$\frac{\partial \pi_{F_i}^{C*}}{\partial e^{C*}} = (M_C - \Delta)(1-\beta)(1-2e^{C*}),$$

which is positive if and only if $e^{C*} < \frac{1}{2}$. From (9) $e^{C*} \leq \frac{1}{2}$ if and only if $k \geq \beta M_C + (1-\beta)\Delta$.

Proof of Lemma 5

Differentiating the Nash-equilibrium effort level, (9), with respect to M_C we obtain $\frac{\partial e^{C*}}{\partial M_C} = \frac{\beta k}{(k+\beta M_C + \Delta(1-\beta))^2}$, which is always positive. Differentiating (9) with respect to Δ, we obtain

$$\frac{\partial e^{C*}}{\partial \Delta} = \frac{(1-\beta)\, k}{(k + \beta M_C + \Delta(1-\beta))^2} > 0.$$

Differentiating (9) with respect to β, we obtain

$$\frac{\partial e^{C*}}{\partial \beta} = \frac{k(M_C - \Delta)}{(k + \beta M_C + \Delta(1-\beta))^2} > 0.$$

Finally, from (9), it is easy to see that e^{C*} is always a decreasing function of k.

Proof of Lemma 6

Differentiating the equilibrium level of employee profits, (10), with respect to respect to M_C we obtain

$$\frac{\partial \pi_{E_i}^{C*}}{\partial M_C} = \frac{k^2(\beta M_C + \Delta(1-\beta))\beta}{(k+\beta M_C + (1-\beta)\Delta)^3} > 0.$$

Similarly, differentiating the equilibrium level of firm profits, (11), with respect to M_C we obtain

$$\frac{\partial \pi_{F_i}^{C*}}{\partial M_C} = (1-\beta)k\frac{k(2\beta(M_C-\Delta)+\Delta)+\Delta\beta M_C + \Delta^2(1-\beta))}{(k+\beta M_C + (1-\beta)\Delta)^3} > 0.$$

Hence, the equilibrium level of total profits are an increasing function of M_C as well.

Proof of Lemma 7

Differentiating the equilibrium level of employee profits, (10), with respect to Δ, we obtain

$$\frac{\partial \pi_{E_i}^{C*}}{\partial \Delta} = \frac{k^2(1-\beta)(\beta M_C + (1-\beta)\Delta)}{(k+\beta M_C + (1-\beta)\Delta)^3} > 0.$$

Differentiating the equilibrium level of firm profits, (11), with respect to Δ, we obtain

$$\frac{\partial \pi_{F_i}^{C*}}{\partial \Delta} = (1-\beta)k\frac{M_C((1-2\beta)k - \beta M_C) - (M_C+2k)(1-\beta)\Delta}{(k+\beta M_C + (1-\beta)\Delta)^3},$$

which is positive for $\beta \leq \overline{\beta}_F \equiv \frac{kM_C - \Delta(M_c+2k)}{(M_c+2k)(M_c-\Delta)}$. Similarly, differentiating the equilibrium level of total profits, (12), with respect to Δ we obtain

$$\frac{\partial \pi_{T_i}^{C*}}{\partial \Delta} = (1-\beta)k\frac{M_C((1-\beta)k - \beta M_C) - (M_C+k)(1-\beta)\Delta}{(k+\beta M_C + (1-\beta)\Delta)^3},$$

which is positive for $\beta \leq \overline{\beta}_T \equiv \frac{kM_C - \Delta(M_c+k)}{(M_c+k)(M_c-\Delta)}$. It is straightforward to show that $\overline{\beta}_F < \overline{\beta}_T$.

Proof of Lemma 8

Differentiating the equilibrium level of employee profits, (10), with respect to β we obtain

$$\frac{\partial \pi_{E_i}^{C*}}{\partial \beta} = k^2\frac{(M_C-\Delta)(\beta M_C + (1-\beta)\Delta)}{(k+\beta M_C + (1-\beta)\Delta)^3} > 0$$

since $M_C \geq \Delta$. Differentiating the equilibrium level of firm profits, (11), with respect to β we obtain

$$\frac{\partial \pi_{F_i}^{C}}{\partial \beta} = k(M_C-\Delta)\frac{kM_C - (2k+M_C)\Delta - (M_C+2k)(M_C-\Delta)\beta}{(k+\beta M_C + (1-\beta)\Delta)^3},$$

which is positive if and only if $\Delta \leq \overline{\Delta}_F \equiv \frac{M_C(k(1-2\beta)-\beta M_C)}{(1-\beta)(2k+M_C)}$. Similarly, differentiating the equilibrium level of total profits, (12), with respect to β we obtain

$$\frac{\partial \pi_{T_i}^{C*}}{\partial \beta} = k(M_C-\Delta)\frac{k(M_C-\Delta) - M_C\Delta - (M_C+k)(M_C-\Delta)\beta}{(k+\beta M_C + (1-\beta)\Delta)^3},$$

which is positive if and only if $\Delta \leq \overline{\Delta}_T \equiv \frac{M_C(k(1-\beta)-\beta M_C)}{(1-\beta)(k+M_C)}$. It is straightforward to show that $\overline{\Delta}_F < \overline{\Delta}_T$.

Proof of Lemma 9

Differentiating the equilibrium level of employee profits, (10), with respect to k we obtain

$$\frac{\partial \pi_{E_i}^{C*}}{\partial k} = -\frac{1}{2}(\beta M_C + (1-\beta)\Delta)^2 \frac{k - \beta M_C - (1-\beta)\Delta}{(k + \beta M_C + (1-\beta)\Delta)^3},$$

which is negative if and only if $k > \beta M_C + (1-\beta)\Delta$. Differentiating the equilibrium level of firm profits, (11), with respect to k we obtain

$$\frac{\partial \pi_{F_i}^{C*}}{\partial k} = -(1-\beta)(\beta M_C + (1-\beta)\Delta)(M_C - \Delta)\frac{k - \beta M_C - (1-\beta)\Delta}{(k + \beta M_C + (1-\beta)\Delta)^3},$$

which is negative if and only if $k > \beta M_C + (1-\beta)\Delta$. Differentiating the equilibrium level of total profits, (12), with respect to k we obtain

$$\frac{\partial \pi_{T_i}^{C*}}{\partial k} = -\frac{1}{2}(\beta M_C + (1-\beta)\Delta)((2-\beta)M_C - (1-\beta)\Delta)\frac{k - \beta M_C - (1-\beta)\Delta}{(k + \beta M_C + (1-\beta)\Delta)^3},$$

which is negative if and only if $k > \beta M_C + (1-\beta)\Delta$, since $(2-\beta)M_C - (1-\beta)\Delta > 0$.

Proof of Proposition 1

Direct comparison of (3) with (9) reveals that the probability of obtaining an innovation under competition is greater than that under monopoly if and only if $\left(\frac{k}{\beta M} - 1\right)(1-\beta)\Delta - \beta M_C + k(\frac{M_C}{M} - 1) > 0$.

Proof of Proposition 2

Given firm profits under monopoly, (4), and under competition, (11), define

$$\Psi(\Delta, \beta, M_C, k) \equiv \pi_{F_i}^{C*} - \pi_{F_i}^{M*} = \frac{k(1-\beta)\left(\beta M_C + (1-\beta)\Delta\right)\left(M_C - \Delta\right)}{\left(k + \beta M_C + (1-\beta)\Delta\right)^2} - \frac{(1-\beta)\beta M^2}{k}.$$

If $\Psi(\Delta, \beta, M_C, k) \geq 0$, firm profits are greater under competition than under monopoly.

Set $M = 1$ and consider the function $\Delta^F(\beta, M_C, k)$ implicitly defined by $\Psi(\Delta, \beta, M_C, k) = 0$. Note first that $\Psi(\Delta, \beta, M_C, k) \geq 0$ if and only if

$$\widehat{\Psi}(\Delta, \beta, M_C, k) \equiv k^2(1-\beta)\left(\beta M_C + (1-\beta)\Delta\right)\left(M_C - \Delta\right) - (1-\beta)\beta\left(k + \beta M_C + (1-\beta)\Delta\right)^2 \geq 0.$$

By direct calculation, we have that

$$\begin{aligned}\widehat{\Psi} \quad = \quad & \left(-k^2(1-\beta)^2 - (1-\beta)^3\beta\right)\Delta^2 + k^2(1-\beta)\beta M_C^2 - (1-\beta)\beta\left(k + \beta M_C\right)^2 \\ & + \left(-k^2(1-\beta)\beta M_C + k^2(1-\beta)^2 M_C - 2(1-\beta)^2\beta\left(k + \beta M_C\right)\right)\Delta.\end{aligned}$$

Thus, $\widehat{\Psi}$ is a quadratic function of Δ. Consider the coefficient of the Δ^2 term given by

$$-k^2(1-\beta)^2 - (1-\beta)^3\beta \leq 0.$$

This implies that $\widehat{\Psi}$ is a concave function of Δ. Setting $\widehat{\Psi}=0$, gives

$$\begin{aligned}\Delta_1^F(\beta,M_C,k) &= \frac{M_C k^2(1-2\beta)-2\beta(1-\beta)(k+\beta M_C)-\sqrt{k^3(kM_C^2-4\beta(1-\beta)(k+M_C))}}{2(1-\beta)(\beta+k^2-\beta^2)},\\ \Delta_2^F(\beta,M_C,k) &= \frac{M_C k^2(1-2\beta)-2\beta(1-\beta)(k+\beta M_C)+\sqrt{k^3(kM_C^2-4\beta(1-\beta)(k+M_C))}}{2(1-\beta)(\beta+k^2-\beta^2)}.\end{aligned}$$

with $\Delta_1^F \le \Delta_2^F$. Let $\widehat{\Delta}^F \equiv \Delta_1^F(\beta_1,M_C,k)=\Delta_2^F(\beta_1,M_C,k)$. Concavity of $\widehat{\Psi}$ in Δ implies that $\widehat{\Psi}\ge 0$ if and only if $\Delta_1^F \le \Delta \le \Delta_2^F$. Note that Δ_1^F and Δ_2^F are real when $kM_C^2-4\beta(1-\beta)(k+M_C) = (4k+4M_C)\beta^2-(4k+4M_C)\beta+kM_C^2>0$, which is a convex function of β. Thus Δ_1^F and Δ_2^F are real when $0\le\beta\le\beta_1^F$ and $\beta>\beta_2^F$ where

$$\begin{aligned}\beta_1^F &= \frac{4k+4M_C-4\sqrt{(k+M_C)(k+M_C-kM_C^2)}}{2(4k+4M_C)},\\ \beta_2^F &= \frac{4k+4M_C+4\sqrt{(k+M_C)(k+M_C-kM_C^2)}}{2(4k+4M_C)}.\end{aligned}$$

For $\beta>\beta_2^F$, Δ_1^F and Δ_2^F are non-positive. Note that β_1^F is real if and only if $\left(k+M_C-kM_C^2\right)>0$, that is, $M_C<\frac{1}{2k}\left(1+\sqrt{(1+4k^2)}\right)$. Hence, when $M_C<\frac{1}{2k}\left(1+\sqrt{(1+4k^2)}\right)$, Δ_1^F and Δ_2^F are real for

$$0\le\beta\le\beta_1^F=\frac{1}{2(4k+4M_C)}\left(4k+4M_C-4\sqrt{(k+M_C)(k+M_C-kM_C^2)}\right).$$

If, instead, $\beta_1^F<\beta\le 1$, then $\widehat{\Psi}<0$ for all Δ. We can now show that $0\le\Delta_1^F\le M_C$. Note that

$$\begin{aligned}\Delta_1^F-M_C &= \frac{M_Ck^2(1-2\beta)-2\beta(1-\beta)(k+\beta M_C)-\sqrt{k^3(kM_C^2-4\beta(1-\beta(k+M_C))}}{2(1-\beta)(\beta+k^2-\beta^2)}-M_C\\ &= -\frac{k(kM_C+2\beta)-2\beta^2(M_C+k)+2\beta M_C+\sqrt{k^3(kM_C^2-4\beta(1-\beta(k+M_C))}}{2(1-\beta)(\beta+k^2-\beta^2)}\\ &< 0,\end{aligned}$$

since $k>0$. Set now $M_C=1$. Note that $1<\frac{1}{2k}\left(1+\sqrt{(1+4k^2)}\right)$ for all k. Differentiating Δ_1^F with respect to β, we obtain

$$\begin{aligned}\frac{\partial\Delta_1^F}{\partial\beta} &= -\frac{\left(2k\beta+2\beta^2+2\beta k^2-2\beta^3-k^2-2k\beta^2+A\right)\left(2k^2-6\beta^2-2+8\beta\right)}{B^2}+\\ &\quad\frac{2k+4\beta+2k^2-6\beta^2-4k\beta+\frac{1}{2A}\left(-4k^4+8k^4\beta-4k^3+8\beta k^3\right)}{B},\end{aligned}$$

where $A\equiv\sqrt{(k^4-4k^4\beta+4\beta^2k^4-4\beta k^3+4\beta^2k^3)}$, and $B\equiv 2\beta k^2-2k^2-2\beta^3-2\beta+4\beta^2$. By direct calculation, it can be shown that $\frac{\delta\Delta_1^F}{\delta\beta}=0$ has no real valued solutions for the relevant parameter values and that it is always positive. This implies that $\Delta_1^F(\beta,M_C,k)$ is a strictly increasing function of β, for

$0 \leq \beta \leq \beta_1^F$, and therefore invertible. Furthermore, by continuity, this will also hold for all M_C in an open neighborhood of $M_C = 1$. Let $[\overline{M}_0, \overline{M}_1]$ be such a neighborhood, with $\overline{M}_0 < 1 < \overline{M}_1$. Let then $\widehat{\beta}_{1F}(M_C, \Delta, k)$ be the inverse function of $\Delta_1^F(\beta, M_C, k)$. Consider now Δ_2^F. Following a similar procedure, differentiating Δ_2^F now with respect to β, we obtain

$$\begin{aligned}\frac{\partial \Delta_2^F}{\partial \beta} &= -\frac{\left(2k\beta + 2\beta^2 + 2\beta k^2 - 2\beta^3 - k^2 - 2k\beta^2 - A\right)\left(2k^2 - 6\beta^2 - 2 + 8\beta\right)}{B^2} + \\ &\quad \frac{2k + 4\beta + 2k^2 - 6\beta^2 - 4k\beta - \frac{1}{2A}\left(-4k^4 + 8k^4\beta - 4k^3 + 8\beta k^3\right)}{B}.\end{aligned}$$

By direct calculation, it can be shown that $\frac{\partial \Delta_2^F}{\partial \beta} = 0$ as well has no real valued solutions for the relevant parameter values and that it is always negative. This implies that $\Delta_2^F(\beta, M_C, k)$ is a strictly decreasing function of β, for $0 \leq \beta \leq \beta_1^F$, and therefore invertible. Furthermore, by continuity, this will also hold for all M_C in an open neighborhood of $M_C = 1$. Let $[\overline{\overline{M}}_0, \overline{\overline{M}}_1]$ be such a neighborhood, with $\overline{\overline{M}}_0 < 1 < \overline{\overline{M}}_1$. Let then $\widehat{\beta}_{2F}(M_C, \Delta, k)$ be the inverse function of $\Delta_2^F(\beta, M_C, k)$. The first part of the proof is concluded by setting $\widehat{\beta}_F(M_C, \Delta, k) = \widehat{\beta}_{1F}(M_C, \Delta, k)$ for $0 \leq \Delta \leq \hat{\Delta}^F$, and $\widehat{\beta}_F(M_C, \Delta, k) = \widehat{\beta}_{2F}(M_C, \Delta, k)$ for $\hat{\Delta}^F \leq \Delta \leq M_0$, and by defining $M_0 \equiv \max\{\overline{M}_0,\ \overline{\overline{M}}_0\}$ and $M_1 \equiv \min\{\overline{M}_1,\ \overline{\overline{M}}_1\}$.

Differentiating Δ_1^F with respect to k and setting again $M_C = 1$, we obtain

$$\begin{aligned}\frac{\partial \Delta_1^F}{\partial k} &= -\frac{\left(2k\beta + 2\beta^2 + 2\beta k^2 - 2\beta^3 - k^2 - 2k\beta^2 + C\right)(4k\beta - 4k)}{D^2} + \\ &\quad \frac{2\beta + 4k\beta - 2k - 2\beta^2 + \frac{1}{2C}\left(4k^3 - 16\beta k^3 + 16\beta^2 k^3 - 12\beta k^2 + 12\beta^2 k^2\right)}{D},\end{aligned}$$

where $C \equiv \sqrt{(k^4 - 4k^4\beta + 4\beta^2 k^4 - 4\beta k^3 + 4\beta^2 k^3)}$ and $D \equiv 2\beta k^2 - 2k^2 - 2\beta^3 - 2\beta + 4\beta^2$. By direct calculation, it can be shown that $\frac{\partial \Delta_1^F}{\partial k} = 0$ has no real valued solutions for $\beta \in [0, 1]$ and $k > 0$, and that it is negative. This implies that $\frac{\partial \Delta_1^F}{\partial k} < 0$,and, since Δ_1^F is a strictly increasing function of β, $\frac{\partial \widehat{\beta}_F}{\partial k} > 0$. Differentiating Δ_2^F with respect to k and setting again $M_C = 1$, we obtain

$$\begin{aligned}\frac{\partial \Delta_2^F}{\partial k} &= -\frac{\left(2k\beta + 2\beta^2 + 2\beta k^2 - 2\beta^3 - k^2 - 2k\beta^2 - C\right)(4k\beta - 4k)}{D^2} \\ &\quad + \frac{2\beta + 4k\beta - 2k - 2\beta^2 - \frac{1}{2C}\left(4k^3 - 16\beta k^3 + 16\beta^2 k^3 - 12\beta k^2 + 12\beta^2 k^2\right)}{D}.\end{aligned}$$

By direct calculation, it can be shown that $\frac{\partial \Delta_2^F}{\partial k} = 0$ has no real valued solutions for $\beta \in [0, 1]$ and $k > 0$, and that it is positive. This implies that $\frac{\partial \Delta_2^F}{\partial k} > 0$, and therefore that $\frac{\partial \widehat{\beta}_F}{\partial k} > 0$.

Proof of Proposition 3

Given firm profits with the firm-specific technology, (4), and the competitive technology, (11), define

$$\Psi(\Delta, \beta, M_C, k) \equiv \pi_{F_i}^{C*} - \pi_{F_i}^{M*} = \frac{k(1-\beta)\left(\beta M_C + (1-\beta)\Delta\right)(M_C - \Delta)}{\left(k + \beta M_C + (1-\beta)\Delta\right)^2} - \frac{(1-\beta)\beta M^2}{k}.$$

If $\Psi(\Delta,\beta,M_C,k)\geq 0$, both firms prefer the competitive technology to their firm-specific technology. In the proof of Proposition 2 above, we already proved that $\Psi(\Delta,\beta,M_C,k)\geq 0$ for $\beta\leq\widehat{\beta}_F(M_C,\Delta,k)$ and for $M_C\in(M_0,M_1)$,. Hence, the firms choose the competitive technology if $\beta\leq\widehat{\beta}_F(M_C,\Delta,k)$, giving (i). If, instead, $\Psi(\Delta,\beta,M_C,k)<0$, which happens for $\beta>\widehat{\beta}_F(M_C,\Delta,k)$ given the equilibrium strategy of firm 2, it is optimal for firm 1 to choose the competitive technology if $M_C\geq 1$, since $\pi_{F_1}^{C*}=\frac{(1-\beta)\beta M_C^2}{k}\geq\frac{(1-\beta)\beta}{k}=\pi_{F_1}^{M*}$ for $M_C\geq 1$, and to choose the firm-specific technology otherwise. Given the choice of firm 1, firm 2 will always find it optimal to choose its firm-specific technology, giving (ii).

Proof of Proposition 4

In the proof of Proposition 2 we have shown that $\widehat{\beta}_F(M_C,\Delta,k)$ is a single valued function for $M_C\in(M_0,M_1)$, $\Delta\in[0,M_C]$, and $k>0$. Thus, the function $\widehat{\beta}_F(M_C,\Delta,k)$ is the unique solution to the system of simultaneous equations

$$\begin{aligned}\phi(\Delta,\beta,M_C,k) &\equiv \Pi-\pi_{F_i}^{C}(\Delta,\beta,M_C,k)=0,\\ \psi(\Delta,\beta,M_C,k) &\equiv \Pi-\pi_{F_i}^{M}(\beta,k)=0.\end{aligned}$$

By the implicit function theorem, we have that

$$\frac{\widehat{\partial\beta}_{F_i}}{\partial M_C}=-\frac{\begin{vmatrix}1 & -\frac{\partial\pi_{F_i}^C}{\partial M_C}\\ 1 & 0\end{vmatrix}}{\begin{vmatrix}1 & -\frac{\partial\pi_{F_i}^C}{\partial\beta}\\ 1 & -\frac{\partial\pi_{F_i}^M}{\partial\beta}\end{vmatrix}}=\frac{\frac{\partial\pi_{F_i}^C}{\partial M_C}}{\frac{\partial\pi_{F_i}^M}{\partial\beta}-\frac{\partial\pi_{F_i}^C}{\partial\beta}},$$

where uniqueness of the solution implies that $\frac{\partial\pi_{F_i}^M}{\partial\beta}-\frac{\partial\pi_{F_i}^C}{\partial\beta}>0$. By direct differentiation, we have

$$\frac{\partial\pi_{F_i}^C}{\partial M_C}=(1-\beta)k\frac{k(2\beta(M_C-\Delta)+\Delta)+M_C\Delta\beta+\Delta^2(1-\beta)}{(k+\beta M_C+(1-\beta)\Delta)^3}>0,$$

proving part (i). Part (ii) of the proposition follows from the part of Proposition 2 which proves that $\frac{\partial\Delta_1^F}{\partial\beta}>0$ and $\frac{\partial\Delta_2^F}{\partial\beta}<0$.

Proof of Proposition 5

The proof of this proposition follows the proofs of Proposition 2 and 3. Given total profits with the firm-specific technology, (4), and the competitive technology, (12), let

$$\Phi(\Delta,\beta,M_C,k)\equiv\pi_{T_i}^{C*}-\pi_{T_i}^{M*}=k(\beta M_C+(1-\beta)\Delta)\frac{M_C(2-\beta)-(1-\beta)\Delta}{2\left(k+\beta M_C+(1-\beta)\Delta\right)^2}-\frac{(2-\beta)\beta}{2k}.$$

If $\Phi(\Delta,\beta,M_C,k)\geq 0$, both firms prefer the competitive technology to their firm-specific technologies. If, instead, $\Phi(\Delta,\beta,M_C,k)<0$, given the equilibrium strategy of Firm 2, it is optimal for Firm 1 to choose the competitive technology if $M_C\geq 1$, as $\pi_{T_1}^{C*}=\frac{3M_C^2}{8k}\geq\frac{3}{8k}=\pi_{T_1}^{M*}$ for $M_C\geq 1$, and to choose the firm-specific technology otherwise. Firm 2, instead, will always choose the firm-specific technology, regardless of whether Firm 1 has chosen its firm-specific technology or the competitive technology.

Consider now the function $\widehat{\Delta}(\beta,M_C,k)$ implicitly defined by $\Phi(\Delta,\beta,M_C,k)=0$. Note first that $\Phi(\Delta,\beta,M_C,k)\geq 0$ if and only if

$$\widehat{\Phi}(\Delta,\beta,M_C,k)\equiv k^2(\beta M_C+(1-\beta)\Delta)(M_C(2-\beta)-\Delta(1-\beta))-(2-\beta)\beta\left(k+\beta M_C+(1-\beta)\Delta\right)^2\geq 0.$$

By direct calculation, we have that

$$\begin{aligned}\widehat{\Phi} &= -(1-\beta)^2(\beta M_C^2(2-\beta)+k^2)\Delta^2\\ &\quad -2M_C(\beta-1)(k^2(1-\beta)-\beta M_C(2-\beta)(M_C\beta+k))\Delta\\ &\quad -\beta^2M_C^3(2-\beta)(M_C\beta+2k).\end{aligned}$$

Thus, $\widehat{\Phi}$ is a quadratic, concave function of Δ since the coefficient of the term Δ^2 is negative. Setting $\widehat{\Phi}=0$, gives

$$\begin{aligned}\Delta_1^T(\beta,M_C,k) &= \frac{k^2M_C(1-\beta)-(2-\beta)\beta(\beta M_C+k)-\sqrt{k^3(kM_C^2-(2-\beta)\beta(k+2M_C))}}{(k^2+\beta(2-\beta))(1-\beta)},\\ \Delta_2^T(\beta,M_C,k) &= \frac{k^2M_C(1-\beta)-(2-\beta)\beta(\beta M_C+k)+\sqrt{k^3(kM_C^2-(2-\beta)\beta(k+2M_C))}}{(k^2+\beta(2-\beta))(1-\beta)}.\end{aligned}$$

with $\Delta_1^T\leq\Delta_2^T$. Let $\widehat{\Delta^T}=\Delta_1^T(\beta_1,M_C,k)=\Delta_2^T(\beta_1,M_C,k)$. Concavity of $\widehat{\Phi}$ in Δ implies that $\widehat{\Phi}\geq 0$ if and only if $\Delta_1^T\leq\Delta\leq\Delta_2^T$. Note that Δ_1^T and Δ_2^T are real when $kM_C^2-(2-\beta)\beta(k+2M_C)=\left(2k^3M_C+k^4\right)\beta^2+\left(-2k^4-4k^3M_C\right)\beta+k^4M_C^2>0$, which is a convex function of β. Thus Δ_1^T and Δ_2^T exist when $0\leq\beta\leq\beta_1^T$ and $\beta>\beta_2^T$ where

$$\begin{aligned}\beta_1^T &= \frac{1}{k+2M_C}(k+2M_C-\sqrt{(k+2M_C)(k+2M_C-kM_C^2)}),\\ \beta_2^T &= \frac{1}{k+2M_C}(k+2M_C+\sqrt{(k+2M_C)(k+2M_C-kM_C^2)})>1.\end{aligned}$$

Note that β_1^T exists if and only if $k+2M_C-kM_C^2>0$, that is, $M_C<\frac{1+\sqrt{1+k^2}}{k}$. Hence when $M_C<\frac{1+\sqrt{1+k^2}}{k}$, Δ_1^T and Δ_2^T are real for

$$0\leq\beta\leq\beta_1^T=\frac{1}{k+2M_C}(k+2M_C-\sqrt{(k+2M_C)(k+2M_C-kM_C^2)}).$$

If $\beta_1^T < \beta \leq 1$, then $\widehat{\Phi} < 0$ for all Δ. We can now show that $0 \leq \Delta_1^T \leq M_C$. Note that

$$\begin{aligned}\Delta_1^T - M_C &= \frac{k^2 M_C(1-\beta) - (2-\beta)\beta(\beta M_C + k) - \sqrt{k^3(kM_C^2 - (2-\beta)\beta(k+2M_C))}}{(k^2+\beta(2-\beta))(1-\beta)} - M_C \\ &= \frac{-\beta(k(2-\beta) + M_C(1-\beta)) - \sqrt{k^3(kM_C^2 - (2-\beta)\beta(k+2M_C))}}{(k^2+\beta(2-\beta))(1-\beta)} < 0,\end{aligned}$$

since $k > M_C$. Set now $M_C = 1$. Note that $1 < \frac{1+\sqrt{1+k^2}}{k}$. Differentiating Δ_1^T with respect to β, we obtain

$$\begin{aligned}\frac{\partial \Delta_1^T}{\partial \beta} &= \frac{(-2(1+k)\beta^4 + (8(1+k) - \sqrt{E})\beta^3 + (9\sqrt{E} - (1+k)(8+2k^2))\beta^2}{2(k^2+\beta(2-\beta))^2(1-\beta)^2} \\ &\quad - \frac{14\sqrt{E} - k^2(4k + \sqrt{E} + 4)\beta - k^2(4k+4+3\sqrt{E}) + 4\sqrt{E}}{2(k^2+\beta(2-\beta))^2(1-\beta)^2},\end{aligned}$$

where $E \equiv k^3\left(\beta\left(k+2\right) - \left(2-\beta\right)\left(k+2\right)\right)$. By direct calculation, it can be shown that $\frac{\delta \Delta_1^T}{\delta \beta} = 0$ has no real valued solutions for the relevant parameter values and that it is always positive. This implies that $\Delta_1^T(\beta, M_C, k)$ is a strictly increasing function of β, for $0 \leq \beta \leq \beta_1^T$, and therefore invertible. Finally, set $\Theta_1 \equiv \Delta_1^T - \Delta_1^F$. By direct calculation, it can be shown that $\Theta_1 = 0$ has solution only for $\beta = 0$ and $\beta = -2k$, and that $\Theta_1 < 0$ for $\beta \in [0,1]$. This implies that $\Delta_1^F > \Delta_1^T$ and thus that $\beta_{2F} < \beta_{2T}$. Furthermore, by continuity, this will also hold for all M_C in an open neighborhood of $M_C = 1$. Let $[\overline{M}_0^T, \overline{M}_1^T]$ be such a neighborhood, with $\overline{M}_0^T < 1 < \overline{M}_1^T$. Let then $\widehat{\beta}_{1T}(M_C, \Delta, k)$ be the inverse function of $\Delta_1^T(\beta, M_C, k)$. Consider now Δ_2^T. Following a similar procedure, differentiating now with respect to β, we obtain

$$\begin{aligned}\frac{\partial \Delta_2^T}{\partial \beta} &= \frac{(-2(1+k)\beta^4 + (8(1+k) + \sqrt{E})\beta^3 + (9\sqrt{E} - (1+k)(8+2k^2))\beta^2}{2(k^2+\beta(2-\beta))^2(1-\beta)^2} \\ &\quad + \frac{(14\sqrt{E} - k^2(4k - \sqrt{E} + 4)\beta - k^2(4k+4-3\sqrt{E}) - 4\sqrt{E}}{2(k^2+\beta(2-\beta))^2(1-\beta)^2}.\end{aligned}$$

By direct calculation, it may be shown that $\frac{\partial \Delta_2^T}{\partial \beta} = 0$ as well has no real valued solutions for the relevant parameter values and that it is always negative. This implies that $\Delta_2^T(\beta, M_C, k)$ is a strictly decreasing function of β, for $0 \leq \beta \leq \beta_1^T$, and therefore invertible. Finally, set $\Theta_2 \equiv \Delta_2^T - \Delta_2^F$. By direct calculation, it can be shown that $\Theta_2 = 0$ has solution only for $\beta = 0$ and $\beta = -2k$, and that $\Theta_2 > 0$ for $\beta \in [0,1]$. This implies that $\Delta_2^F < \Delta_2^T$ and thus that $\beta_{2F} < \beta_{2T}$. Furthermore, by continuity, this will also hold for all M_C in an open neighborhood of $M_C = 1$. Let $[\overline{\overline{M}}_0^T, \overline{\overline{M}}_1^T]$ be such a neighborhood, with $\overline{\overline{M}}_0^T < 1 < \overline{\overline{M}}_1^T$. Let then $\widehat{\beta}_{2T}(M_C, \Delta, k)$ be the inverse function of $\Delta_2^T(\beta, M_C, k)$. The proof is concluded by setting $\widehat{\beta}_T(M_C, \Delta, k) = \widehat{\beta}_{1T}(M_C, \Delta, k)$ for $0 \leq \Delta \leq \hat{\Delta}^T$, and $\widehat{\beta}_T(M_C, \Delta, k) = \widehat{\beta}_{2T}(M_C, \Delta, k)$ for $\hat{\Delta}^T \leq \Delta \leq M_C$, and by defining $M_0^T \equiv \max\{\overline{M}_0^T,\ \overline{\overline{M}}_0^T\}$ and $M_1^T \equiv \min\{\overline{M}_1^T,\ \overline{\overline{M}}_1^T\}$.

Proof of Proposition 6

The first order condition of (21) with respect to I is

$$\frac{2(1-\beta)\beta f^{'}(I^{M*})}{k}-1=0.$$

The first order condition of (22) with respect to I_i is

$$\begin{aligned}&\frac{f'(I_i)k\Theta\left(1-\beta\right)\left[\Delta\left(1-2\beta\right)\left(k-\check{\Delta}\right)+2\beta f(I_i)\left(k-\beta f(I_j)\right)-\Delta\beta\left(2f(I_i)+f(I_j)\right)+2\Delta\beta^2\left(f(I_i)+f(I_j)\right)\right]}{\left[\beta^2 f(I_j)f(I_i)-\left(k+\check{\Delta}\right)\left(k-\check{\Delta}\right)+\beta\check{\Delta}f(I_i)+f(I_j)\right]^2}\\&-\frac{2f'(I_i)\beta k\Theta(\beta f(I_j)+\check{\Delta})\left(1-\beta\right)\left(f(I_i)-\Delta\right)\left[\check{\Delta}\left(k-\check{\Delta}\right)+f(I_i)\beta\left(k-\beta f(I_j)\right)-\beta\check{\Delta}\left(f(I_i)+f(I_j)\right)\right]}{\left[\beta^2 f(I_j)f(I_i)-\left(k+\check{\Delta}\right)\left(k-\check{\Delta}\right)+\beta\check{\Delta}\left(f(I_i)+f(I_j)\right)\right]^3}-1\\=&\ 0,\end{aligned}$$

where $\Theta\equiv k-\beta f(I_j)-\check{\Delta}$. By direct examination of the first order condition under monopoly, it is immediate to verify that I^{M*} tends to zero as β tends to zero. Similarly, it is immediate to verify that, if $k>\Delta$, the symmetric Nash-equilibrium for the investment level, I^{C*}, which is given by setting $I_i^*=I_j^*=I^{C*}$, is bounded away from zero as β tends to zero. This implies that there is a $\tilde{\beta}>0$ such that $I^{C*}>I^{M*}$ for all $\beta\in[0,\tilde{\beta})$.

Proof of Proposition 7

To prove this proposition, we need to characterize the Nash-equilibrium effort levels as a function of firms's choice on whether or not to impose a *no-compete* agreement. Let firm i choose $\Delta_i\in\{0,\Delta\}$, $i=1,2$, where $\Delta_i=0$ if firm i imposes a *no-compete* agreement on its employee, and $\Delta_i=\Delta$ if it chooses not to impose one. Employee i then solves the following problem

$$\max_{e_i}\ \pi_{E_i}(\Delta_1,\Delta_2)\equiv e_i(1-e_j)(\beta+(1-\beta)\Delta_i)-\frac{1}{2}e_i^2;\quad i=1,2;\ \ i\neq j. \tag{A1}$$

Firm $i's$ expected profits are given by

$$\pi_{F_i}(\Delta_1,\Delta_2)\equiv e_i(1-e_j)(1-\beta-(1-\beta)\Delta_i);\quad i=1,2;\ \ i\neq j. \tag{A2}$$

Taking the first order condition of (A1) with respect to e_i and solving for e_i, after setting $e_j=e_i$, gives the Nash-equilibrium level of effort $e_i^*(\Delta_i,\Delta_j)$

$$e_i^*(\Delta_i,\Delta_j)=\frac{(\beta+(1-\beta)\Delta_i)(1-\Delta_i)}{1+\beta(1-\Delta_i-\Delta_j)-(1-\beta)\,\Delta_i\Delta_j};\quad i=1,2;\quad i\neq j. \tag{A3}$$

Plugging (A3) into (A1) and (A2) and setting $\Delta_1 \in \{0, \Delta\}$ and $\Delta_2 \in \{0, \Delta\}$ gives the equilibrium value of expected profits for both firms and employees, as follows. First, when both firms set $\Delta_i = 0$, $i = 1, 2$, we have

$$\pi^*_{E_i}(0,0) = \frac{1}{2}\frac{\beta^2}{(\beta+1)^2}; \quad \pi^*_{F_i}(0,0) = \frac{\beta(1-\beta)}{(\beta+1)^2}; \quad \pi^*_{Ti}(0,0) = \frac{1}{2}\frac{\beta(2-\beta)}{(\beta+1)^2}, \quad i = 1, 2.$$

When Firm 1 sets $\Delta_1 = \Delta$ and Firm 2 sets $\Delta_2 = 0$ we have

$$\begin{aligned}
\pi^*_{E_1}(\Delta,0) &= \frac{(\Delta+\beta(1-\Delta))^2}{2(1+\beta(1-\Delta))^2}, \quad \pi^*_{F_1}(\Delta,0) = \frac{(\Delta+\beta(1-\Delta))(1-\Delta)(1-\beta)}{(1+\beta(1-\Delta))^2}; \\
\pi^*_{T_1}(\Delta,0) &= \frac{(\Delta+\beta(1-\Delta))\,(2-\Delta-\beta(1-\Delta))}{2(1+\beta(1-\Delta))^2}; \\
\pi^*_{E_2}(\Delta,0) &= \frac{\beta^2\,(1-\Delta)^2}{2(1+\beta(1-\Delta))^2}; \quad \pi^*_{F_2}(\Delta,0) = \frac{\beta(1-\beta)(1-\Delta)^2}{(1+\beta(1-\Delta))^2}; \\
\pi^*_{T_2}(\Delta,0) &= \frac{\beta(2-\beta)(1-\Delta)^2}{2(1+\beta(1-\Delta))^2}.
\end{aligned}$$

Finally, when both firms sets $\Delta_i = \Delta$, we have that:

$$\begin{aligned}
\pi^*_{E_i}(\Delta,\Delta) &= \frac{1}{2}\frac{(\Delta+\beta(1-\Delta))^2}{(1+\Delta+\beta(1-\Delta))^2}; \quad \pi^*_{F_i}(\Delta,\Delta) = \frac{(\Delta+\beta-\Delta\beta)(1-\Delta-\beta(1-\Delta))}{(1+\Delta+\beta(1-\Delta))^2}, \\
\pi^*_{Ti}(\Delta,\Delta) &= \frac{1}{2}\frac{(\Delta+\beta-\Delta\beta)(2-\Delta-\beta(1-\Delta))}{(1+\Delta+\beta(1-\Delta))^2}, \quad i = 1, 2.
\end{aligned}$$

Consider first a candidate equilibrium in which both firms impose in equilibrium a *no-compete* agreement on their employees. $(\Delta_1 = 0, \Delta_2 = 0)$ is an equilibrium if and only if $\pi^*_{F_i}(\Delta, 0) \leq \pi^*_{F_i}(0, 0)$ for $i = 1, 2$. By direct comparison, we have that $\pi^*_{Fi}(\Delta, 0) < \pi^*_{Fi}(0, 0)$ if and only if $\Delta \geq \Delta^{NC} \equiv \frac{1-\beta^2}{1+\beta(1-\beta)}$.

Consider now a candidate equilibrium in which neither firm imposes a *no-compete* agreement on its employee. $(\Delta_1 = \Delta, \Delta_2 = \Delta)$ is an equilibrium if and only if $\pi^*_{F_i}(\Delta,\Delta) \geq \pi^*_{F_i}(0,\Delta)$, for $i = 1, 2$. By direct comparison, it can immediately be seen that $\pi^*_{F_i}(\Delta,\Delta) \geq \pi^*_{F_i}(0,\Delta)$ for all values of $0 \leq \Delta \leq 1$ and $0 \leq \beta \leq 1$. Hence, $(\Delta_1 = \Delta, \Delta_2 = \Delta)$ is an equilibrium.

Finally, comparing the equilibrium profits of both firms in the equilibrium without *no-compete* agreements, that is, with $(\Delta_1 = \Delta, \Delta_2 = \Delta)$, and the one with *no-compete* agreements, that is, with $(\Delta_1 = 0, \Delta_2 = 0)$, reveals that $\pi^*_{F_i}(\Delta,\Delta) \geq \pi^*_{F_i}(0,0)$ if and only if $\Delta \leq \Delta^P_F \equiv \frac{3\beta^2+2\beta-1}{3\beta^2-2\beta-1}$. It is straightforward to show that $\Delta^P_F < \Delta^{NC}$ for all $0 \leq \Delta \leq 1$ and $0 \leq \beta \leq 1$.

Proof of Proposition 8

Using the equilibrium total profits obtained in the proof Proposition 7, we have that $(\Delta_1 = 0, \Delta_2 = 0)$ is an equilibrium if and only if $\pi^*_{T_i}(\Delta, 0) \leq \pi^*_{T_i}(\Delta_1 = 0, \Delta_2 = 0)$, for $i = 1, 2$. By direct comparison, we have that $\pi^*_{T_i}(\Delta, 0) > \pi^*_{T_i}(0,0)$ for all $0 \leq \Delta \leq 1$ and $0 \leq \beta \leq 1$. Hence, when the equilibrium decision is made on the basis of maximizing the total profits, $(\Delta_1 = 0, \Delta_2 = 0)$ is never an equilibrium.

Consider now an equilibrium where neither firm imposes a no-compete agreement on its employee. $(\Delta_1 = \Delta, \Delta_2 = \Delta)$ is an equilibrium if and only if $\pi^*_{T_i}(\Delta, \Delta) \geq \pi^*_{T_i}(\Delta, 0)$, for $i = 1, 2$. By direct comparison, we have that $\pi^*_{T_i}(\Delta, \Delta) \geq \pi^*_{T_i}(\Delta, 0)$ for all $0 \leq \Delta \leq 1$ and $0 \leq \beta \leq 1$. Hence, $(\Delta_1 = \Delta, \Delta_2 = \Delta)$ is an equilibrium.

Comparing the equilibrium value of total profits in the equilibrium with $(\Delta_1 = \Delta, \Delta_2 = \Delta)$ to that in the equilibrium with $(\Delta_1 = 0, \Delta_2 = 0)$ reveals that $\pi^*_{T_i}(\Delta, \Delta) \geq \pi^*_{T_i}(0,0)$ if and only if $\Delta \leq \Delta^P_T \equiv 2\frac{2\beta^2+\beta-1}{4\beta^2-3\beta-1}$.

Proof of Lemma 10

From the reaction function equation (25), the equilibrium value of e^{HM*} is obtained by setting $e^{HM} = \frac{\beta(2-e^{HM}(2-\beta))}{2k}$, and solving for e^{HM}. Substituting (26) into (23) and (24) gives (27) and (28).

Proof of Lemma 11

The proof follows from direct comparison of (26) with (9).

Proof of Proposition 9

Given firm profits under the horizontal merger, (28), and firm profits under the competitive technology, (11), let

$$\Gamma(\Delta, \beta) \equiv \pi^{C*}_{F_i} - \frac{1}{2}\pi^{HM*}_F = \frac{(1-\beta)\left(\beta + (1-\beta)\Delta\right)(1-\Delta)}{\left(1+\beta+(1-\beta)\Delta\right)^2} - \frac{\beta(1-\beta)(2+\beta)}{2\left(1+\beta(1-\frac{\beta}{2})\right)^2}.$$

If $\Gamma(\Delta, \beta) < 0$, both firms prefer the horizontal merger to the competitive technology. Consider now the function $\widetilde{\Delta}(\beta)$ implicitly defined by $\Gamma(\Delta, \beta) = 0$. Note first that $\Gamma(\Delta, \beta) \geq 0$ if and only if

$$\widehat{\Gamma}(\Delta, \beta, k) \equiv \left(\beta + (1-\beta)\Delta\right)\left((2-\beta) - (1-\beta)\Delta\right) 2\left(1 + \beta(1-\frac{\beta}{2})\right)^2$$
$$-2\beta(1-\beta)(2+\beta)(1+\beta+(1-\beta)\Delta)^2 \geq 0.$$

By direct calculation, we have that

$$\widehat{\Gamma} = \frac{1}{2}\left(2-\beta^2\right)\left(\beta^2 - 8\beta - 2\right)\left(\beta - 1\right)^2\Delta^2 - \left(8\beta + 8\beta^2 - 16\beta^3 - 5\beta^4 + 10\beta^5 - \beta^6 - 4\right)\Delta + 2\beta^4 - 2\beta^3 + 5\beta^5 - \frac{1}{2}\beta^6.$$

Thus, $\widehat{\Gamma}$ is a quadratic and concave function of Δ since the coefficient of the term Δ^2 is always negative.

Setting $\widehat{\Gamma}=0$, gives two solutions $\Delta_1^{HM}(\beta,k)$ and $\Delta_2^{HM}(\beta,k)$, with $0<\Delta_1^{HM}\leq\Delta_2^{HM}$, where

$$\begin{aligned}
\Delta_1^{HM}(\beta) &= \frac{\beta\left(2\beta^4-13\beta^3-4\beta^2+20\beta+8\right)-4+\sqrt{\left(\beta^2+10\beta-2\right)\left(\beta^2+2\beta-2\right)\left(\beta^2-2\beta-2\right)^2}}{2\left(\beta-1\right)\left(\beta\left(\beta-6\right)\left(\beta^2-2\right)+4\right)}, \\
\Delta_2^{HM}(\beta) &= \frac{\beta\left(2\beta^4-13\beta^3-4\beta^2+20\beta+8\right)-4-\sqrt{\left(\beta^2+10\beta-2\right)\left(\beta^2+2\beta-2\right)\left(\beta^2-2\beta-2\right)^2}}{2\left(\beta-1\right)\left(\beta\left(\beta-6\right)\left(\beta^2-2\right)+4\right)}.
\end{aligned}$$

This implies that the two firms will not merge and thus stay separate if and only if $\Delta_1^{HM}<\Delta<\Delta_2^{HM}$, and will merge otherwise. Note that the roots Δ_1^{HM} and Δ_2^{HM} are real for $0\leq\beta\leq-5+3\sqrt{3}$. By direct calculation it is possible to show that $\frac{\partial\Delta_1^{HM}}{\partial\beta}>0$ and that $\frac{\partial\Delta_2^{HM}}{\partial\beta}<0$ for $0\leq\beta<-5+3\sqrt{3}$. This implies that $\Delta_1^{HM}(\beta)$ is invertible; define then $\beta_1^{HM}(\Delta)$ the inverse function of $\Delta_1^{HM}(\beta)$. Similarly, $\Delta_2^{HM}(\beta)$ is invertible, and define $\beta_2^{HM}(\Delta)$ the inverse function of $\Delta_2^{HM}(\beta)$. Let $\hat{\Delta}^{HM}$ be the value at which $\Delta_1^{HM}(\beta)=\Delta_2^{HM}(\beta)$. Thus, let $\beta^{HM}(\Delta)\equiv\beta_1^{HM}(\Delta)$ for $0\leq\Delta\leq\hat{\Delta}^{HM}$, and let $\beta^{HM}(\Delta)\equiv\beta_2^{HM}(\Delta)$ for $\hat{\Delta}^{HM}\leq\Delta\leq 1$, concluding the proof.