CMCDS Premia Implicit in the Term Structure of Corporate CDS Spreads*

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Abstract

Credit default risk for an obligor can be hedged away with either a credit default swap (CDS) contract or a constant maturity credit default swap (CMCDS) contract. An investor should be indifferent to the instrument used since both cover the same risk with identical payoffs. On a large universe of obligors, we find strong evidence that there is persistent difference in the hedging cost associated with the two comparable contracts. Between 2001 and 2006 it would have been more profitable to sell CDS and buy CMCDS. In addition, the implied forward CDS rates are unbiased estimates of the future spot CDS rates.

Keywords: Constant Maturity Credit Default Swaps, Forward Credit Spreads, Convexity Adjustment, Forward Rate Unbiasedness Hypothesis

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Abstract

Credit default risk for an obligor can be hedged away with either a credit default swap (CDS) contract or a constant maturity credit default swap (CMCDS) contract. An investor should be indifferent to the instrument used since both cover the same risk with identical payoffs. On a large universe of obligors, we find strong evidence that there is persistent difference in the hedging cost associated with the two comparable contracts. Between 2001 and 2006 it would have been more profitable to sell CDS and buy CMCDS. In addition, the implied forward CDS rates are unbiased estimates of the future spot CDS rates.

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Credit default swaps (CDS) have been instrumental in the increased trading in structured credit financial markets until the beginning of 2007 when the sub-prime crisis has started to develop. The British Bankers Association reported an exponential evolution of the total notional amount traded on global credit derivatives reaching $20 trillion by the end of 2006, British Bankers' Association (2006). The single-name credit default swaps volume as a percentage of total credit derivatives volume was 33% in 2006, being by far the most important instrument in credit markets.

Following the analogy with the constant maturity swap (CMS) contract, another traded credit derivative is the constant maturity credit default swap (CMCDS). In such a contract the buyer pays a premium (spread) in exchange for protection. While in a CDS the spread is fixed, in a CMCDS the spread is floating and calculated according to an indexing mechanism. In particular, the spread is set equal to the observed reference CDS spread at each reset date, multiplied by a factor known as the participation rate (PR). The reference constant maturity CDS spread may have different maturity to the one of the contract itself and hence one could trade for instance a 5-year CMCDS referenced by the 3-year or 7-year CDS spread.

The CMCDS instrument allows economic agents to take views on the future shape of the CDS curve. Moreover, combining a CDS and a CMCDS with the same reference entity leads to the complete elimination of credit default risk for that obligor, allowing investors to isolate spread risk (i.e. the risk of changes in the premium not related to an actual credit event) and to hedge default risk. In addition, CMCDS are useful for protection sellers to hedge against spread widening risk. The liquidity of this contract is largely unknown but the above features guarantee its survival and further development alongside the single-name CDS contract.

One stream of the literature on CDS has focused on issues like the validity of the theoretical equivalence of CDS prices and credit bond spreads and the determinants of credit default swap changes[1]. Duffie (1999) and Hull and White (2000) point out that the credit default swap spread for a corporate should be very close to the spread of a par yield bond issued by the reference entity over the par yield risk-free rate to avoid arbitrage between the cash and the synthetic markets. The validity of the theoretical equivalence of CDS spreads and bond yield spreads is tested in Blanco, Brennan, and Marsh (2005). Using a dataset of 33 U.S. and European investment-grade firms, authors find that the parity relation holds on average over time for most companies, implying that the bond and CDS markets may price credit risk equally. Deviations from parity are found only for three European firms, for which CDS prices are substantially higher than credit spreads for long periods of time. These cases are attributed to a combination of imperfections in the contract specification of CDS and measurement errors in computing the credit spread. For all the other companies they find only short-lived deviations from parity in the sample. A possible explanation is that the CDS market leads the bond market in determining the price of credit risk.

The relationship between credit default swaps and corporate spreads is investigated also in Longstaff, Mithal, and Neis (2005). The authors use information on credit default swaps to obtain direct measures of the size of the default and nondefault components in corporate spreads. From CDS premia for 5-year contracts and the corresponding corporate bond prices for 68 firms traded during the period March 2001–October 2002, the default component accounts for more than 50% of the total corporate spread, even for the highest-rated investment-grade firms.

Norden and Weber (2004) apply traditional event study methodology to show that CDS markets anticipate rating downgrades and that anticipation starts approximately 60-90 days before the announcement day. This result is consistent with Hull, Predescu, and

[1] The relevant literature on the determinants of credit default swap changes includes Tang and Yan (2008); Ericsson, Jacobs, and Oviedo (2008); Zhang, Zhou, and Zhu (2006) and Cao, Yu, and Zhong (2006).

White (2004) who confirm that reviews for downgrade contain also significant information. Pan and Singleton (2008) use a full term structure of sovereign CDS spreads to derive the market-implied default intensity and also the implicit loss rate. They argue that a lognormal process for the default intensity (as opposed to a square-root process used in Longstaff, Mithal, and Neis, 2005) is capable of capturing most of the variation in the term structure of spreads.

An important research issue is the identification of the credit instrument to use for protection against default risk. If supply and demand conditions lead to an imbalanced market, it would be useful to know whether it is more cost effective to pay a floating premium spread rather than fixed[2]. This paper provides compelling evidence that this is the case in credit derivatives markets for corporates in the US. In a nutshell, we show that it is possible to have a CDS curve built from no-arbitrage prices and at the same time have fair CMCDS prices that provide statistical arbitrage. However, CMCDS prices are traded over the counter and the data necessary to extract the market's behavioral masterprint is not widely available. In this paper, we calculate the fair CMCDS prices for a large database of obligors for which market CDS premia is available. The calculation requires bootstrapping the survival probability curve from the observed CDS quotes. To this end both nonparametric (e.g. piecewise constant hazard rates) and parametric (Nelson-Siegel interpolation and a method driven by an Ornstein-Uhlenbeck (OU) process for the hazard rates) methods common in practice are implemented. On a large universe of obligors one should expect *ex ante* that there is no difference which contract is used to hedge default risk. Nevertheless, we identify, *ex post*, the statistical credit arbitrage that had been available between 2001 and 2006, in terms of the number of obligors, size of profits that could have been made and the timing of the opportunities.

[2]This situation has been already encountered in interest rate markets as documented by Brooks (2000) who showed *ex post* that for the interest rate swap market in the 1990s it was profitable net to pay floating and receive fixed.

Our work differs from Pan and Singleton (2008), where the focus is on sovereign credit risk, and from Blanco, Brennan, and Marsh (2005) and Longstaff, Mithal, and Neis (2005), where the comparison is between the synthetic and cash credit markets, in that we investigate arbitrage between two synthetic credit markets for corporates. In addition, our sample of corporate reference entities is larger than used previously in the literature, with market panel data for approximately 200 obligors covering investment grade and non-investment grade companies. A final point we address in this paper is about testing the forward unbiasedness hypothesis for the forward credit default swap curves calibrated as an intermediary step to determine the CMCDS fair price.

The remainder of this paper is organized as follows. The next section describes the pricing of CDS and CMCDS in practice and the convexity adjustment for the latter contract. The dataset used for calibration and an illustrative example regarding calibration issues are shown in Section II. The results of the statistical arbitrage analysis based on a type of buy and hold trading (static) strategy and also on a dynamic day by day investment are reported in Section III. In Section IV we test the forward unbiasedness hypothesis and the final section concludes.

I Market Models for CDS and CMCDS Pricing

In this section we describe how premia for CDS and CMCDS contracts are derived. The survival probabilities are inferred from the market CDS spreads and subsequently used to determine the participation rate driving the CMCDS premium.

A The Pricing Framework for CDS and CMCDS

There is a huge body of literature on the pricing of credit risk, spanning from the class of structural models initiated by Merton (1974) and Black and Cox (1976) to the reduced form framework described by Jarrow and Turnbull (1995), Jarrow, Turnbull, and Lando (1997), and Duffie and Singleton (1999). An advanced formalisation for valuation of single-name credit derivatives is presented in Jamshidian (2004), where the general subfiltration approach of Jeanblanc and Rutkowski (2000) to modelling default risk, containing the Cox-process setting of Lando (1998), is integrated with a numéraire invariant approach.

The methodology of for the valuation of CDS of Hull and White (2000) is widely applied. Consider a CDS contract with periodic premium $S(0,T)$ to be paid at times $s_1 < s_2 < \ldots < s_N = T$ or until default, in exchange for a single protection payment to be made at the default time τ, provided that $\tau \in (s_0, s_N]$. Let θ_t be the risk neutral default probability density at time t, so that the probability of default in $[0,T]$ is $\int_0^T \theta_t \mathrm{d}t$. The probability that no credit event occurs up to time t is $\pi_t = 1 - \int_0^t \theta_u \mathrm{d}u$. Denoting by R the recovery rate upon default, the periodic premium to be paid by the buyer of the CDS when the risk-free rate is constant and equal to r is

$$S(0,T) = \frac{(1-R)\int_0^T DF(u)\theta_u \mathrm{d}u}{\sum_{i=1}^N \Delta(s_i, s_{i-1})\pi_{s_i} DF(s_i) + \int_0^T a_u DF(u)\theta_u \mathrm{d}u} \tag{1}$$

where a_u is the accrual payment at time u, $\Delta(s_i, s_{i-1})$ is the time accrual between the market paying coupon times s_{i-1} and s_i, which are quarterly, and $DF(u) = \exp\left(-\int_0^u r_t dt\right)$ is the discount factor calculated from deterministic interest rate curve $\{r_t, t \geq 0\}$ calibrated from market Libor and swap rates. The denominator is the risky PV01, the value of the premium leg assuming a premium of 1 basis point, with the first term indicating the value of a risky annuity and the second term representing the present value of the accrual payments. The numerator is the expected present value under the risk-neutral measure of the payoff received by the protection buyer.

In practice the integrals in (1) are approximated. We assume that the default intensity is driven by a hazard rate λ. Let us assume a monthly grid $\{u_j : j \text{ non negative integer}\}$

for the time of default, and that the default arrives on average in the middle of the time interval. Thus, the CDS premium spread is calculated as

$$S(0,T) = \frac{(1-R)\sum_{j=1}^{n} DF(u_j)[SP(u_{j-1}) - SP(u_j)]}{\sum_{i=1}^{N} \Delta(s_i, s_{i-1}) DF(s_i) \frac{1}{2}[SP(s_{i-1}) + SP(s_i)]}. \tag{2}$$

where $SP(\cdot)$ denotes the survival probability and n is the number of months up to maturity T.

In what follows, we discuss how to derive a CMCDS premium participation rate on single obligors based on the information from CDS markets. A more advanced theoretically pricing framework is available in Brigo (2005). Closed-form solutions for constant maturity credit default swaps, as well as credit default swaps and credit default swaptions, are derived also in Krekel and Wenzel (2006), where a Libor market model with default risk is used. Further developments on CMCDS pricing can be found in Brigo and Mercurio (2006) and more recently in Li (2007) and Jonsson and Schoutens (2008).

The participation rate impacts on the magnitude of the premia that will be paid under the terms of this contract and its size is strongly related to the slope of the CDS curve. A participation rate not exceeding 100%, reflects the fact that the CDS curve is upward sloping. On the other hand the participation rate can be bigger than 100%, indicating a downward slope for the term structure of CDS spreads. To derive the PR, we exploit the fact that the loss leg from a CMCDS is identical to the loss leg from a CDS on the same obligor and same maturity and thus the the fixed payment legs ought to coincide in their NPV. Hence, when the reference CDS has maturity m, we have

$$\begin{aligned}
\mathrm{PR}\sum_{i=1}^{N} \mathbb{E}_0[S(s_{i-1}, s_{i-1}+m)]\Delta(s_i, s_{i-1}) DF(s_i)\frac{1}{2}[SP(s_{i-1}) + SP(s_i)] \\
= S(0,T)\sum_{i=1}^{N} \Delta(s_i, s_{i-1}) DF(s_i)\frac{1}{2}[SP(s_{i-1}) + SP(s_i)],
\end{aligned}$$

where the right hand side term comes from (2). Therefore the formula for PR that is applied for all corporates in our sample is

$$\mathrm{PR} = \frac{S(0,T)\sum_{i=1}^{N}\Delta(s_i,s_{i-1})DF(s_i)[SP(s_{i-1})+SP(s_i)]}{\sum_{i=1}^{N}\mathbb{E}_0[S(s_{i-1},s_{i-1}+m)]\Delta(s_i,s_{i-1})DF(s_i)[SP(s_{i-1})+SP(s_i)]}. \quad (3)$$

The major problem with (3) is the evaluation of the expected value of future spreads in the denominator. It is clear that, when spreads evolve in a completely deterministic setting, future realised spreads are completely determined from today's spread curve and thus the expected value equals the corresponding forward spread. However, for high volatility names or long maturities a convexity adjustment is required in addition to the forward CDS spread calculation, as described next.

The Forward CDS Spread and the Convexity Adjustment

A long position in a forward default swap gives a credit protection that is active for a period of time in the future at a premium agreed upon today, but paid only during the active period of the contract. The price for a forward contract for default protection during the time period $(t, t+m)$ can be calculated as in Berd (2003):

$$FS(t,t+m) = \frac{S(0,t+m) - \delta(t,t+m)S(0,t)}{1-\delta(t,t+m)} \quad (4)$$

where

$$\delta(t,t+m) \equiv \frac{\mathrm{RiskyPV01}(0,t)}{\mathrm{RiskyPV01}(0,t+m)}.$$

In practice there is a discrepancy between the realised future rate and the implied forward rate. The difference is attributed mainly to a convexity adjustment. This issue has been investigated in mathematical finance especially in interest rate derivatives pricing (see Pelsser, 2003; Benhamou, 2000, 2002; Henrard, 2005a,b). It plays an important role for CMCDS pricing as discussed also in Li (2007) and Jonsson and Schoutens (2008).

Our approach for taking into account a convexity adjustment is to use the default intensity described by the following OU process

$$\mathrm{d}\lambda_t = (k - \alpha\lambda_t)\mathrm{d}t + \sigma \mathrm{d}B_t. \tag{5}$$

Calamaro and Nassar (2004) derive an approximate formula for the expected value of the future spread, when the default intensity follows (5):

$$\mathbb{E}_0[S(s_i, s_i + m)] \approx FS(s_i, s_i + m) + \frac{1}{2}\sigma^2 C_i[FS(s_i, s_i + m) - S(0, m)] \tag{6}$$

with $C_i = \frac{1-\mathrm{e}^{-\alpha s_i}}{k\alpha}$. Then the PR can be rewritten as

$$\mathrm{PR} = \frac{S(0,T)}{\overline{FS}(0,T) + \frac{\sigma^2}{2}\frac{C(0,T)}{D(0,T)}} \tag{7}$$

where

$$D(0,T) = \sum_{i=1}^{n} \Delta(s_i, s_{i-1}) DF(s_i) \frac{1}{2}[SP(s_{i-1}) + SP(s_i)]$$
$$C(0,T) = \sum_{i=1}^{n} \Delta(s_i, s_{i-1}) DF(s_i) \frac{1}{2}[SP(s_{i-1}) + SP(s_i)] C_i[FS(s_{i-1}, s_{i-1} + m) - S(0, m)]$$

and $\overline{FS}(0,T)$ is a weighted average of the forward CDS spreads over the reset dates:

$$\overline{FS}(0,T) = \frac{\sum_{i=1}^{n} \Delta(s_i, s_{i-1}) DF(s_i)[SP(s_{i-1}) + SP(s_i)] FS(s_{i-1}, s_{i-1} + m)}{\sum_{i=1}^{n} \Delta(s_i, s_{i-1}) DF(s_i)[SP(s_{i-1}) + SP(s_i)]}.$$

In this paper, we employ the most popular methods in the industry during our period of investigation rather than the latest credit pricing models in the literature to approximate the fair CMCDS prices that come out of trading over the counter. In the following section we briefly describe the alternative methods used in this paper.

B Extracting Survival Probability Curves

The schedule of fixed payments is quarterly as this is the dominating market standard. The number of quarters fitting into the pricing time grid until maturity T is equal to $k = \left[\frac{n}{3}\right]$, where $[x]$ denotes the integer part of x and n corresponds to the number of months until maturity. It is evident that $k = \frac{n}{3}$ only if $t_v = t_0 \equiv 0$, that is the settlement of the credit contract (t_v) coincides with a credit market quarterly coupon paying date (t_0). The first premium is paid at time t_{n-3k+3} (which coincides with t_3 when n is a multiple of 3). A cash flow diagram is reported in Figure 1 for both the standard CDS contract and the CMCDS contract referencing the same entity. We take into account whether the trading day is within a month of the next credit market paying date or not.

[Figure 1 about here.]

There are four methods (OU process with and without convexity adjustment, piecewise constant hazard rates, Nelson-Siegel interpolation) underpinning our results that are commonly used in practice to infer survival probabilities from CDS market quotes and which are presented next. We refer to Brigo and Mercurio (2006) and O'Kane and Turnbull (2003), for more technical details.

B.1 Fitting the CDS Curve Using an OU Process for the Hazard Rate

With stochastic hazard rates the survival probability up to a time t under the risk-neutral measure is given by

$$SP(t) = \mathbb{E}_0 \left[\exp \left(- \int_0^t \lambda_s \mathrm{d}s \right) \right]. \tag{8}$$

When the hazard rate follows an OU process as in (5) the expectation can be derived in closed form (see for instance Vasicek, 1977; Luciano and Vigna, 2006)

$$SP(t) = \exp[a(t) + b(t)\lambda_0], \tag{9}$$

$$a(t) = -\frac{(b(t)+t)(\alpha k - \frac{\sigma^2}{2})}{\alpha^2} - \frac{\sigma^2}{4\alpha} b(t)^2; \qquad b(t) = \frac{e^{-\alpha t} - 1}{\alpha}. \tag{10}$$

One way to derive this formula is to express the stochastic intensity λ as a function Λ of an affine process X whose dynamics is given by the SDE:

$$\mathrm{d}X_t = f(X_t)\mathrm{d}t + g(X_t)\mathrm{d}\tilde{B}_t$$

where $\tilde{B}$ is a multidimensional Brownian motion and the drift $f(X_t)$ and the covariance matrix $g(X_t)g(X_t)'$ have affine dependence on X_t (see Duffie, Filipović, and Schachermayer, 2003). It can be shown that, under technical conditions (see Duffie and Singleton, 2003), for any $w \in \mathbb{R}$

$$\mathbb{E}_t \left[e^{\int_t^T -\Lambda(X_u)\mathrm{d}u + wX_T} \right] = e^{a(T-t)+b(T-t)X_t} \tag{11}$$

where the coefficients $a(\cdot)$ and $b(\cdot)$ satisfy generalized Riccati ODEs. If we assume that the intensity itself is an affine process as in (5), then we can apply (11) with $w = 0$ and $\Lambda(x) = x$. In this case the ODEs can be solved analytically yielding (10).

Note that the condition $SP(0) = 1$ is automatically satisfied. There are four parameters to calibrate (k, α, σ and λ_0). We follow the standard market practice and we estimate the obligor individual parameters by minimising the squared residual error between the model implied and market CDS spreads.

B.2 Piecewise Constant Hazard Rates

The survival probabilities can be bootstrapped from (2) when there are sufficient maturities for traded contracts to cover the entire set of time points for which survival probabilities must be calculated. One common approach, feasible also in presence of a small number of maturities, advocated by O'Kane and Turnbull (2003) is to assume that the hazard rate curve is piecewise constant. Suppose that the CMCDS contract we are interested in is traded at time t_v and there are CDS market spreads for the same obligor for maturities $T_1, \ldots, T_M$. Denoting $\lambda_1 = \lambda_{0,T_1}$, $\lambda_i = \lambda_{T_{i-1},T_i}$, $i = 2, \ldots, M$, the survival function $SP(T - t_v)$ is then given by[3]

$$-\log SP(T-t_v) = \lambda_1(T-t_v)\mathbb{1}_{[0,T_1)}(T-t_v) + \sum_{i=1}^{M-2}\left[\sum_{j=1}^{i}(\lambda_j - \lambda_{j+1})T_j + \lambda_{i+1}(T-t_v)\right] \times \mathbb{1}_{[T_i,T_{i+1})}(T-t_v) + \left[\sum_{j=1}^{M-1}(\lambda_j - \lambda_{j+1})T_j + \lambda_{i+1}(T-t_v)\right]\mathbb{1}_{[T_{M-1},\infty)}(T-t_v). \quad (12)$$

For each maturity expressed in months a numerical searching algorithm is applied to determine λ_i, $i = 1, \ldots, M$.

B.3 Calibration with Nelson-Siegel Interpolation

Another possibility is to consider a deterministic time-varying hazard rate such that $\int_0^t \lambda(s)\mathrm{d}s = \Psi(t)t$. The role of function $\Psi(t)$ is to capture any term structure variation. One of the common choices for function $\Psi(t)$ is the Nelson and Siegel (1987) function[4]

$$\Psi(t) = \alpha_0 + (\alpha_1 + \alpha_2)\left(\frac{1 - \exp(-\frac{t}{\alpha_3})}{\frac{t}{\alpha_3}}\right) - \alpha_2 \exp\left(-\frac{t}{\alpha_3}\right) \quad (13)$$

[3] $\mathbb{1}_{\{A\}}(x)$ denotes the indicator function that is equal to one if $x \in A$ and zero otherwise.

[4] Markit, the leading data provider from which we obtained our data, employs a similar approach based on Nelson-Siegel interpolation to produce theoretical credit curves when liquidity of data is very low.

This function can generate many different curve shapes. The parameter α_0 is the long term mean of the default intensity. Parameter α_1 is the deviation from the mean, with $\alpha_1 > 0$ implying a downward sloping intensity and $\alpha_1 < 0$ implying an upward sloping term structure. In addition, the reversion rate toward the long-term mean is negatively related to $\alpha_3 > 0$. The parameter α_2 is responsible for generating humps when it is different from zero. Bluhm, Overbeck, and Wagner (2003) argue against using humps as this may lead to overfitting problems. We therefore assume that $\alpha_2 = 0$ and estimate $\alpha_0, \alpha_1, \alpha_3$ only from CDS spread data using a nonlinear optimization algorithm for a suitable minimization function such as sum of squared errors or sum of absolute errors. See Appendix A for details.

II Data Description and Some Examples

A Single Name CDS Data

Our dataset consists of daily single-name composite spreads covering the period January 2001–November 2006 for all corporates traded in US dollar and for maturities 6m, 1y, 2y, 3y, 4y, 5y, 7y, 10y, 15y, 20y, 30y downloaded from Markit, the industry standard provider in credit markets. The composite spread is the average spread for a credit contract from price information provided to Markit by its contributors. Markit applies a series of data quality tests to remove unreliable information from the sample set[5]. For each day and for each obligor there is also a recovery rate reported that we use later in our analysis. Additional information like sector, rating and country are reported as well. Only the CDS market spreads related to senior tier of debt have been retained for liquidity reasons.

Since the CDS prices were followed through according to the quarterly schedule of

[5]Markit only builds composites when there are at least three contributors to each composite. The cleaning process includes testing for stale, flat curves and outlying data. On average Markit rejects approximately 45% of the CDS data received due to failure under any combination of the three criteria above.

payments, we have selected only those reference entities for which at least one coupon payment was scheduled in 2001. After this first filtering, 446 obligors were left. We kept in our sample only the names for which there was data for recovery rates and spreads covering the entire calendar of payments until the end of the survey period. At this stage 295 companies remained in the sample. A further reduction was due to the elimination of obligors with either low liquidity (only one or two maturities traded) or for which we faced numerical convergence problems when fitting the data. The final sample consists of 198 companies for the static analysis and 207 names for the dynamic analysis. Table I describes the cross-classification of the reference entities by the average rating and sectors[6].

[Table I about here.]

B Reference Rate Yield Data

For our empirical analysis, we also need the calculation, at daily frequency and over the entire sample period, of discount curves. While traditionally the government bond yields were the obvious choice, more recently the yield curve build from Libor and swap contracts has been employed as a proxy for the riskless curve. The next best proxy would be the general collateral or repo rates as recommended by Duffie (1999) and Houweling and Vorst (2005) but the maturities for these rates are mainly up to one year. This does not fit our style of analysis which needs discounting from much longer maturities. Sundaresan (1991) pointed out that swap rates have credit premia embedded and moreover there is counterparty risk, although Duffie and Huang (1996) indicate that this is only about one or two basis points. Further discussion on this issue can be found in McCauley (2002) and Collin-Dufresne and Solnik (2001).

[6]Our sample excludes obligors classified as Government.

The discount factor curves are constructed daily from Libor rates with maturities 1 month to 11 months and swap rates with maturities 1y, 2y, 3y, 4y, 5y, 7y, 10y, 20y, 30y. A continuum of discount factors is obtained with log-linear interpolation. The discount factor for $t \in [\mathcal{T}_j, \mathcal{T}_{j+1}]$, $DF(t)$, is given by

$$\log\left(DF(t)\right) = \frac{\mathcal{T}_{j+1} - t}{\mathcal{T}_{j+1} - \mathcal{T}_j} \log\left(DF(\mathcal{T}_j)\right) + \frac{t - \mathcal{T}_j}{\mathcal{T}_{j+1} - \mathcal{T}_j} \log\left(DF(\mathcal{T}_{j+1})\right).$$

Data has been downloaded from Bloomberg and it is widely available for the USD interest rates.

C A Preliminary Illustration

As an illustration of the methods mentioned above to determine the survival probabilities, we now consider the term structure of CDS spreads on a particular day (October 3rd 2005) for three obligors, representative of the companies in our sample: i) a firm with large CDS premia and spreads available for all maturities (Abitibi Consol Inc); ii) a firm with small CDS premia and few maturities available (Microsoft Corp); iii) a firm with small CDS premia and all maturities available (Tesco PLC). Table II reports the market quotes for all three companies.

[Table II about here.]

In order to calculate the CMCDS premium for a particular corporate we need to determine the Libor-swap discount curve and the survival probabilities using one of the methods described in Section I; calculate the entire family of CDS forward curves using (4) and apply (3) to calculate the PR, an indicator of the magnitude of premium to be paid.

Example 1: Nelson Siegel and Piecewise constant hazard rates methods. The parameter estimates $\widehat{\boldsymbol{\alpha}}$ and $\widehat{\boldsymbol{\alpha}}_w$ for the Nelson Siegel approach[7] and the hazard rate underpinning the piecewise constant model are reported in Table III along with the participation rates for a CMCDS contract with maturity five years, where the maturity of the reference floating spread is five years as well. In Figure 2 the corresponding survival probabilities together with the observed CDS spreads are also plotted. Microsoft presents more difficulties for fitting due to lack of data for the entire spectrum of maturities and therefore we cannot apply Nelson-Siegel method with weights for this type of obligor. The main conclusion is that, when data is available for the entire term structure, the three methods have similar accuracy.

[Table III about here.]

[Figure 2 about here.]

Example 2: OU process and Convexity adjustment. Here we assume an OU process for the default intensity and evaluate the impact of the convexity adjustment on the participation rate for the same three obligors mentioned above. The calibrated CDS curves and observed CDS market values are plotted in Figure 3. Given the limited number of observations for Microsoft it is not surprising that the fit is poor and numerical problems are indicated by $\hat{\sigma} = 0$ and $\hat{\lambda}_0 = 0$. The estimated parameters and participation rates are reported in Table IV. The convexity adjustment seems to have a significant impact on the calculation of participation rate for Tesco, with a substantial reduction from 64.82% down to 49.89%, but with no relevant impact for the other two obligors. Therefore, methods that do not take into account convexity are still useful for comparison purposes.

[7] Technical details regarding the estimation of the vector of parameters, with and without weights, are provided in Appendix A.

[Table IV about here.]

[Figure 3 about here.]

To summarise, all three methods provide good fit for CDS spreads. However, though more computationally intensive, the piecewise constant hazard rates is the method which performs best when obligors present a reduced number of maturities.

III Arbitrage Evidence in Credit Markets

The main aim of this section is to explore arbitrage opportunities when CDS and CMCDS contracts are two alternative instruments. A market participant should be indifferent to which instrument to use to hedge default risk. Here, we show that the above conjecture was not true for the period 2001–2006. We base our analysis on a quarterly comparison between the two competing contracts for a large sample of corporates in our dataset. Statistical arbitrage is explored by pairing the two contracts in opposite directions (buy CDS and sell CMCDS) and by looking at what would have been the net cumulative profit and loss for an investor employing a static strategy, similar to buy and hold. Then, we analyse the outcome of a dynamic trading strategy that assumes the investor enters the same trade every day between the 20th September 2001 and the 19th December 2001 and carries until November 2006. With a large universe of obligors we can explore, *ex post*, the credit arbitrage in terms of the number of obligors and the size of profits that could have been made and the timing of the opportunities.

The relative value arbitrage position is monitored across all 20 quarters in the period of study. For each obligor in our sample the quarterly time series for the static strategy $\{y_i : i = 0, \ldots, 19\}$ is calculated as

$$y_i = \mathrm{PR}^{t_0} \times S(t_{3i}, t_{3i} + m) - S(t_0, t_0 + T),$$

where t_0 is the settlement date, PR^{t_0} is the participation rate on the day t_0, and S is the periodic premium of the CDS contract.

For illustration purposes, let us first consider two obligors with liquid CDS curves, namely AT&T and Goldman Sachs Gp Inc. In our analysis, the settlement date is the 20th September 2001, while the maturity of both the CDS and CMCDS contracts is five years ($T = 5$) with the CMCDS contract indexed to a five year CDS ($m = 5$). Table V reports descriptive statistics for y. The two obligors show a contrasting situation. AT&T has a positive mean net spread payments and negative median. The graphs of the time series y and its empirical density depicted in the top of Figure 4 suggest both negative and positive payments with the empirical density centered around zero. This is the typical outcome expected if there is no statistical credit arbitrage. Goldman Sachs however is quite the opposite, with all values between the 5% quantile and 95% quantile negative, under all methods. The graph of the spread payment series and the empirical density (see bottom of Figure 4) confirm that this name provided a great arbitrage opportunity. The illustration using AT&T and Goldman Sachs as reported above is exemplary. In what follows, we investigate whether the synthetic credit universe of corporates in our sample is closer to AT&T than to Goldman Sachs.

[Table V about here.]

[Figure 4 about here.]

A Static Investment Analysis

For all companies in our sample we compute the net cumulative profit/loss (NCPL) that an investor would have realized by selling protection with CMCDS and buying protection via CDS. Both contracts are issued with five years maturity and same settlement date (the 20th September 2001). For each company $j = 1, 2, \ldots, 198$, we compute the NCPL as

$$z_j = \sum_{i=0}^{k-1} \Delta(t_{n-3i}, t_{n-3(i+1)}) \left[\mathrm{PR}_j^{t_0} \times S_j(t_{n-3(i+1)}, t_{n-3(i+1)} + m) - S_j(t_0, t_0 + T) \right] \quad (14)$$

where t_i denotes a payment date, $S_j(u, u+m)$ denotes the CDS spread at time u with maturity term m for company j, and $\mathrm{PR}_j^{t_0}$ is the participation rate for company j at t_0[8].

Table VI reports the summary statistics for the NCPL calculated using the four methods described in Section I indicating that, not only on average the NCPL is negative, but its distribution is skewed towards the range of negative values.

[Table VI about here.]

[Figure 5 about here.]

The plots representing the outcome distribution of z_j and described in Figure 5 confirm for all methods this conclusion. The analysis shows that there has been statistical arbitrage between the CDS market and the CMCDS market overall. The direction of the arbitrage is clearly in favor of *selling* CDS and *buying* CMCDS. Nonetheless, as can be seen from Figure 5, there were also obligors for which it would have been profitable to buy single-name CDS and sell CMCDS with same maturity.

Table VII reports the number of obligors with a positive (negative) NCPL at various threshold. For all methods, we observe that approximately the same percentage of companies has negative NCPL ranging from 85% when using the Nelson-Siegel interpolation,

[8] Following market practice, we applied a cap on the floating payment and we computed the NCPL as

$$z_j^{t_v,\mathrm{cap}} = \sum_{i=0}^{k-1} \Delta(t_{n-3i}, t_{n-3(i+1)}) \Big[\min\left\{800bps, \mathrm{PR}_j^{t_0} \times S_j(t_{n-3(i+1)}, t_{n-3(i+1)} + m)\right\} - S_j(t_0, t_0 + T) \Big].$$

The results obtained in this case do not differ from those calculated using (14).

to 88% with piecewise constant hazard rates, to 90% when we assume a OU process for the evolution of the hazard rate. Finally, when we use the convexity-adjusted version of the OU, we observe 85.5% of companies with negative NCPL.

[Table VII about here.]

From a portfolio analysis point of view, obligors can be ranked according to their NCPL z_j. Table VIII shows a list of the companies with $z_j < -250$ bps and those with $z_j > 250$ bps for all methods applied here; similarly for the -500 and 500 bps and -1000 and 1000 bps bounds. Appendix B lists the five companies with the largest negative and positive NCPL.

[Table VIII about here.]

The large possible difference in cumulative realised profit and loss is somehow surprising, given that both trades cover the same risk of default. The CMCDS financial product is not so much sensitive to the levels of the premia but to the shape of the CDS curve or alternatively the survival curve.

Hasbro Inc is the company with the largest negative NCPL for all methods apart from Nelson-Siegel. On the other hand, with the only exception of the piecewise constant hazard rate approach, all methods indicate that Global Marine Inc is the company with maximum positive NCPL. It is perhaps not very surprising to see that every method indicated General Motors and Ford[9] among the five companies with the largest NCPL z_j.

[9]It is well known that both names have been the source of the credit spreads widening in 2005. Ford Motor Credit Corp has also been discussed in Blanco, Brennan, and Marsh (2005). In our database there are two reference entities related to Ford, one called Ford Motor Co and another one called Ford Motor Credit Co. Although the latter is a subsidiary of the former the two entities are legally separated vis-a-vis a credit event and as such we have treated them separately.

B Dynamic Investment Analysis

In this section, we report the results of the dynamic trading strategy for the two credit protection contracts (CDS and CMCDS), by following the paired trades on a daily basis. Our sample contains only the obligors for which there is data available until the maturity of the five year contract. For each company $j = 1, \ldots, 207$, we calculate the NCPL

$$z_j^{t_v} = \sum_{i=0}^{k-1} \Delta(t_{n-3i}, t_{n-3(i+1)}) \left[\mathrm{PR}_j^{t_v} \times S_j(t_{n-3(i+1)}, t_{n-3(i+1)} + m) - S_j(t_v, t_v + T)\right]$$

for all settlement days t_v between 20th September 2001 and 19th December 2001[10]. For a given company j, n_j denotes the number of days t_v for which we can determine $z_j^{t_v}$. The yardstick measure for comparison is the average net cumulative profit and loss (ANCPL) denoted by $\bar{z}_j$. Each paired trade that starts on any given day within the above period is followed through maturity and the profit and loss is calculated and reported comparatively on an average basis.

Table IX reports descriptive statistics for the ANCPL for the paired trade. The results suggest that overall there was credit arbitrage during the five year period. The magnitude of the ANCPL varies according to the method applied and it seems that the convexity correction could play a major role. The graphical representation of the ANCPL $\bar{z}_j$ corresponding to all obligors in the sample is illustrated in Figure 6. There is a confirmation of the presence of large arbitrage opportunities in both sides of the trade. In particular for the Nelson-Siegel method and the piecewise constant hazard rates 85% of sample has negative ANCPL, with 87% for the OU hazard rate approach without

[10]Following the credit market convention compute the first term of the summation as follows:

$$\begin{aligned} &\Delta(t_3, t_v)\mathbb{1}_{\{t_3 - t_v > 1mth\}} \left[\mathrm{PR}_j^{t_v} \times S_j(t_3, t_3 + m) - S_j(t_v, t_v + T)\right] \\ &+\Delta(t_3, t_v)\mathbb{1}_{\{t_3 - t_v \leq 1mth\}} \left[\mathrm{PR}_j^{t_v} \times S_j(t_6, t_6 + m) - S_j(t_v, t_v + T)\right] \\ &= \Delta(t_3, t_v) \left[\mathrm{PR}_j^{t_v} \left(S_j(t_3, t_3 + m)\mathbb{1}_{\{t_3 - t_v > 1mth\}} + S_j(t_6, t_6 + m)\mathbb{1}_{\{t_3 - t_v \leq 1mth\}}\right) - S_j(t_v, t_v + T)\right] \end{aligned}$$

to take into account the different behavior of the first coupon.

convexity adjustment and only 70% for the one with convexity adjustment.

[Table IX about here.]

[Figure 6 about here.]

As in the previous section in Table X we report the number of companies at various levels of ANCPL. Our analysis clearly indicates the skewness towards the negative levels in the results pointing out to the existent arbitrage during that period.

[Table X about here.]

In Appendix B we list for each computational method the five companies with extreme (negative or positive) values of ANCPL. In Table XI we list obligors with ANCPL in absolute value larger than 500 bps (1000 bps) for all methods of calculation. Once again the two Ford entities and General Motors appear in the list, likely because of the widening of the spreads associated with the rating downgrade in 2005. More surprising is the longer side of the list with companies that would have provided negative ANCPL, with three of them (Hasbro Inc, J C Penney Co Inc and Pennzoil Quaker St Co) showing statistical arbitrage of magnitude over 10%. There seems to be little explanation for this empirical finding other than the fact that the implied forward CDS curves at the time when the participation rate is considered were far away from the realised CDS rates later on.

[Table XI about here.]

From the full set of results reported in this section, there is evidence of the existence of arbitrage opportunities between CDS and CMCDS markets. This conclusion calls for a further investigation on whether the forward CDS rates are unbiased estimators of future spot CDS rates. If the forward rates are biased, this could explain the statistical arbitrage opportunities identified in this paper. On the other hand, if the forward CDS rates are unbiased then the cause of arbitrage may lie elsewhere. In addition, unbiased estimators will give more confidence to apply our analysis with different data. In the next section we test the forward rate unbiasedness hypothesis.

IV Testing the Forward Unbiasedness Hypothesis for CDS Rates

The forward rate unbiasedness hypothesis (FRUH) postulates that the forward rate is an unbiased predictor of the corresponding future spot rate. This hypothesis has been extensively tested for exchange rates (see Liu and Maddala, 1992; Maynard, 2003; Westerlund, 2007, among the others), either by regressing the future spot rate, s_{t+k}, on a constant and the forward rate, f_t, or by checking for a unit slope in a regression of the spot return $s_{t+k} - s_t$ on the forecasting error, $f_t - s_t$, which ought be stationary under the FRUH (see Froot and Frankel, 1989, for instance). An alternative approach could be testing s_{t+k} and f_t for cointegration like in Baillie and Bollerslev (1989) and Hai, Mark, and Wu (1997).

In this paper we adopt the latter approach to a panel data analysis. Therefore we test for cointegration as implied by the FRUH and test for stationarity in the resulting panel forecasting errors $F_j(t_i, t_i + m|\mathcal{F}_{t_{i-1}}) - S_j(t_i, t_i + m)$. The FRUH cannot be rejected if the panel of forecasting errors is found to be stationary. Consistent with the notation used in Section III, t_0 refers to the 20th September 2001, $i = 1, \ldots, 20$ and m is five years. This means that at each coupon paying day, we calculate the forward spread for a contract entered into the next quarter and five years maturity.

We apply the panel unit root test of Pesaran (2007) suitable in presence of a number of obligors much larger than the number of time observations and to take into account the cross-sectional dependence in the data. Let y_{it} ($i = 1, \ldots, M$, $t = 1, \ldots, Q$) denote a variable observed both cross-sectionally and over time. The Pesaran (2007) test statistic we use is defined as $\text{CIPS}(M,Q) = M^{-1}\sum_{i=1}^{M} t_i(M,Q)$, representing the mean of the t-ratios of b_i in the OLS cross-sectionally augmented Dickey-Fuller regression

$$\Delta y_{it} = a_i' \boldsymbol{d}_t + b_i y_{i,t-1} + c_i \bar{y}_{t-1} + d_i \Delta \bar{y}_t + e_{it}, \tag{15}$$

where $\bar{y}_t = M^{-1}\sum_{i=1}^{M} y_{it}$, $\Delta\bar{y}_t = M^{-1}\sum_{i=1}^{M}\Delta y_{it} = \bar{y}_t - \bar{y}_{t-1}$. The vector $\boldsymbol{d}_t$ represents the deterministic component. The relevant case for us is $\boldsymbol{d}_t = 0$, equivalent to no intercept and no trend, but for completeness of our econometric analysis we also consider $\boldsymbol{d}_t = 1$ when there is intercept and no trend, and $\boldsymbol{d}_t = (1,t)'$ with intercept and individual specific time trends. Cross-section dependence is controlled by including the cross-sectional means $\bar{y}_{t-1}$ and $\Delta\bar{y}_t$, in (15). The critical values are obtained from Pesaran (2007, Table II(a)–(c)). We apply the CIPS test statistics when y_{it} represents the realized spreads (S), the forward spreads (FS) and the forecasting error ($S-F$).

[Table XII about here.]

The results are reported in Table XII and they indicate that both the realized spreads and the calculated forward rates are non-stationary. When we look at the panel of forecasting errors, the test lead to a rejection of the null of non-stationarity, with only two exceptions, i.e. the two Nelson Siegel methods when both the intercept and a deterministic trend are included in the panel regression (15). This means that, in general the forward CDS spread calculated using the four methods considered is an unbiased estimator for the future CDS rates. This result is not in contradiction with the fact that the forward CDS rate is not an unbiased estimate of the future CDS rate under the risky forward measure, with the difference given by the convexity adjustment. Our testing is done under the real or physical measure and the results can differ.

V Concluding Remarks

This paper explored the credit statistical arbitrage opportunities that might have existed on US corporate credit markets over the period January 2001–November 2006. A large database of market single-name credit default swap premia was employed to produce the corresponding fair constant maturity credit default prices.

We used four methods to price CMCDS and our results are consistent across all four. We measured the size of the statistical arbitrage through a buy and hold type of static strategy and also its dynamic version consisting of investing daily between two market reset dates. The main conclusion is that it would have been more profitable to sell CDS and buy CMCDS.

We observed trading gains from CDS spreads widening for companies such as Ford and General Motors as a consequence of the rating downgrade in 2005. However, the majority of credit arbitrage identified was in the opposite direction, with most names benefiting from spreads tightening beyond the expected levels implied by the forward curves. A possible explanation is that the strategy of buying CMCDS and selling CDS was very profitable *ex-post* for almost all names because it is implemented in a period of significant decrease in the CDS spreads in the market. The decrease is beyond what the forward rates imply, pointing to the conjecture that there was too much liquidity pumped up in the financial system through various channels, well beyond the actual needs of the real economy. This may have triggered the current crisis in credit markets. If the same strategy would be studied in a period of significant upward trend in CDS spreads, then the reverse strategy (buying CDS and selling CMCDS) may prove profitable *ex-post*.

We also tested whether the forward CDS spreads calculated as part of the CMCDS pricing process were unbiased estimates of the future spot CDS spreads. Using panel data tests we fail to reject the unbiasedness hypothesis. Accepting that the forward CDS rates are unbiased estimates of future spot rates means that the statistical arbitrage identified cannot be attributed to an estimation bias. One may argue that the dynamics of the credit curve was more accelerated than the forward credit curves simply implied.

Our research opens up several avenues to further analysis. An obvious one is a similar investigation for sovereign names. This market has its own characteristics due to the fact that bankruptcy is not a possible event for a sovereign obligor. Additionally, we have not considered reference CDS spreads other than the 5-year maturity: an exploration of other liquid maturities such as 3-year and 10-year may reveal additional empirical linkages between the family of forward CDS curves and the market CDS curves. Finally, the forward unbiasedness hypothesis can be tested on other panel data based on different reference rates and horizons.

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Appendix A Details on Parameter Estimation

OU Process

Given a set of CDS spreads with maturities $\{t_n\}_{n\in\mathcal{M}}$, in order to estimate the vector of parameters $\boldsymbol{\theta} = (\lambda_0, \sigma, k, \alpha)'$, we first compute the theoretical CDS premium spread, $S(0, t_n; \boldsymbol{\theta})$ following formula (2) and then solve the optimization problems[11]

$$\arg\min_{\boldsymbol{\theta}} \sum_{n\in\mathcal{M}} [S(0, t_n) - S(0, t_n; \boldsymbol{\theta})]^2 \quad \text{or} \quad \arg\min_{\boldsymbol{\theta}} \sum_{n\in\mathcal{M}} |S(0, t_n) - S(0, t_n; \boldsymbol{\theta})| \,.$$

subject to the constraints

$$\theta > 0$$

$$SP'(T_M) < 0$$

where T_M is the last maturity (20yr) of the available CDS data and

$$SP'(t) = \frac{k}{\alpha}(1 - \mathrm{e}^{-\alpha t}) + \lambda_0 \mathrm{e}^{-\alpha t} - \frac{\sigma^2}{2\alpha^2}(1 - \mathrm{e}^{-\alpha t})^2$$

Nelson-Siegel

Given $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \alpha_3)'$ in the parameter space $U_{\boldsymbol{\alpha}} \subset \mathbb{R}^3$, we solve the minimization problems

$$\tilde{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}\in U_{\boldsymbol{\alpha}}} \sum_{n\in\mathcal{M}} [S(0, t_n) - S(0, t_n; \boldsymbol{\alpha})]^2 = \arg\min_{\boldsymbol{\alpha}\in U_{\boldsymbol{\alpha}}} \tilde{f}(\boldsymbol{\alpha})$$

or

$$\breve{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}\in U_{\boldsymbol{\alpha}}} \sum_{n\in\mathcal{M}} |S(0, t_n) - S(0, t_n; \boldsymbol{\alpha})| = \arg\min_{\boldsymbol{\alpha}\in U_{\boldsymbol{\alpha}}} \breve{f}(\boldsymbol{\alpha})$$

[11] Also the weighted objective functions of the form (19)–(20) have been considered.

where $S(0,t_n;\boldsymbol{\alpha})$ denotes the theoretical CDS spread maturing at time t_n with a Nelson-Siegel function with parameter $\boldsymbol{\alpha}$. The optimization should be done under the following constraints which identify $U_{\boldsymbol{\alpha}}$:

$$\alpha_0 > 0, \quad \alpha_3 \;>\; 0 \tag{16}$$

$$SP(t) - SP(t+1) \;\geq\; 0 \quad \text{for any } t > 0. \tag{17}$$

The condition (17) is equivalent to

$$\alpha_0 + \alpha_1 \exp\left(-\frac{t}{\alpha_3}\right) \geq 0 \tag{18}$$

which is obtained by imposing that the function $\Psi(t) \times t$ is not increasing. As far as the choice of the function to be minimized, in practice we set $\hat{\boldsymbol{\alpha}} = \tilde{\boldsymbol{\alpha}}$ unless $\tilde{f}(\breve{\boldsymbol{\alpha}}) < \tilde{f}(\tilde{\boldsymbol{\alpha}})$ or $\breve{f}(\breve{\boldsymbol{\alpha}}) < \breve{f}(\tilde{\boldsymbol{\alpha}})$. To reflect the fact that CDS contracts with different maturities may have different levels of liquidity we also consider *weighted* minimization :

$$\tilde{\boldsymbol{\alpha}}_w = \arg\min_{\boldsymbol{\alpha}\in U_{\boldsymbol{\alpha}}} \sum_{n\in\mathcal{M}} w_n \left[S(0,t_n) - S(0,t_n;\boldsymbol{\alpha})\right]^2 = \arg\min_{\boldsymbol{\alpha}\in U_{\boldsymbol{\alpha}}} \tilde{f}_w(\boldsymbol{\alpha}) \tag{19}$$

or

$$\breve{\boldsymbol{\alpha}}_w = \arg\min_{\boldsymbol{\alpha}\in U_{\boldsymbol{\alpha}}} \sum_{n\in\mathcal{M}} w_n \left|S(0,t_n) - S(0,t_n;\boldsymbol{\alpha})\right| = \arg\min_{\boldsymbol{\alpha}\in U_{\boldsymbol{\alpha}}} \breve{f}_w(\boldsymbol{\alpha}). \tag{20}$$

In this case $\hat{\boldsymbol{\alpha}}_w = \tilde{\boldsymbol{\alpha}}_w$ or $\hat{\boldsymbol{\alpha}}_w = \breve{\boldsymbol{\alpha}}_w$.

In particular, provided that the number, $\mathcal{N}$, of contracts at some point in time is more than six we attach to each CDS market spread the following weights:

Maturity t_n (Months)	w_n
60	40%
36	30%
12	15%
84	6%
120	4%
24	3%
All the Other	$\frac{2}{\mathcal{N}-6}$%

Appendix B Reference entities with arbitrage

Panel A reports the top five companies with negative NCPL for each method of calculation. Panel B reports the top five companies with the largest NCPL. "NS" denotes the Nelson-Siegel interpolation, "Piecewise Constant" the bootstrapping procedure with piecewise constant hazard rates, "OU" and "OU conv" are the methods with the OU process without and with convexity adjustment, respectively.

Panel A

NS	Piecewise Constant	OU	OU conv
Lucent Tech Inc	Hasbro Inc	Hasbro Inc	Hasbro Inc
LA Pac Corp	Pennzoil Quaker St Co	LA Pac Corp	LA Pac Corp
Pennzoil Quaker St Co	LA Pac Corp	Pennzoil Quaker St Co	Pennzoil Quaker St Co
Hasbro Inc	CNA Finl Corp	Cap One Bk	Lucent Tech Inc
CNA Finl Corp	Cap One Bk	Cap One Finl Corp	Cap One Bk

Panel B

NS	Piecewise Constant	OU	OU conv
Finl Sec Assurn Inc	Wyeth	Wyeth	Wyeth
Ford Mtr Co	Gen Mtrs Corp	Ford Mtr Co	Ford Mtr Co
Gen Mtrs Corp	Gillette Co	Gen Mtrs Corp	Gen Mtrs Corp
Gillette Co	Global Marine Inc	Gillette Co	Gillette Co
Global Marine Inc	Ford Mtr Co	Global Marine Inc	Global Marine Inc

Panel A reports the top five companies with negative ANCPL for each method of calculation. Panel B reports the top five companies with the largest ANCPL. “NS” denotes the Nelson-Siegel interpolation, “Piecewise Constant” the bootstrapping procedure with piecewise constant hazard rates, “OU” and “OU conv” are the methods with the OU process without and with convexity adjustment, respectively.

Panel A

NS	Piecewise Constant	OU	OU conv
J C Penney Co Inc	Hasbro Inc	J C Penney Co Inc	J C Penney Co Inc
Hasbro Inc	Pennzoil Quaker St Co	Hasbro Inc	Pennzoil Quaker St Co
Pennzoil Quaker St Co	J C Penney Co Inc	Pennzoil Quaker St Co	Hasbro Inc
Aetna Inc.	ServiceMaster Co	ServiceMaster Co	ServiceMaster Co
ServiceMaster Co	Aetna Inc.	Aetna Inc.	Aetna Inc.

Panel B

NS	Piecewise Constant	OU	OU conv
Ford Mtr Cr Co	Ford Mtr Cr Co	Ford Mtr Cr Co	Textron Inc
Toys R Us Inc	Toys R Us Inc	Toys R Us Inc	Wyeth
Ford Mtr Co	Ford Mtr Co	Ford Mtr Co	Gen Mtrs Corp
Williams Cos Inc	Williams Cos Inc	Williams Cos Inc	Williams Cos Inc
Gen Mtrs Corp	Gen Mtrs Corp	Gen Mtrs Corp	Wells Fargo & Co

Figure 1: Comparison of premia calculations for CDS and CMCDS referenced by the same obligor. At each quarterly market payment time $t_0, t_3, t_6, ...$ the fixed CDS rate $S(t_v, t_v + T)$ is paired with the floating premium given by the product of participation rate PR^{t_v} and the realised reference market spot rate $S(t_{3i}, t_{3i} + m)$. The actual coupon is calculated by multiplying those rates to the quarter period using the market money count conventions. The time t_v shows the day when the trading is realised, which may not coincide with a market scheduled coupon paying day t_0; if t_v is within one month of t_3 then the first coupon is paid at t_6, otherwise it is paid at t_3.

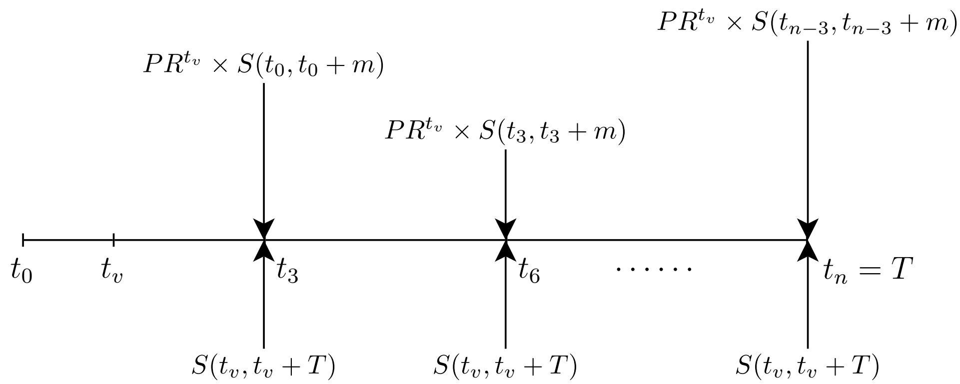
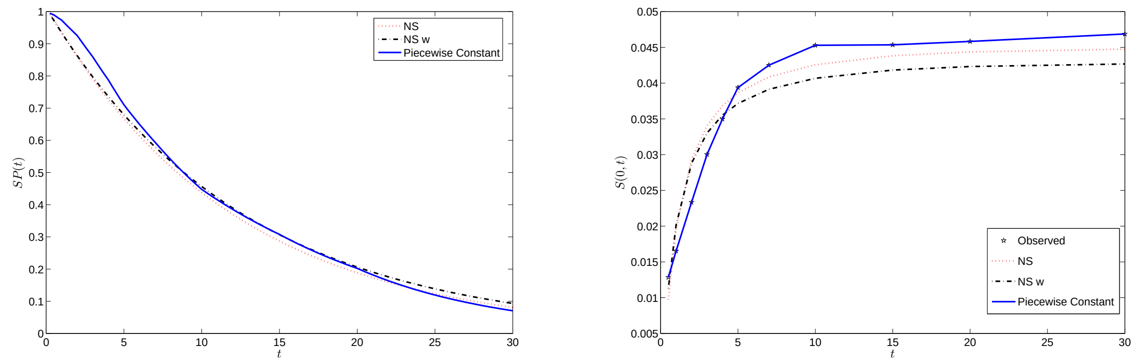
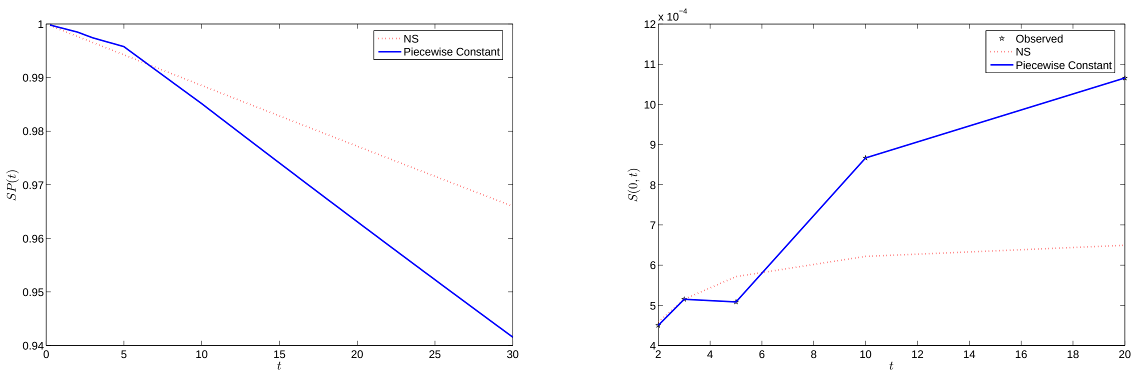


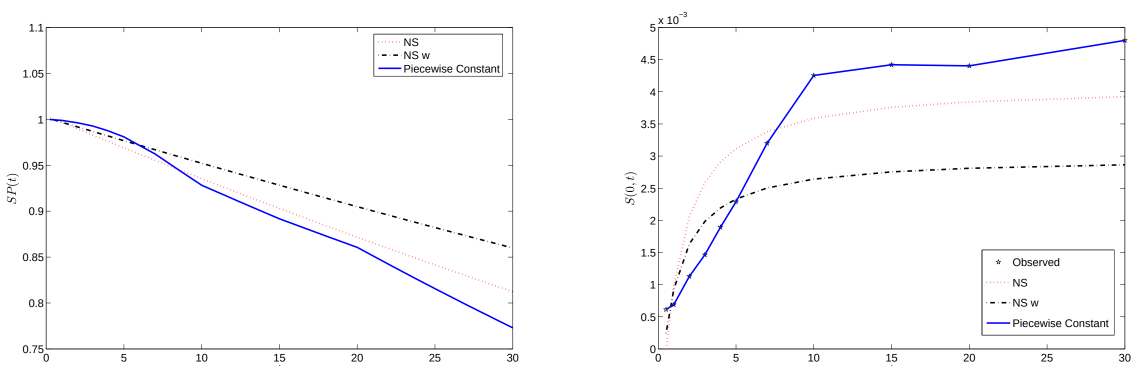
Figure 2: Bootstrapped survival probabilities and CDS curves for October 3rd 2005 for Abitibi Consol Inc, Microsoft Corp and Tesco PLC. "Piecewise Constant" denotes the bootstrapping procedure with piecewise constant hazard rates, "NS" the Nelson-Siegel interpolation and "NS w" the Nelson-Siegel interpolation with weights in the objective function.



(a) Abitibi Consol Inc: Bootstrapped Survival Probabilities (left) and CDS spreads (right)

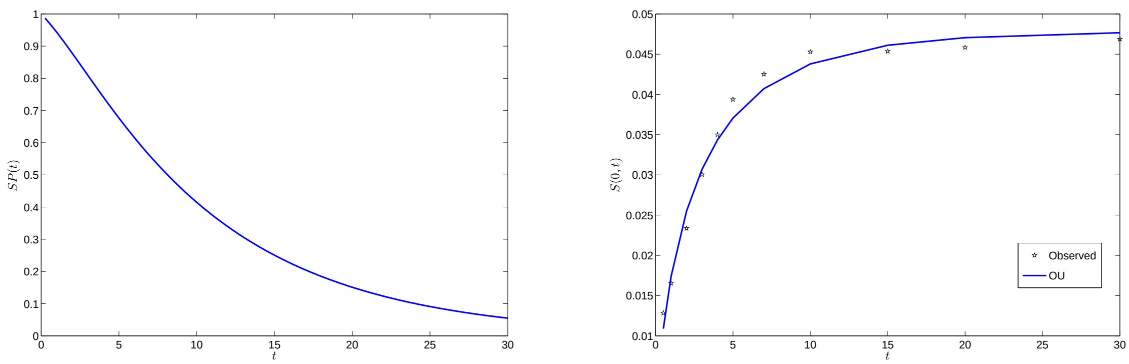


(b) Microsoft Corp: Bootstrapped Survival Probabilities (left) and CDS spreads (right)

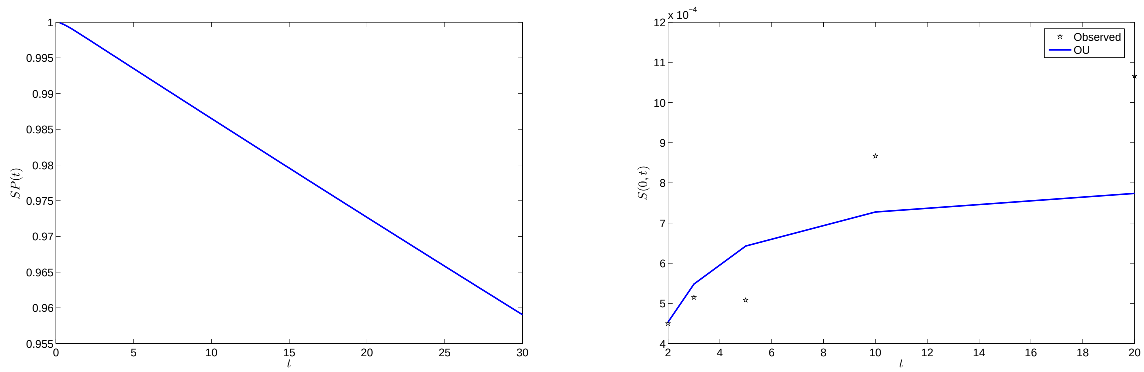


(c) Tesco PLC Bootstrapped Survival Probabilities (left) and CDS spreads (right)

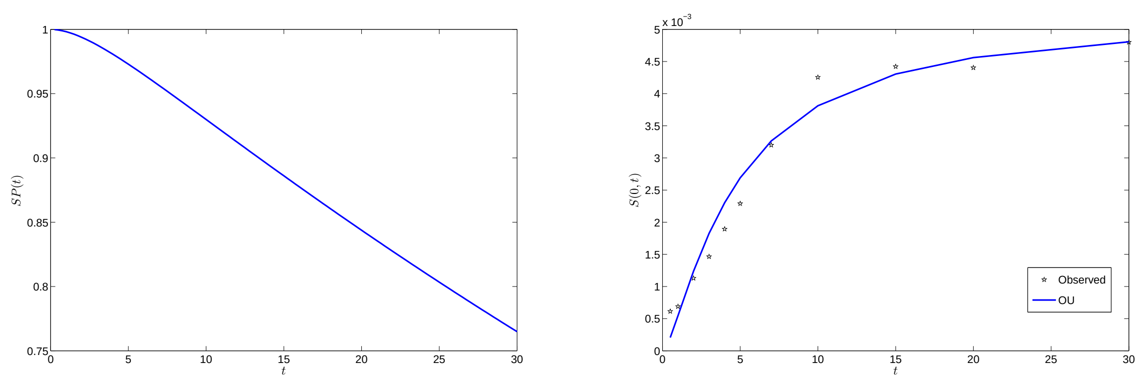
Figure 3: Bootstrapped survival probabilities and CDS spreads for October 3rd 2005 for Abitibi Consol Inc, Microsoft Corp and Tesco PLC using the OU process.



(a) Abitibi Consol Inc: Bootstrapped Survival Probabilities (left) and CDS spreads (right)

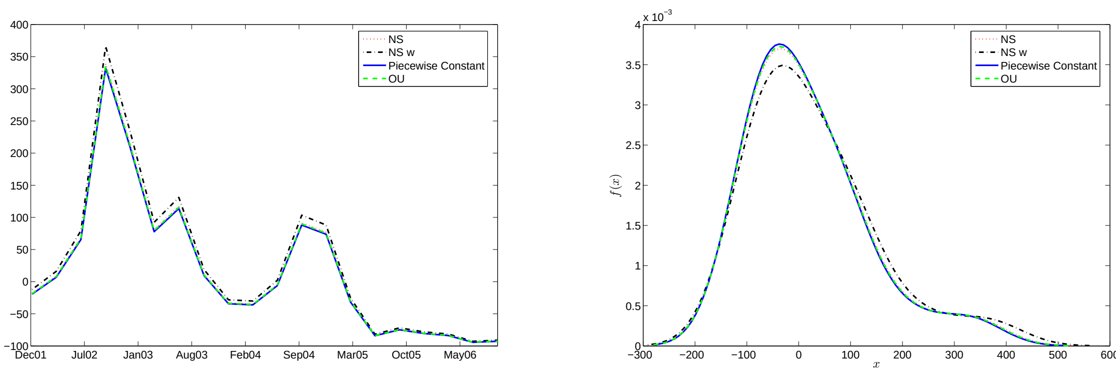


(b) Microsoft Corp: Bootstrapped Survival Probabilities (left) and CDS spreads (right)

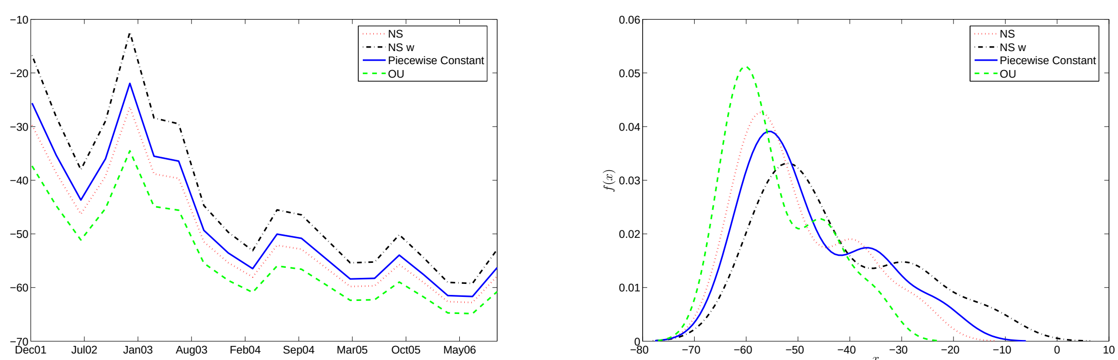


(c) Tesco PLC Bootstrapped Survival Probabilities (left) and CDS spreads (right)

Figure 4: Comparative series of net coupon spreads (basis points) for the paired strategy short CDS long CMCDS settled on 20 September 2001; "Piecewise Constant" is for the bootstrapping procedure with piecewise constant hazard rates, "NS" the Nelson-Siegel interpolation, "NS w" the Nelson-Siegel interpolation with weights in the objective function and "OU" the method with the OU process.

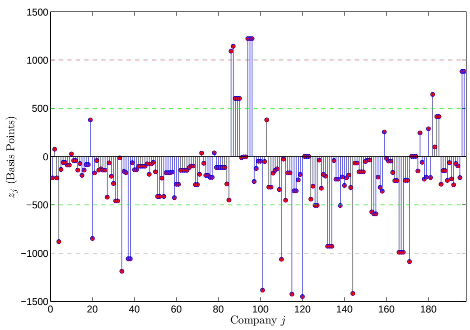


(a) AT&T: Series of net coupon spreads payments (left) and smoothed empirical density (right)

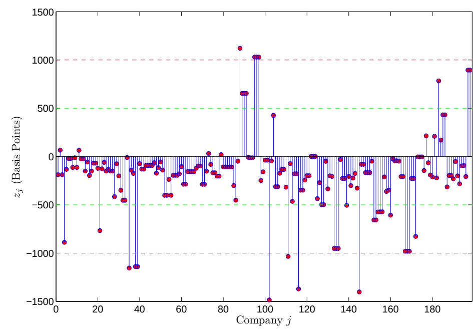


(b) Goldman Sachs Gp Inc: Series of net coupon spreads payments (left) and smoothed empirical density (right)

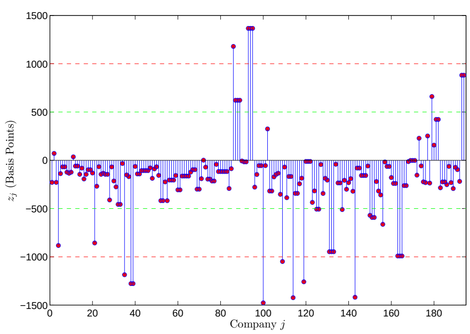
Figure 5: The calculated net cumulative profit/loss (NCPL), z_j, on the paired trade short CDS long CMCDS for all obligors.



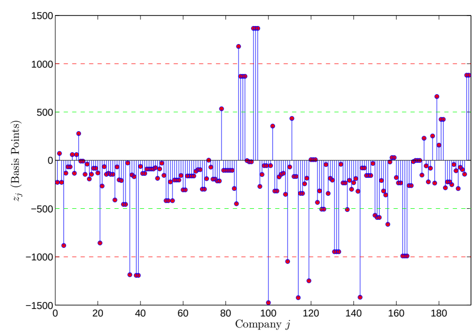
(a) Nelson Siegel



(b) Piecewise Constant

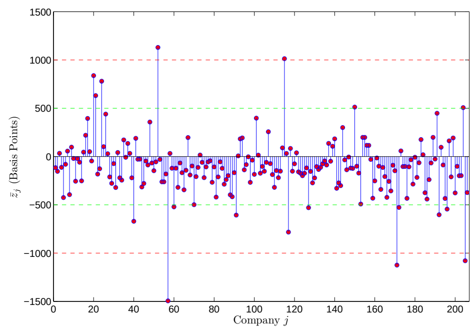


(c) OU process

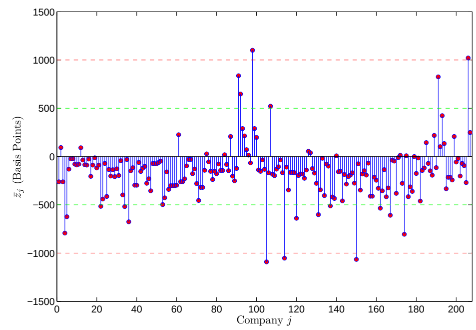


(d) OU process with convexity adjustment

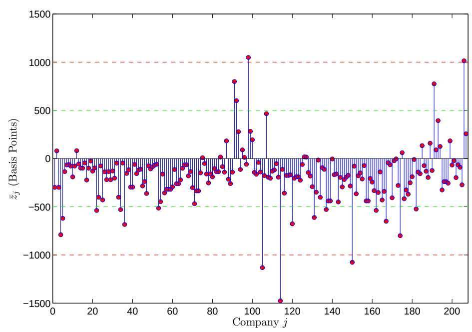
Figure 6: The calculated average net cumulative profit/loss (ANCPL), $\bar{z}_j$, on the paired trade long CDS short CMCDS for all obligors.



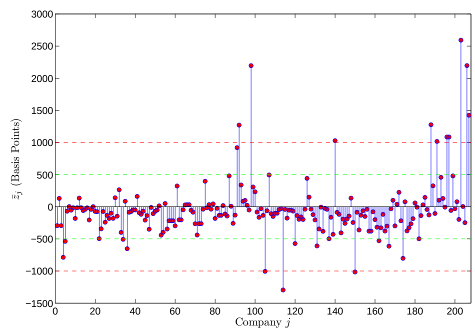
(a) Nelson Siegel



(b) Piecewise Constant



(c) OU process



(d) OU process with convexity adjustment

Table I: Number of reference entities in our sample for 20th September 2001 by sector and average rating

	AAA	AA	A	BBB	BB	B	CCC	NA	Total
Basic Materials	0	0	7	6	1	1	0	0	15
Consumer Goods	0	3	9	17	4	7	0	1	41
Consumer Services	0	1	5	15	5	1	1	0	28
Financials	4	8	15	10	1	1	0	0	39
Government	2	3	1	0	0	0	0	0	6
Health Care	0	1	3	1	1	0	0	1	7
Industrials	1	0	10	17	0	1	0	0	29
Oil & Gas	1	1	5	4	0	1	0	0	12
Technology	0	0	3	3	1	1	0	1	9
Telecommunications	0	0	1	2	1	1	0	0	5
Utilities	0	0	3	8	2	0	0	0	13
Total	8	17	62	83	16	14	1	3	204

Table II: Market quotes (basis points) on October 3rd 2005 (Source: Markit).

Maturity (Months)	Abitibi Consol Inc	Microsoft Corp	Tesco PLC
6	128.41	–	6.13
12	165.26	–	6.89
24	233.47	4.50	11.29
36	300.28	5.15	14.65
48	349.90	–	18.93
60	393.73	5.08	22.90
84	425.10	–	31.99
120	452.91	8.67	42.54
180	453.55	–	44.20
240	458.28	10.66	44.03
360	468.68	–	48.00
R	0.3929	0.4	0.394

Table III: Parameter estimation and calculation of participation rates. Panel A reports $\hat{\boldsymbol{\alpha}}$ and Panel B $\hat{\boldsymbol{\alpha}}_w$, the estimated parameters for the Nelson-Siegel method and the Nelson-Siegel with weights in the objective function, respectively. Panel C shows the participation rates for a CMCDS with $T = m = 5$ using the bootstrapping procedure with piecewise constant hazard rates, the Nelson-Siegel interpolation (NS) and the Nelson-Siegel interpolation with weights in the objective function (NS w)

Panel A

	Abitibi Consol Inc	Microsoft Corp	Tesco PLC
$\hat{\alpha}_0$	0.0844	0.0012	0.0071
$\hat{\alpha}_1$	-0.1790	0.0000	-0.0230
$\hat{\alpha}_3$	0.1001	0.1550	0.1631

Panel B

	Abitibi Consol Inc	Microsoft Corp	Tesco PLC
$\hat{\alpha}_{w0}$	0.0795	—	0.0051
$\hat{\alpha}_{w1}$	-0.1443	—	-0.0141
$\hat{\alpha}_{w3}$	0.0793	—	0.1420

Panel C

PR	Abitibi Consol Inc	Microsoft Corp	Tesco PLC
NS	0.7788	0.7393	0.5429
NS w	0.8254	—	0.7497
Piecewise Constant	0.7727	0.5900	0.5191

Table IV: Parameter estimation (Panel A) and calculation of participation rates (Panel B) for the method with the OU process with and without convexity adjustment.

Panel A

	Abitibi	Microsoft	Tesco
k	0.0379	0.0037	0.0040
α	0.3727	2.6324	0.2902
λ_0	0.0498	0.0000	0.0001
σ	0.0088	0.0000	0.0258

Panel B

PR	Abitibi	Microsoft	Tesco
With convexity adjustment	0.7408	0.6090	0.6482
Without convexity adjustment	0.7390	0.6090	0.4989

Table V: Summary statistics for the net coupon spreads (basis points) of the paired strategy short CDS long CMCDS across all four methods.

AT&T

Method	mean	median	std	min	max	5% Percentile	95% Percentile
Nelson Siegel	18.95	-11.24	112.38	-94.02	338.99	-93.19	276.84
Nelson Siegel weighted	26.95	-5.05	119.09	-92.76	366.10	-91.89	300.25
OU process	18.04	-11.95	111.61	-94.16	335.90	-93.34	274.18
Piecewise Constant	16.92	-12.81	110.68	-94.33	332.12	-93.52	270.92

Goldman Sachs Gp Inc

Method	mean	median	std	min	max	5% Percentile	95% Percentile
Nelson Siegel	-50.13	-54.11	11.00	-62.79	-26.36	-62.71	-28.05
Nelson Siegel weighted	-42.95	-48.07	14.14	-59.24	-12.38	-59.14	-14.56
OU process	-54.32	-57.63	9.16	-64.87	-34.51	-64.80	-35.92
Piecewise Constant	-47.86	-52.19	11.99	-61.67	-21.94	-61.58	-23.78

Table VI: Summary statistics for the calculated net cumulative profit/loss (NCPL) on the paired trade short CDS long CMCDS.

Method	mean	median	std	min	max	5% Percentile	95% Percentile
Nelson Siegel	-176.56	-153.79	416.17	-1450.21	1221.59	-991.74	602.12
Piecewise Constant	-185.48	-157.05	387.73	-1485.07	1121.22	-969.47	565.17
OU Process	-188.80	-160.52	413.96	-1476.73	1366.11	-991.23	582.15
OU Process with convexity adjustment	-169.55	-157.21	428.35	-1474.86	1366.11	-991.23	634.84

Table VII: Number of obligors with a positive (negative), larger than 250 bps (smaller than -250 bps), larger than 500 bps (smaller than -500 bps) and larger than 1000 bps (smaller than -1000 bps) NCPL by different methods of calculation. "NS" denotes the Nelson-Siegel interpolation, "Piecewise Constant" the bootstrapping procedure with piecewise constant hazard rates, "OU" and "OU conv" are the methods with the OU process without and with convexity adjustment, respectively.

	NS	Piecewise Constant	OU	OU conv
pos	29	23	19	28
neg	168	175	175	166
> 250 bps	17	13	14	17
> 500 bps	11	10	10	11
> 1000 bps	5	4	4	4
< -250 bps	53	53	55	54
< -500 bps	23	23	23	23
< -1000 bps	9	7	8	8

Table VIII: List of obligors with NCPL in absolute value larger than 500 bps for all methods of calculation. Those marked with * have NCPL larger than 1000 bps in absolute value.

$z_j < -500$ bps	$z_j > 500$ bps
Aetna Inc.	Ford Mtr Co *
Arrow Electrs Inc	Ford Mtr Cr Co
CNA Finl Corp *	GA Pac Corp
Cap One Bk *	GATX Finl Corp
Cap One Finl Corp *	Gen Mtrs Corp *
Hasbro Inc *	Gillette Co *
J C Penney Co Inc *	Global Marine Inc *
LA Pac Corp *	Toys R Us Inc
Motorola Inc	Williams Cos Inc
NOVA Chems Corp	Wyeth
Nabors Inds Inc	
Nordstrom Inc	
Pennzoil Quaker St Co *	
Raytheon Co	
Reebok Intl Ltd	
Roche Hldgs Inc	
ServiceMaster Co	
Shaw Comms Inc	
Sherwin Williams Co	

Table IX: Summary statistics for average net cumulative profit/loss (ANCPL) on the paired trade long CDS short CMCDS.

Method	mean	median	std	min	max	5% Percentile	95% Percentile
Nelson Siegel	-105.43	-109.61	296.00	-1494.80	1129.92	-522.32	395.29
Piecewise Constant	-156.20	-152.14	277.61	-1090.60	1100.49	-544.99	229.40
OU Process	-173.20	-160.07	280.55	-1475.95	1048.46	-549.41	204.11
OU Process with convexity adjustment	-42.73	-82.53	451.02	-1294.75	2592.18	-530.53	934.47

Table X: Number of obligors with a positive (negative), larger than 250 bps (smaller than -250 bps), larger than 500 bps (smaller than -500 bps) and larger than 1000 bps (smaller than -1000 bps) ANCPL by different methods of calculation. "NS" denotes the Nelson-Siegel interpolation, "Piecewise Constant" the bootstrapping procedure with piecewise constant hazard rates, "OU" and "OU conv" are the methods with the OU process without and with convexity adjustment, respectively.

	NS	Piecewise Constant	OU	OU conv
pos	52	30	26	55
neg	154	177	181	152
> 250 bps	14	9	10	22
> 500 bps	7	6	5	11
> 1000 bps	2	2	2	10
< -250 bps	49	61	64	45
< -500 bps	11	14	16	12
< -1000 bps	3	3	3	3

Table XI: List of obligors with ANCPL in absolute value larger than 500 bps for all methods of calculation. Those marked with * have ANCPL larger than 1000 bps in absolute value.

$\bar{z}_j < -500$ bps	$\bar{z}_j > 500$ bps
Aetna Inc.	Ford Mtr Co
Agrium Inc	Ford Mtr Cr Co
CNA Finl Corp	Gen Mtrs Corp *
Hasbro Inc *	Toys R Us Inc
J C Penney Co Inc *	Williams Cos Inc *
LA Pac Corp	
Mattel Inc	
Pennzoil Quaker St Co *	
SUPERVALU INC	
ServiceMaster Co	

Table XII: CIPS test statistics. Here S is for the spot five year CDS spread and F is the forward CDS spread corresponding to the spot spread. No asterisk denotes lack of significance at 5% level and two asterisks denote significance at 1% level.

No intercept and no trend

	S	FS	$S - FS$
Nelson Siegel	-1.27	-1.13	-2.19**
Nelson Siegel weighted	-1.25	-1.54	-2.08**
OU process	-1.25	-1.20	-2.51**
Piecewise Constant	-1.31	-1.21	-2.80**

Intercept only

	S	FS	$S - FS$
Nelson Siegel	-1.68	-1.60	-2.43**
Nelson Siegel weighted	-1.58	-1.66	-2.17**
OU process	-1.58	-1.49	-2.54**
Piecewise Constant	-1.67	-1.51	-2.88**

Intercept and trend

	S	FS	$S - FS$
Nelson Siegel	-2.03	-2.00	-2.49
Nelson Siegel weighted	-1.93	-1.78	-2.27
OU process	-1.93	-1.93	-2.74**
Piecewise Constant	-2.04	-1.85	-2.90**