Testing alternative theories of financial decision making: a survey study with lottery bonds

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Abstract

In this paper, we present the results of a simple, easily replicable, survey study based on lottery bonds. It is aimed at testing whether agents make investment decisions according to expected utility, cumulative prospect theory (Tversky-Kahneman, 1992) or optimal expectations theory (Brunnermeier and Parker, 2005, Brunnermeier *et al.*, 2007) when they face skewed distributions of returns. We show that more than 56% of the 245 participants obey optimal expectations theory. They choose a distribution of payoffs which is dominated for second-order stochastic dominance and which would not be chosen according to cumulative prospect theory, for a large range of parameter values.

Our results first cast doubt on the relevance of variance as a measure of risk; they show the importance of skewness in decision making and, more precisely, they emphasize the attractiveness of the best outcome, an essential feature of optimal expectations theory. The ranking of outcomes, used in cumulative prospect theory, seems insufficient to characterize the way people distort beliefs. As by-products of this study, we illustrate that agents use heuristics when they choose numbers at random and have, in general, a poor opinion about the rationality of others.

JEL classification: D03, D81

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I Introduction

Standard economic theory assumes relatively simple rules to describe human behavior. Agents are supposed to manage any quantity of information they receive, according to Bayes' rule, and to take decisions without emotions or distorted beliefs. Their objective is to maximize the expectation of their utility function (henceforth EU model). There is now some evidence that agents make systematic "errors", especially in assessing probabilities. The probability of very good outcomes tends to be overvalued (Alpert and Raiffa, 1982; Buehler *et al.*, 1994; Weinstein, 1980), introducing an optimistic bias. The consequence is a sub-optimal allocation of wealth. For example, households' portfolios are not well diversified. A part of their wealth is invested in mutual funds (and then well diversified) but another part, in general non negligible, is concentrated on a few stocks (Calvet *et al.*, 2007, Goetzmann and Kumar, 2008, Mitton and Vorkink, 2007, Polkovnichenko, 2005). Moreover, portfolios are often biased toward lottery-type stocks with positive skewness. Barberis and Huang (2008), Bali *et al.*(2009), Kumar (2009) and Mitton and Vorkink (2007) recently published papers focused on that problem. Barberis and Huang (2008) show that stocks with positively skewed returns can be overpriced on markets populated by investors obeying cumulative prospect theory (CPT in the following). It is especially the case if the return on skewed securities is independent of the returns on other securities and if the supply of skewed stocks is small relative to the global market supply. Kumar (2009) shows the existence of significant links between investment behavior and lottery play behavior. He shows that investors used to play (unfair) state-lotteries also prefer lottery-like stocks. This observation is reinforced during economic downturns. Bali *et al.* (2009) show that stocks exhibiting at least one very high return in the past month are overpriced. This effect is robust when controlling for idiosyncratic volatility. Mitton and Vorkink (2007) analyze the behavior of more than 60,000 retail investors and show that those who are not diversified select a few highly skewed stocks. It then seems that gambling and investment behaviors cannot be disentangled because of the preference for skewness and/or the attractiveness of the best outcome.

Brunnermeier and Parker (2005) and Brunnermeier *et al.* (2007) developed a theory of optimal expectations (OET in the following) to take into account this optimistic bias. In the second paper, Brunnermeier *et al.* (2007) consider a simple one-period, two-dates model; they assume that agents behave optimally given their beliefs, and choose portfolios maximizing the expected present value of future utility flows. Roughly speaking, the felicity of agents is composed of *ex ante* and *ex post* utility. *Ex ante*, it is optimal to distort beliefs in an optimistic way. However, this distortion comes at a cost, lying in a sub-optimal portfolio choice and a lower *ex post* expected utility. The authors call “optimal beliefs” the subjective assessment of probabilities which maximizes an average of *ex ante* and *ex post* utilities. In a complete market framework with a finite number of states of nature, they show that the optimal portfolio contains the risk-free asset, and the most skewed asset. Concerning optimal beliefs, they prove that the probability of only one state is overvalued, the probabilities of the other states being undervalued.

In cumulative prospect theory, distortion of beliefs is linked to payoffs, only through the ranking of gains and losses. Consequently, in a finite state space, the outcomes of two comonotonic prospects are weighted identically and independently of the values of the outcomes; only ranking matters. Concerning the distortions of beliefs, our results show that a large proportion of agents not only take into account the ranking, but also the values of the outcomes, especially the largest one.

In this paper, we present a survey study to test whether the attractiveness of the best outcome is really an important component of the decision making process or if agents behave according to the EU or CPT models. The test is based on a questionnaire asking participants to choose among different random outcomes of lottery bonds. These securities are well suited to address the question we are dealing with. First, they exist in many countries for more than two centuries, and are, even today, very popular (Green and Rydqwist, 1997, Guillen and Tschoegl, 2002, Lévy-Ullmann, 1896, Millar and Gentry, 1980, Pfiffelmann and Roger, 2005, Ridge and Young, 1998, Tufano, 2008). Second, contrary to the distribution of stock returns which is unknown, the distribution of lottery bond returns is, in general, perfectly known. The possible outcomes are given objective probabilities. Third, most people who never invested in lottery bonds easily understand how payoffs are defined because they bet, at least occasionally, on state lotteries like the lotto game[2].

We consider two designs for the lottery bonds. They differ by the way the amount distributed through the lottery is defined. The first bond is designed to study the decision making process and to answer our main question concerning the choice of EU, CPT or OET. The individual amount received by winners through the lottery is known in advance and the remaining amount to be shared among all subscribers (including the winners) is random. The second bond is aimed at controlling for the “minimum” required level of rationality, that is, the compliance to the first-order stochastic dominance principle. In this case, the global amount

[2] Skewness-seeking investors are labeled “Lotto investors” by Mitton and Vorkink (2007).

paid through the lottery is known in advance (but the individual gain is random due to a pari-mutuel feature). The remaining amount shared by all subscribers is then not random. In fact, as we cannot define an incentive compatible payment scheme for the participants (without assuming that one theory is better than the others), the questions related to this second bond allow to "select" respondents that provide answers compatible with first-order stochastic dominance.

Our results show that more than 55% of participants behave like OET investors, exhibiting a preference for the random payoff with the highest possible outcome. It is important to notice that the possible random payoffs of our first bond have the same expected value and that the variance of returns is the highest for the payoff with the largest outcome. Moreover, our design is such that the payoff including the highest possible outcome is dominated by all other choices in the sense of second-order stochastic dominance. It then reinforces our result in favor of OET.

Our analysis also provides two by-products. The first one concerns the random choice of numbers. We illustrate that, at the aggregate level, people do not choose numbers at random, even when they are expected to do so[3]. This result is in line with most studies on state lotteries. These papers show that the distribution of numbers actually chosen by players is not uniform (see, for example, Farrell *et al.*, 2000, Roger and Broihanne, 2007, among others) because they use common heuristics to select numbers.

The second side-result is linked to the assumption that rationality is common knowledge (Aumann, 1976). It is a strong assumption and many examples show that it does not represent the way people are thinking. The most famous example is the beauty contest (first introduced by J.M. Keynes, 1936, chapter 12, p. 156[4]), translated by H. Moulin (1986) in numerical terms. Players have to choose a number between 0 and 100 and the winner is the one who chooses the number closest to a given percentage (say a) of the mean choice of players. The

[3] For example, Boland and Pawitan (1999) show that people have difficulties to choose numbers randomly, even in very simple tasks.

[4] As stated by John Maynard Keynes (1936): "Or, to change the metaphor slightly, professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees."

Nash equilibrium of the game is that everybody chooses 0 when $a < 1$[5]. However, all experiments show that most people are far from choosing 0 (Thaler, 1998, Nagel, 1995). Our results also confirm that a non negligible percentage of participants have a poor opinion about the rationality of others.

To sum up, we test three assumptions in this paper:

1) When facing positively skewed distributions, investors do not behave like risk averse expected utility maximizers. In particular, they do not use a mean-variance criterion. Positive skewness is a highly weighted decision criterion and, more precisely, the probability of the highest possible outcome is overvalued.
2) People are not choosing numbers at random even when they are expected to do so. They have common preferred numbers, a sub-optimal characteristic in a pari-mutuel game.
3) When the distribution of payoffs depends on the decision of others, agents have a tendency to consider that other people are not fully rational (and they seem right!).

The paper is organized as follows. Section II describes the two lottery bonds used in the survey study. This section is written in such a way that the reader can think about his/her possible answers and can "participate" to the survey. Section III presents the theoretical analysis of the bonds and explains what theoretical choices should be, according to the three theories under examination. Section IV presents the empirical results and section V concludes.

II Design of the lottery bonds

Lottery bonds are in general fixed-rate bonds (with coupon rate r) issued by a state or a firm. However, r applies to the global issue, not to the individual subscribers. If N one-year bonds are issued, each with a \$1 face value, the issuer repays $B = (1+r)N$ at the maturity date. A part of this amount is redistributed by means of a lottery. For example, B is divided in two parts such that $B = B_1+B_2$. $n < N$ bonds are drawn at random and their holders share B_1 (equally or not, depending on the design of the lottery). The remaining amount, B_2, is then shared equally among the N subscribers, or among the $N – n$ losers.

[5] Some guessing games involve negative feedback. Players have to find the closest number to 100 – p x *mean*. For $p = 2/3$ it is easy to see that the equilibrium choice is 60 (see Sutan and Willinger, 2009)

In most cases, the issuer bears no risk since it repays *B* whatever happens in the random draw. The risk is entirely borne by subscribers. Lottery bonds are then unusual financial assets since issuers voluntarily introduce randomness in payoffs.

In the two following subsections, we describe and characterize the two lottery bonds used in the survey study. The two bonds differ only by the way the amount paid through the lottery is defined. In the second subsection, we formalize the payoffs and introduce the notations used in the theoretical analysis of section 3.

II-1 Description of the lottery bonds

The two bonds are designed as follows.

A bank issues N (equal to 1,000,000 in the questionnaire) units of a lottery bond, each bond being sold \$1. The subscriber of one bond has to choose a number between 1 and 10. At the maturity date, the bank pays an interest rate r (5% in the questionnaire) on the global amount issued, then repaying $(1+r)N$ (\$1,050,000) . However, the bank first draws one number at random between 1 and 10, say j (we say that *series j has been drawn*).

The two bonds differ in the way gains of winning subscribers are defined.

- For the first bond, the issuer pays \$1 to each of the subscribers of series j and shares equally the remaining amount among **all** subscribers, including the winning ones. For example, if r = 5%, N = 1,000 and if 150 subscribers have chosen the winning series, they will receive \$1.90. After having paid the winners \$1 each, the bank shares the remaining \$900 among the 1,000 subscribers, each one receiving \$0.9.
- The second bond follows different rules. 10% of the initial amount issued is devoted to winners, the remaining being shared among **all** subscribers, including the winning ones. With the same data as before, a winner would receive \$100/150 + \$0.95 because the 150 winners have to share \$100. The remaining amount is constant because the global amount won through the lottery is independent of the number of winners. Consequently, all losing series receive \$0.95, whatever the number of winners is.

The main question addressed in this paper is to know how people choose a series when they get information about choices of former subscribers. Table 1 shows the individual payoffs received by subscribers of the first lottery bond, depending on the series they invested in and on the series which has been drawn. In this example, the number of bonds is 1,000,000 and the interest rate paid by the bank is 5 %, so the bank repays \$1,050,000 at the maturity date. The first line indicates the number of subscribers in each series and the first column identifies the possible states of nature (the series number drawn at random by the issuer). There are then 10 states of nature. The following columns give the payoffs received by a subscriber of a given series in each state. For example, 1.95 is the final payoff obtained by a series-1 subscriber if number 1 is drawn. As there are 100,000 subscribers in this series, each of them first receives $1 and the remaining $950,000 are shared equally among all the participants, so each subscriber receives $0.95. It explains the amounts appearing in the corresponding line. When number 2 is drawn, the series-1 subscriber receives $0.9 because there were 150,000 subscribers in series 2. The remaining amount is $900,000 shared by the 1,000,000 subscribers. The same calculations justify the other amounts in the table.

Table 1: Payoffs of bond 1

The "series*k*" column contains the payoffs received at the maturity date by a subscriber of series *k* when the number drawn at random is the one appearing in the first column and the same line.

	Series1	Series2	Series3	Series4	Series5	Series6	Series7	Series8	Series9	Series10
	100000	150000	80000	120000	60000	140000	70000	50000	130000	100000
1	1.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95
2	0.9	1.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
3	0.97	0.97	1.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97
4	0.93	0.93	0.93	1.93	0.93	0.93	0.93	0.93	0.93	0.93
5	0.99	0.99	0.99	0.99	1.99	0.99	0.99	0.99	0.99	0.99
6	0.91	0.91	0.91	0.91	0.91	1.91	0.91	0.91	0.91	0.91
7	0.98	0.98	0.98	0.98	0.98	0.98	1.98	0.98	0.98	0.98
8	1	1	1	1	1	1	1	2	1	1
9	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	1.92	0.92
10	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	1.95

Table 2 shows the individual payoffs received by the subscribers of the second lottery bond, depending on the series they invested in and on the series which has been drawn. Table 2 is built as table 1, the number of subscribers in each series and the coupon rate being the same. For example, 1.95 is the final payoff obtained by a series-1 subscriber if series 1 is drawn. As there are 100,000 subscribers in this series, each of them first receives $1 ($100,000 shared by 100,000 winners) and the remaining $950 000 are shared equally among all the participants. Each losing subscriber then receives $0.95. It explains the amounts appearing in the corresponding line. When a different number is drawn, the series-1 subscriber receives $0.95, because the remaining amount is still $950 000, shared among the 1,000,000 subscribers. The essential difference between the two bonds is that the payoff received by a "losing-series" doesn't depend on the losing number, all losers receiving $0.95. In other words, each bond 2 is characterized by only two possible payoffs, a winning one or a losing one. Only the winning amount is linked to the number of subscribers in the corresponding series.

Table 2: Payoffs of bond 2

The "series*k*" column contains the payoffs received at the maturity date by a subscriber of series *k* when the number drawn at random is the one appearing in the first column and the same line.

	Series1	Series2	Series3	Series4	Series5	Series6	Series7	Series8	Series9	Series10
	100000	150000	80000	120000	60000	140000	70000	50000	130000	100000
1	1.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95
2	0.95	1.62	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95
3	0.95	0.95	2.2	0.95	0.95	0.95	0.95	0.95	0.95	0.95
4	0.95	0.95	0.95	1.78	0.95	0.95	0.95	0.95	0.95	0.95
5	0.95	0.95	0.95	0.95	2.62	0.95	0.95	0.95	0.95	0.95
6	0.95	0.95	0.95	0.95	0.95	1.66	0.95	0.95	0.95	0.95
7	0.95	0.95	0.95	0.95	0.95	0.95	2.38	0.95	0.95	0.95
8	0.95	0.95	0.95	0.95	0.95	0.95	0.95	2.95	0.95	0.95
9	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	1.72	0.95
10	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	1.95

II-2 Formal presentation of payoffs

The payoffs of bond 1 can be formalized as follows.

Let $X^i(j)$ the payoff received by a series-*i* subscriber when the number *j* is drawn. Denote *N* the global number of subscribers. $N = \sum_{k=1}^{10} N_k$ where N_k is the number of series-*k* subscribers (first line of tables 1 and 2). The issuer repays $(1+r)N$ but, according to the preceding rule, the individual payoff X^i is defined by:

$$X^i(j) = \begin{cases} (1+r) - \dfrac{N_j}{N} \text{ if } j \neq i \\ 2 + r - \dfrac{N_i}{N} \text{ if } j = i \end{cases} \tag{1}$$

Denote *Z* the random variable defined on the ten states of nature by:

$$Z(k) = (1+r)\mathbf{1}_\Omega - \theta(k), k = 1,...,10 \tag{2}$$

where $\theta(k) = N_k / N$ and $\mathbf{1}_\Omega$ is the indicator function of the set of states of nature. X^i can be written as $X^i = Z + \mathbf{1}_{\{i\}}$ with $\mathbf{1}_{\{i\}}$ being the indicator function of state *i*. This decomposition will be useful to study the moments of payoffs in the next section.

Suppose that you have to choose a series to invest in. Obviously, if the numbers N_j are unknown, one can reasonably expect indifference between the 10 series, which all generate an expected payoff (1 + *r*) because the indicator functions $\mathbf{1}_{\{i\}}, i = 1,\ldots,10$, have the same probability distribution.

But what would be your choice if you are told the numbers N_j and you are the last one-unit subscriber? For example, with the data of table 1, would you choose series 8 with 50,000 subscribers, series 2 with 150,000 or a series with an intermediate number of subscribers?

The random payoffs of bond 2 can be formalized in the same way.

Let $Y^i(j)$ denote the payoff received by a series-*i* subscriber when the number *j* is drawn and still denote *N* the global number of subscribers. Y^i is defined by:

$$Y^i(j) = \begin{cases} 0.9 + r \text{ if } j \neq i \\ 0.9 + r + \dfrac{0.1N}{N_i} \text{ if } j = i \end{cases} \tag{3}$$

Y^i can then be written as:

$$Y^i = (0.9 + r)\mathbf{1}_{\Omega} + \frac{0.1}{\theta}\mathbf{1}_{\{i\}}.$$

Suppose you have to choose a series to invest in. As before, if the numbers of subscribers N_j are unknown, one can expect indifference between the 10 series. But what would be your choice if you are told the distribution of θ and you are the last one-unit subscriber?

III Theoretical analysis

III-1 Lottery bond 1

We remarked before that if the numbers N_j are unknown, the potential subscribers should be indifferent between series. What is changed when the information about the distribution θ becomes available? The following proposition shows that, if the distribution of frequencies is given, the expected return on bond 1 remains equal across series.

Proposition 1

1) *The expected payoff of an investment in any series of bond 1 is equal to 1+r.*
2) *The variance of series-i return, conditional on a distribution $D = (N_1,\ldots,N_{10})$ of bonds already sold, is decreasing with N_i and given by:*

$$V_D\left[X^i\right]=\frac{1}{10}\left[\sum_{j=1}^{10}\left(\frac{N_j}{N}\right)^2+\left(1-\frac{2N_i}{N}\right)\right] \tag{4}$$

Proof

1) Denote $E_D\left[X^i\right]$ the expected payoff received by a subscriber of series *i*, conditional on a given distribution *D*.

$$E_D\left[X^i\right]=E_D\left[Z+\mathbf{1}_{\{i\}}\right]=\frac{1}{10}\left[\sum_{j\neq i}\left(1+r-\frac{N_j}{N}\right)+2+r-\frac{N_i}{N}\right]=(1+r)$$

The first moment is independent of frequencies and is then not a criterion for a rational investor to decide.

2) Using result of point (1), we can write $Z=E_D(X^i)-\theta$ with $\theta(i)=N_i/N$. We then get:

$$\begin{aligned}V_D\left[X^i\right]&=E_D\left[\left(\mathbf{1}_{\{i\}}-\theta\right)^2\right]\\&=E_D\left[\mathbf{1}_{\{i\}}\right]+E_D\left[\theta^2\right]-2E_D\left[\theta\mathbf{1}_{\{i\}}\right]\\&=\frac{1}{10}\left[\left(1-2\frac{N_i}{N}\right)+\sum_{i=1}^{10}\left(\frac{N_i}{N}\right)^2\right]\end{aligned} \tag{5}$$

The last term of the second equality, $2E_D\left[\theta\mathbf{1}_{\{i\}}\right]$, is equal to $0.2\theta(i)$. Consequently, $V_D\left[X^i\right]$ is a decreasing function of N_i.

Point 2 of proposition 1 shows that a mean-variance investor would choose to “play with the crowd”, a not so intuitive result. But, if we consider the case where all subscribers choose the same number, the issue becomes a risk-free asset, paying 1 + *r*, whatever the number drawn by the bank. It explains why playing with the crowd is variance reducing.

Expected utility maximization

Consider now the general case of a risk-averse investor and denote *U* her utility function, assumed strictly increasing and strictly concave. The following proposition generalizes the preceding results and shows that this investor always chooses the most popular number, that is the one for which N_i is maximum.

Proposition 2

Let U denote a strictly increasing and strictly concave utility function. If N_i < N_j then $E_D\left[U(X^i)\right] < E_D\left[U(X^j)\right]$

Proof:

$$X^i = Z + \mathbf{1}_{\{i\}}$$

Assume without loss of generality that the values of *Z* are ranked in increasing order, corresponding to a ranking of the N_i in decreasing order. We know that $X^i(i)$ is the maximum possible value of X^i. It means that selecting number *i*, when buying a bond, transfers the *i*-th outcome of *Z* at the right tail of the probability distribution of X^i (since winning always generates a better outcome than losing, whatever the losing number is!). It implies that transferring the lowest possible outcome to the right tail by adding $1 is always preferred by a risk averse agent, due to the decreasing marginal utility assumption. But, as the lowest value of *Z* corresponds to the highest value of N_i, a risk averse investor always prefer to bet with the crowd. Proposition 2 then indicates that risk averse expected utility maximizers should choose unambiguously n°2 if they are facing the distribution of frequencies given in tables 1 and 2.

Optimal expectations

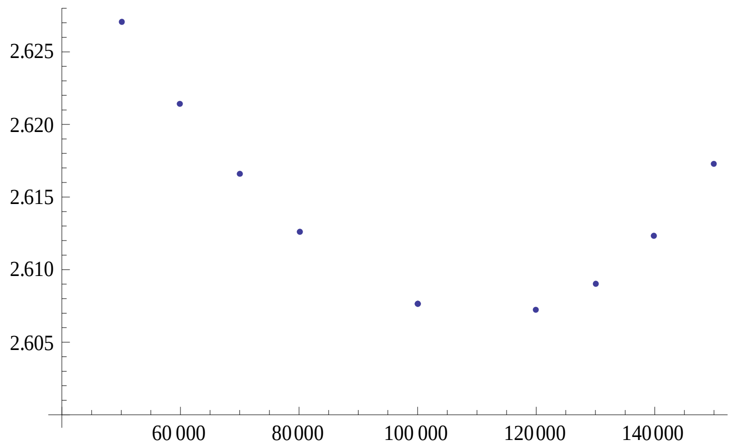
According to optimal expectations theory, described here in a one-period framework with a finite number of states (Brunnemeier *et al.*, 2007), agents maximize the following expectation:

$$\frac{1}{2}\left(E_{\Pi^*}\left[U(C^*)\right] + E_{\Pi}\left[U(C^*)\right]\right) \tag{5}$$

where the decision variables are Π^* and C^*. Π^* is the vector of subjective probabilities (used to calculate *ex ante* utility) and C^* is the optimal consumption vector. Π stands for the vector of objective probabilities, corresponding to *ex post* utility. Maximizing expression (5) means that agents choose simultaneously $\left(\Pi^*, C^*\right)$ in order to optimize an average of *ex ante* and *ex post* expected utility. There is then a trade-off between an optimistic distortion of beliefs which maximizes the *ex ante* utility and the penalty coming from a sub-optimal allocation. In fact, with distorted beliefs, choosing C^* does not lead to a maximum of $E_{\Pi}\left[U(C^*)\right]$.

Brunnemeier *et al.* (2007) get a two-fund separation result when the ratio of Arrow-Debreu security prices divided by state probabilities is constant across states of nature. The optimal portfolio consists to invest a part of wealth in the risk-free asset and the remaining amount in one and only one of the most skewed securities. In our choice context, the prices of all series are equal, the probabilities are equal and there is no aggregate risk. Therefore, the prices of Arrow-Debreu securities are identical. According to this two-fund separation result, agents should choose number 8 which is the most positively skewed portfolio. It also corresponds to the lowest N_i. Figure 1 illustrates the non monotonic link between the skewness of payoffs and the number of subscribers using the data of table 1. It appears that series 8 is the most positively skewed series. As we saw that variance is also higher for series 8, choosing this series means a strong preference for skewness and illustrates the attractiveness of the highest outcome.

Figure 1: Skewness of bond-1 payoffs for the ten series, as a function of the number of subscribers (data of table 1)



Cumulative prospect theory

Suppose now that agents obey CPT. They maximize a value function, depending on gains and losses, calculated with respect to a reference point. Two natural choices are available for the reference point; the initial price of the bond, or the initial price capitalized at rate *r*. The latter is often used when prospect theory is applied to financial decisions (see, for example, Barberis *et al.*, 2001). The two reference points lead to the same results for the problem at hand, simply because all series payoffs include 9 possible losses and 1 possible gain, whatever the reference point is. In CPT, gains and losses are loaded by decision weights, obtained by distorting the cumulative (or decumulative) distribution function of payoffs. The weighting function *w* is defined by Tversky and Kahneman (1992) as:

$$w(p,\beta)=\frac{p^{\beta}}{\left(p^{\beta}+(1-p)^{\beta}\right)^{\frac{1}{\beta}}}$$

with $\beta < 1$. The weighting function is different for gains and losses. For losses, decision weights are obtained by applying *w* to the cumulative distribution function. For gains, it is applied to the decumulative distribution function. Moreover, the parameter β may be different for gains and losses. The values estimated by Tversky and Kahneman (1992) were $\beta^{+}=0.61$ and $\beta^{-}=0.69$. In our framework, all states have the same probability 0.1. When outcomes are ranked in increasing order, the weight of the *k*-th outcome is then $\pi_k = w\left[0.1\times k,\beta^{-}\right]-w\left[0.1\times(k-1),\beta^{-}\right]$. The weight of the unique gain is equal to $w(0.1,\beta^{+})$. We observe here that weights are determined only by the ranking of outcomes, not by their values. It is also important to notice that the unique gain would allow to use a unique weighting function, as in rank-dependent utility models. In this framework, the weight of the gain is $1-w(0.9,\beta^{-})$ and the decision weights define a probability measure on the set of states of nature. This case will be examined hereafter.

The value function is defined, for a gain/loss *x* by:

$$v(x)=\begin{cases} x^{\alpha} \text{ if } x>0 \\ -\lambda(-x)^{\alpha} \text{ elsewhere} \end{cases}$$

$\lambda > 1$ is the loss aversion coefficient and $\alpha < 1$ characterizes the curvature of the value function. *v* is then concave for gains and convex for losses. Investors then choose number *i* to maximize $\sum_{k=1}^{10}\pi_k v(X^i(k)-x^*)$ where x^* is the reference point.

To understand what would be the choice of a CPT investor, we report on table 3 some elements of comparison for the two series with the lowest number of former subscribers, $N_i = 50,000$ (series 8) and $N_j = 60,000$ (series 5). We observe that the two cumulative distribution functions are equal for the 8 lowest payoffs, as shown in the two first lines of table 3.

Table 3: Payoffs of series 5 and 8

The payoffs of the two series are ranked in increasing order and the corresponding weights are calculated with the parameters estimated by Tversky and Kahneman (1992), except β which is equal for gains and losses. We then have $\alpha = 0.88; \lambda = 2.25; \beta = 0.65$.

Series5	0.9	0.91	0.92	0.93	0.95	0.95	0.97	0.98	1	1.99
Series8	0.9	0.91	0.92	0.93	0.95	0.95	0.97	0.98	0.99	2
Weight	0.17872	0.08121	0.06432	0.05798	0.05872	0.05654	0.06493	0.07759	0.10545	0.17872

The difference between the two series comes from the two highest payoffs which are (1;1.99) for series 5 and (0.99; 2) for series 8. The third line gives the decision weights for $\beta = 0.65$. The ratio of weights for the two highest payoffs is around 1.7. It is then an incentive to choose series 8 with the highest payoff. However, two other variables are important in the choice. The loss aversion coefficient, equal to 2.25 in the table, favors series 5 and largely compensates the lower weight. Moreover, the gain in the state with the highest payoff (0.94 or 0.95 with a reference point of 1.05) is much higher than the absolute value of the loss in the other state (0.05 or 0.06). As $\alpha < 1$, the increase of the value function between 0.05 and 0.06 is much larger than the absolute value of the decrease between 0.95 and 0.94. The consequence is that a CPT agent would prefer series 5 instead of series 8 for a large range of parameters. Moreover, if α is decreased, then increasing the curvature of the value function, the optimal CPT choice goes to a series with a larger number of subscribers. If the loss aversion coefficient is decreased, the result is unchanged (the optimal choice is not series 8) as long as loss aversion stays above 1.2, an unusual value in experimental studies. Figure 2 shows the CPT evaluation of the 10 series with the initial parameters. Using a weighting function only based on the distortion of cumulative distribution functions, as in the rank-dependent expected utility model, induces a weight of 0.25 for the gain which favors series n°8. However, it is not enough to compensate loss aversion and the effect of decreasing marginal utility. Series 5 with 60000 subscribers is the optimal choice in this case, as long as the loss aversion coefficient is greater than 1.72.

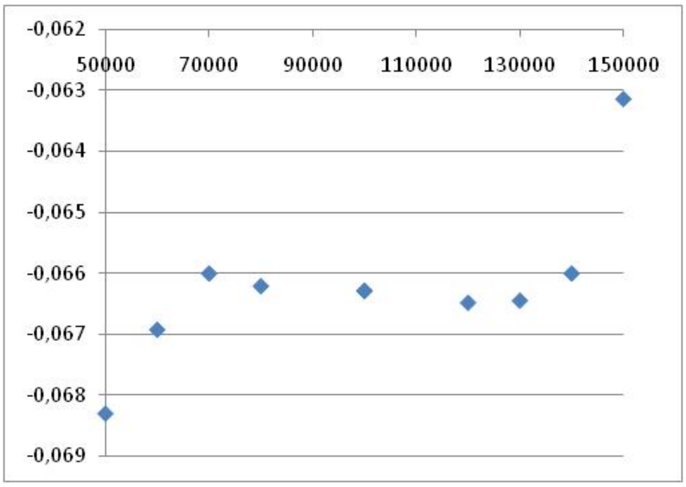


Figure 2: Value function of CPT for each series.

The horizontal axis gives the number of subscribers and the vertical axis the value function of CPT with $\alpha = 0.88$, $\lambda = 2.25$, $\beta = 0.65$. The reference point is $r = 1.05$.

To summarize the analysis, using the data of table 1, agents should choose series 2 if they are risk averse expected utility maximizers, series 8 if their behavior is driven by optimal expectations theory and, finally, series 2 or 5 if they obey cumulative prospect theory with usual parameter values. For CPT, changing the parameters could lead to other choices but keeping reasonable values leads to a choice different from series 8. This result may seem surprising since CPT is often used to justify the participation to unfair state lotteries. However, in state lotteries, the probability of winning is very low, giving a more important role to the distortion of beliefs. Here, the objective probability of the winning state is only multiplied by 1.7. It is not enough to make the choice of the most skewed alternative optimal.

III-2 Lottery bond 2

The analysis of bond 2 is much more simple. We saw in equation (3) that:

$$Y^i(j) = (0.9 + r)\mathbf{1}_\Omega + \frac{0.1N}{N_i}\mathbf{1}_{\{i\}} = \begin{cases} 0.9 + r \text{ if } j \neq i \\ 0.9 + r + \dfrac{0.1N}{N_i} \text{ if } j = i \end{cases}$$

Losing series generate the same payoff 0.9+r but the payoff of the winning series is inversely proportional to the number of subscribers of this series. Being given a distribution of frequencies, the optimal choice of the last subscriber of the issue is always the series with the lowest number of former subscribers. In fact, the corresponding payoff dominates the others in the sense of first order stochastic dominance. Looking more closely at two series i and j with frequencies N_i and N_j with $N_i < N_j$, we observe that the payoffs are 0.9+r with probability 0.9 but series *i* pays $0.1 \times N \times \left(\frac{1}{N_i} - \frac{1}{N_j} \right)$ more than series *j* with probability 0.1. It is as if you were given for free a lottery ticket paying this amount with probability 0.1. Whatever your preferences are (obeying first order stochastic dominance), you accept the lottery ticket. Bond 2 is used to introduce a screening process in the survey study. Due to the problem addressed in the paper, there is no incentive compatible payment scheme because we have no a priori about which choice is the "right" one for bond 1. However, we can suspect that the answers of participants which do not obey first-order stochastic dominance are highly questionable. Simply, some students may not be motivated by the exercise. They then answer at random. The empirical section is then focused on participants having provided answers compatible with first order stochastic dominance. Nevertheless, we provide in the appendix the table of answers for the complete sample.

III-3 Rationality as common knowledge

Assume now that you have to choose a series to invest in bond 2, after one million other subscribers. You also know that one more million subscribers will choose after you, with updated information about sales. If rationality is common knowledge, it is not difficult to see that the equilibrium sharing of bonds at the end of the process should be an equal sharing across series. In fact, for bond 2, it is always optimal to choose the series with the lowest frequency when you are the last subscriber. It implies that, as long as the frequency of a given series is lower than 200,000, you can choose this series. The following rational subscribers will stop choosing a given series when the frequency will reach 200,000. Beyond this threshold, this choice becomes sub-optimal because there is at least another series with a lower number of subscribers. Consequently, if you believe that rationality is common knowledge, you can choose at random if the current sharing of bonds is the one given in table 1 or 2.

For bond 1, the story is a little bit different. If you think that others are like you, you should choose the same answer to questions 3 and 5 if you are a risk-averse expected utility maximize, assuming that the following subscribers will also bet with the crowd. The optimal choice if you obey OET is, as for bond 2, to invest at random, assuming the others also obey OET. Obviously, if you consider that a proportion of agents is risk-averse, you will never choose the highest frequency series, anticipating that it will be chosen by these expected utility maximizers.

III-4 The random choice of numbers

Finally assume that you have no information about the sharing of bonds across former subscribers. You are only told the way the bank will reimburse the issue. In this case, the probability distribution of returns is equal in each series, either for bond 1 or for bond 2. Your choice then should be random in the set of ten series. Therefore, the distribution of choices at the aggregate level should be uniform. We show in the next section that it is not the case.

IV The survey study

IV-1 The questionnaire

The study was realized during different finance courses in two French universities (University of Strasbourg and University of Clermont-Ferrand) and two business schools, EM Strasbourg Business School (France) and HEC Lausanne (Switzerland). 337 students participated, enrolled in economics, finance or accounting programs at the MSc level[6]. The complete questionnaire is provided in the appendix. The English version was used in Lausanne (all courses being taught in English at the MSc level) and a French version in the other programs. The numbers of students in the different locations are provided in table 4.

Table 4: Origin of participants

University or Business School	Number of students
EM Strasbourg Business School	41
University of Strasbourg	126
HEC Lausanne	74
University of Clermont-Ferrand	96
TOTAL	337

The participants had to answer 6 questions, divided into three groups of 2 questions, related to the lottery bonds presented in the preceding section. In each pair of questions, the first one is related to bond 1 and the second to bond 2. The required answers were simply numbers between 1 and 10 corresponding to the choice of a series number.

For the first two questions, participants were only told the characteristics of the bonds, without any other information, either on the table of payoffs or on the choice of former subscribers. For example, bond 1 was presented as follows.

[6] For the University of Strasbourg, 86 (40) students come from the MSc in Finance (Actuarial Studies), in Lausanne, 69 (5) come from the MSc in Finance (Actuarial Studies), in Clermont Ferrand, there were 43 students from the MSc in Finance, 39 from the MSc in Accounting and Control and 14 from the MSc in Economic Analysis.

A bank (called bank1) issues 1,000,000 one-year bonds at a price of 1€ each. The bank repays 1,050,000€ at the end of the year (a 5% interest rate on the issue). When buying one bond, subscribers choose an integer number between 1 and 10. On the repayment date, the bank draws at random a (lucky) number between 1 and 10 and first repays 1€ to each subscriber having chosen the lucky number. The remaining amount is equally shared among ***all*** *subscribers, including the winning ones.*

As mentioned before, with no information other than the way payoffs are defined, investors should be indifferent between the series; we then expect a random choice for the two first questions.

After having answered the two first questions, participants received the information summarized in table 5 (it corresponds to the first line of tables 1 and 2). It was provided on a slide, so all participants knew that everybody was receiving the same information. For questions 3 and 4, it was specified that the respondent was about to buy the last bond of the issue. In other words, everybody was able to infer the final distribution of payoffs.

Table 5: Information about choices of former subscribers (shown publicly on a Powerpoint slide)

Quantities already bought in each series				
Series1	Series2	Series3	Series4	Series5
100 000	150 000	80 000	120 000	60 000
Series6	Series7	Series8	Series9	Series10
140 000	70 000	50 000	130 000	100 000

A slight modification was introduced for the students at university of Clermont-Ferrand. They were shown table 5 for questions related to bond 1 (questions 3 and 5) and table A2 (see the appendix) for questions 4 and 6. In table A2, the quantities of bonds are the same but the numbers associated with these quantities are different. We then deal with the same set of cumulative distributions of returns but the number identifying a given distribution is not the same. The idea was to control for a possibility of "inertia" in the answers. We saw, in the preceding section, that the rational answer for question 4 is series n°8 but, it is also the answer to question 3 for agents obeying optimal expectations theory. Consequently, we had to check if some students were choosing the same number to answer the two questions, simply by applying a law of least effort. In fact, nobody in the subsample (96 students) chose (8, 8) to answer questions 3 and 4. There is then no reason to think that it is different for the other subsamples. In the subsample of 96 students, the rational answer to question 4 was n°4. We then also checked whether some students could have been considered rational "by inertia", that is by selecting the same answer for questions 3 and 4. More precisely, we counted the number of students having chosen the same number for questions 3 and 4, whatever this number was. Only 3 students made such a choice over the subsample of 96 students. Consequently, the results presented in the following cannot be invalidated with this argument.

In the third sequence of two questions (questions 5 and 6), the rule was that participants had to choose a series number with the same information as in questions 3 and 4, but they were told that one million bonds were still to be sold to other subscribers after their own choice. Moreover, participants were also informed that the future subscribers would get updated information about sales at the time of their own purchase. Therefore, participants had to build expectations about the decision rules of future subscribers. These two last questions are devoted to analyze the opinion of participants about the rationality of others, as in the usual beauty contest.

To present the results concerning questions 3 to 6 in a simple way, we use the numbering of series in table 5 for all the subsamples. Obviously, for questions 1 and 2, participants were not shown table 5 or table A2, so we keep the numbers they used to answer.

IV-2 Results

IV-2-1 Attractiveness of the best outcome

As mentioned before, the data provided in table 5 imply that agents obeying first-order stochastic dominance must choose series n°8 at question 4. 245 students over 337 made this choice. They are called “rational” in the following even if “wrong” answers to this question can simply be due to a lack of motivation to participate. Table 6 shows the percentage of rational answers in each training program.

Table 6: Numbers of « rational » participants across training programs

	Rational answers	Total	Percentage "rational"
HEC Lausanne	60	74	81,08%
EM Strasbourg Business School	34	41	82,93%
University of Strasbourg	90	126	71,43%
University of Clermont-Ferrand	61	96	63,54%
Total	245	337	72,70%

The lower percentage in Clermont-Ferrand is possibly due to the more complicated task students had to manage with different data for bonds 1 and 2. Apart from this, the results are not really surprising. We find the highest percentages in the two Business School programs in Strasbourg and Lausanne. The other programs are part of Departments of Economics and Management Science. University students are less used (and possibly less motivated) to participate in such surveys than students in Business Schools[7]. It may explain the significant difference between the proportions.

The detailed answers to the other 5 questions are given in table 7[8], panel A. The lines of the table are ranked according to the number of hypothetical subscribers in the series, not according to the series number. The main point concerns question 3. There is a clear preference for series n°8 since 56.61% of respondents chose this series. The second important observation is the 22.31% of participants having chosen series 2. They are either risk averse expected utility maximizers or CPT investors. The eight other possible answers have almost negligible frequencies since they gather around 20%. However, as mentioned in section 3, these answers could possibly be attributed to CPT investors with different parameters or weighting functions.

Our results show a strong preference, not only for skewed returns (all returns are skewed in our study) but for the most skewed return, and for the random payoff with the highest outcome. This result is then clearly in line with the predictions of optimal expectations theory.

It is also interesting to come back to the comparison of cumulative distribution functions of series 8 and 5 (provided in table 3), corresponding to the two first lines of table 7. It is remarkable that 56.61% of participants chose series 8 and only 2.89% series 5 when the difference between the two is only a swap of one cent between the two highest outcomes. It confirms the attractiveness of the highest outcome, as predicted in OET.

Panel B of table 7 provides the proportions of choices 2 and 8 in the different locations. The proportions in the two universities are very close to each other. No significant difference appears in the distributions. On the contrary, we observe a higher (lower) proportion of expected utility maximizers in the sample of HEC Lausanne (EM Strasbourg) but it is difficult to interpret these differences, taking into account the number of students in each subsample.

[7] For example, students in Business Schools are used to evaluate courses and teachers, a not so common practice in French universities.

[8] Table A3 in the appendix gives (for completeness of information) the answers of the global sample, including the participants qualified as “irrationals”.

Table 7

Panel A: Answers of « rational » participants

The figures are provided in percentage but the number of answers in each column varies from 241 and 244 (a few students left some questions unanswered). The answers for Q4 are not provided because, by construction, all "rational" respondents chose series n°8 for this question (except those in Clermont-Ferrand for which the rational answer was n°4).

BONDS BOUGHT	Q1	Q2	Q3	Q5	Q6
50000	8,30	8,26	56,61	20,25	27,05
60000	12,03	13,22	2,89	4,13	4,10
70000	16,60	15,29	3,72	8,68	9,02
80000	10,37	9,09	4,13	7,85	5,74
100000	15,77	15,29	2,89	6,61	6,15
100000	7,47	9,09	1,24	2,48	5,74
120000	7,05	5,37	1,24	2,48	2,46
130000	7,88	7,44	2,07	2,48	1,64
140000	3,73	4,96	2,89	6,61	7,38
150000	10,79	11,98	22,31	38,43	30,74
TOTAL	100,00	100,00	100,00	100,00	100,00

Panel B: Percentages of choices 2 and 8 in the different locations

EMS= EM Strasbourg Business School, LAU = HEC Lausanne, UDS = University of Strasbourg, UCF = University of Clermont-Ferrand. N is the number of "rational" participants in each subsample. The last line gathers all answers that do not correspond to choices 2 and 8.

	EMS	LAU	UDS	UCF
N	34	60	90	61
Series 8	82,35%	46,67%	54,44%	52,46%
Series 2	5,88%	31,67%	23,33%	19,67%
Other	11,76%	21,67%	22,22%	27,87%

IV-2-2 Opinions about the rationality of others

Questions 5 and 6 were designed as questions 3 and 4, except that participants were told that one million bonds were still to be sold after their own choice, the next subscribers choosing with updated information about sales.

As mentioned in the preceding section, concerning Q6, if participants were thinking that other subscribers are rational they should be indifferent between all solutions. We then expect a uniform distribution of choice. However, assuming that other subscribers are not completely rational, and have a "one-step" reasoning, leads you to play with the crowd, expecting that the others will stay in the low frequency series. In the same way, with a two-step reasoning, you should stay in the low frequency series. Figure 3 shows that answers to question 6 corresponds to agents mainly using a one or two-step reasoning. 141 participants (58%) chose numbers 2 or 8 corresponding to the two highest frequencies. Obviously, the uniform distribution hypothesis is rejected at conventional levels.

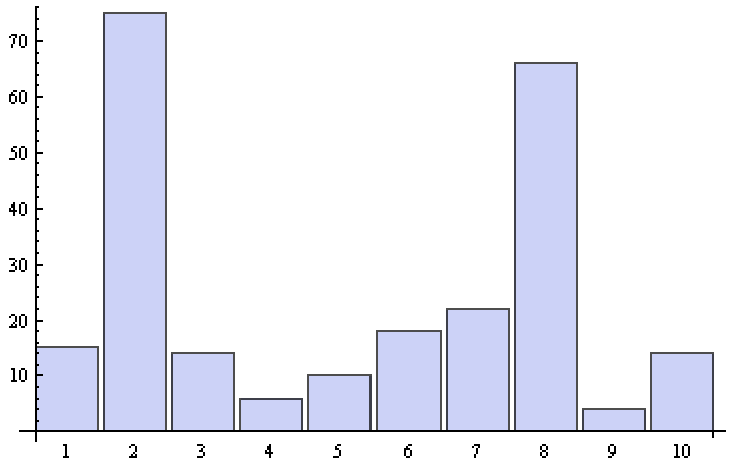


Figure 3: Bar chart of answers to question 6

Concerning questions 3 and 5, the panel A of table 8 summarizes the frequencies of choice of the 242 participants having answered the two questions. The two series with 100,000 former subscribers have been aggregated since they pay exactly the same payoffs. The first column (row) gives the answer to question 3 (5). We observe that among the 54 participants having chosen the highest frequency at question 3, 39 (72.2%) stay on that choice for question 5. It is rational since they want to bet with the crowd and hope that the others will do the same. We also observe the behavior already mentioned for question 6. Around one third of participants (47) obeying OET use a one step reasoning and switch to the highest frequency series at question 5. They expect that the others will continue to bet on the lowest frequency series (which then becomes a high frequency series!). 35 participants (25%) stay on their choice, using a two-step reasoning. The remaining 40% have a more sophisticated behavior by choosing other series. One more time, it is clear that the conditional distribution of answers to question 5 (conditioned on the choice of the lowest frequency series to question 3) is not uniform. The deviation from the uniform distribution ($\chi^2 > 140$)[9] is essentially due to the participants using a one-step or a two-step reasoning. They account for more than 75% of the χ^2 value.

[9] To calculate the χ^2 statistic, we grouped the two series with 120,000 and 130,000 subscribers due to the low frequencies of these series.

If we compare now the answers at questions 3 and 6 (table 8, panel B), almost the same comments can be done. We also reject the uniform distribution assumption for question 6, conditional on the choice of the lowest frequency at question 3 ($\chi^2 \approx 116$).

Table 8: Answers to the pairs of questions (3, 5) and (3, 6)

Panel A: Frequencies of pairs of choices for questions 3 and 5

Q3\5	50000	60000	70000	80000	100000	120000	130000	140000	150000
50000	35	5	14	7	13	4	3	9	47
60000	1	2	0	0	0	0	0	1	3
70000	0	0	7	1	1	0	0	0	0
80000	1	0	0	6	1	0	0	1	1
100000	4	0	0	1	5	0	0	0	0
120000	1	0	0	0	0	2	0	0	0
130000	0	0	0	0	0	0	2	0	3
140000	1	0	0	1	0	0	1	4	0
150000	6	3	0	3	2	0	0	1	39

Panel B: Frequencies of pairs of choices for questions 3 and 6

Q3\6	50000	60000	70000	80000	100000	120000	130000	140000	150000
50000	30	6	13	8	14	3	3	12	48
60000	1	0	0	1	0	0	0	1	4
70000	3	0	4	0	1	0	0	1	0
80000	3	1	1	1	0	0	0	1	3
100000	5	0	0	1	1	0	0	1	2
120000	0	0	1	0	0	0	0	0	2
130000	4	0	1	0	0	0	0	0	0
140000	2	0	0	0	2	2	0	1	0
150000	17	3	2	3	11	1	1	1	15

To sum up, it appears that the choices of participants do not confirm the assumption that rationality is common knowledge. The most frequent answers correspond to a one-step or two-step reasoning. This result is indeed not surprising since it is the most common values found in the literature for the depth of reasoning[10].

IV-2-3 Heuristics in random choice of numbers

We expect random choices to questions 1 and 2 because participants have no information about choices of former subscribers. Figure 4 shows the bar chart cumulating choices for these two questions. In case of a uniform distribution, we should have frequencies around

[10] See Nagel (1999) for a survey on beauty contest games.

48.3 which is the mean frequency of the uniform distribution. It appears that the actual answers are far from a uniform distribution. This hypothesis is clearly rejected at the 1% level. The χ^2 value is 63.97 for a 1% critical value of 21.66.

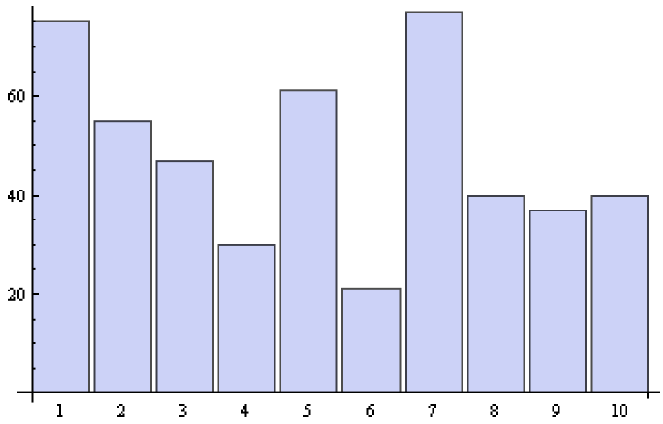


Figure 4: Bar chart of answers to questions 1 and 2

The popularity of numbers 7 and 5, representing respectively 15.94% and 12.62% of choices for questions 1 and 2, can be explained by comparable results for state lotteries. For example, in the French lotto game, players have to choose 5 numbers between 1 and 49 and (independently) a lucky number between 1 and 10. The sponsor of the game draws at random the winning combination and the lucky number. Since the start of this version of the game in October 2008, number 7 has been drawn 14 times[11] (see the detailed results in table 9). For these particular draws, the mean proportion of winners of the lucky number is 16.53% when 10% is expected if players choose their numbers at random[12]. For number 5, the mean proportion is 13% in a set of 10 draws, the minimum and maximum proportions being 12.16% and 13.79%.

The main difference between the results of the lotto game and ours concern number 1. However, this difference can possibly be justified by the fact that participants, considering all choices as equivalent, select number 1 which is obviously the first in the list of possible choices. In some sense, selecting number 1 could be interpreted as a random choice. But even if we share these answers between the ten numbers, the difference with a uniform distribution remains significant.

[11] For the 155 first draws up to 09/26/2009.
[12] The data on French lotto draws are provided on www.fdjeux.com and the percentage of lucky number winners is reported on www.sojah.com.

Table 9: Relative frequencies of numbers chosen by French lotto players

The column "Mean" gives the proportion of players having chosen the number in the first column over the number of draws in the fifth column. "Min" and "Max" are the corresponding minimum and maximum proportion over the same number of draws.

Number	Mean	Min	Max	Number of draws	Standard deviation
1	7,85%	7,36%	9,29%	17	0,56%
2	8,29%	7,77%	9,40%	14	0,38%
3	11,30%	10,76%	11,68%	18	0,22%
4	10,67%	9,72%	11,28%	20	0,35%
5	13,00%	12,16%	13,79%	10	0,51%
6	10,20%	9,82%	10,77%	14	0,27%
7	16,53%	15,75%	17,10%	14	0,37%
8	10,27%	9,92%	10,75%	17	0,22%
9	9,41%	8,94%	9,86%	15	0,22%
10	6,46%	5,98%	6,94%	16	0,26%

V Conclusion

In this paper, we analyzed the way people manage a simple financial decision making problem based on lottery bonds. Our purpose was to compare three theories, expected utility, cumulative prospect theory, and the more recent optimal expectations theory. The latter implies that agents are especially attracted by the best outcome and overvalue its probability of occurrence. To distinguish between skewness seeking and attractiveness of the best outcomes we designed a lottery bond in such a way that all choices generate positively skewed distributions. We show that more than 55% of participants to the survey select the choice with the highest outcome, controlling for the expected return. Moreover, this choice is dominated by all other choices when expected utility is used as the decision making tool. Our results show that gambling and investing cannot be treated separately, an idea that appeared in several recent papers. These results are then consistent with the theoretical analysis of Brunnemeier *et al.* (2007) and with the empirical study of Bali *et al.* (2009).

We also showed that people use heuristics to choose numbers at random, leading to non random choices at the aggregate level. Numbers 7 and 5 are especially popular and it comes with no surprise[13]. Finally, by introducing a kind of beauty contest in the questionnaire, we

[13] For example, Roger and Broihanne (2007) already showed that these numbers are among the most popular in the French lotto game.

observed that respondents do not assume that rationality is common knowledge, either because they recognize their own limited rationality or because they consider that other are not fully rational. The questions used in this paper are very simple and make the study easily replicable. We then hope that it will be replicated on other populations with different cultural backgrounds to test if our results can be generalized to alternative environments.

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APPENDIX: Questionnaire

BOND n°1

A bank (called bank1) issues 1,000,000 one-year bonds at a price of 1 € each. The bank repays 1,050,000 € at the end of the year (a 5% interest rate on the issue). When buying one bond, subscribers choose an integer number between 1 and 10. On the repayment date, the bank draws at random a (lucky) number between 1 and 10 and first repays 1 € to each subscriber having chosen the lucky number. The remaining amount is equally shared among **all** subscribers, including the winning ones.

- **Q1: Suppose that you buy one bond, which number do you choose?** ☐

BOND n°2

A bank (called bank2) issues 1,000,000 one-year bonds at a price of 1 € each. The bank repays 1,050,000 € at the end of the year (a 5% interest rate on the issue). When buying one bond, subscribers choose an integer number between 1 and 10. On the repayment date, the bank draws at random a (lucky) number between 1 and 10 and first shares equally 100,000 € among subscribers having chosen the lucky number. The remaining amount is equally shared among **all** subscribers, including the winning ones.

- **Q2: Suppose that you buy one bond, which number do you choose?** ☐

The table on the screen gives the number of bonds already sold for each choice of the lucky number.

- **Q3:** If you are about to buy the **last** bond issued by bank 1, which number do you choose? ☐
- **Q4:** If you are about to buy the **last** bond issued by bank 2, which number do you choose? ☐
- **Q5:** Assume now that 2,000,000 bonds were issued by bank 1. If you buy a bond, knowing that 1,000,000 more bonds will still be sold after your choice (the next subscribers being fully informed about the evolution of choices by the updating of the table on the screen), which number do you choose? ☐
- **Q6:** Assume now that 2,000,000 bonds were issued by bank 2. If you buy a bond, knowing that 1,000,000 more bonds will still be sold after your choice (the next subscribers being fully informed about the evolution of choices by the updating of the table on the screen), which number do you choose? ☐

Table A1

Table shown (on a Powerpoint slide) to the participants after they answered questions 1 and 2

Quantities already bought in each series				
Series1	Series2	Series3	Series4	Series5
100 000	150 000	80 000	120 000	60 000
Series6	Series7	Series8	Series9	Series10
140 000	70 000	50 000	130 000	100 000

Table A2

Table shown (on a Powerpoint slide) to the participants in Clermont Ferrand for questions 4 and 6

Quantities already bought				
Series1	Series2	Series3	Series4	Series5
140 000	70 000	130 000	50 000	100 000
Series6	Series7	Series8	Series9	Series10
100 000	120 000	80 000	150 000	60 000

Table A3

Complete results for the sample of 337 participants

SERIES NUMBER	BONDS BOUGHT	Q1	Q2	Q3	Q4	Q5	Q6
1	100000	56	58	14	10	30	25
2	150000	33	35	75	41	116	106
3	80000	38	33	19	8	24	21
4	120000	24	24	8	5	11	10
5	60000	48	45	16	12	20	15
6	140000	13	14	13	9	17	21
7	70000	49	46	12	4	25	26
8	50000	25	25	160	245	70	84
9	130000	21	19	9	1	8	8
10	100000	26	33	8	2	12	19