

# Real option financing under asymmetric information\*

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## Abstract

We extend a standard model of financing under asymmetric information to the case where the investment opportunity is a real option. An initial investment gives access to a public signal that takes the form of a poisson process of unknown parameter. Observing the realization of this process through time generates information on the value of implementing a project, but is costly because it delays cash-flows. The project is owned by a cash-constrained entrepreneur who needs an outside investor to finance the initial investment, as well as a potential future development. An adverse selection problem arises, as the entrepreneur receives some private information about the profitability of the project and enjoys private benefits from the moment where it is fully implemented. This gives him an incentive to hurry implementation by overstating the project prospects. In line with common practices in venture capital, we show that it is optimal to include investment timing in the financial contract (“ex-ante staging”) as an instrument to induce information revelation. This creates however a distortion towards late investment. Furthermore the adverse selection problem may lead to a complete market breakdown where the initial investment cannot be financed. We show that cash holdings of the entrepreneur accelerate investment and increase risk-taking. We derive empirical predictions about the relationships between pay, performance, investment timing and corporate governance.

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# 1 Introduction

Learning plays a crucial role in the development of a wide range of projects. Because the eventual profitability of a venture is uncertain, gathering information in the early stages of the implementation may significantly improve the efficiency of future choices. Among those choices, a critical decision is whether to engage further in the project development by making additional investments. A project implementation can therefore be described as a sequential investment problem where early investments serve as tests of the project viability. The empirical literature on venture capital suggests that investment sequentiality is a common practice for small and innovative firms (Gompers (1995), Sahlman (1990)). The high degree of uncertainty inherent to entrepreneurial projects imposes indeed information acquisition through staged investment procedures (Bergemann and Hege (1998) and (2005)). More generally, firms which rely heavily on research and development are characterized by that same sequentiality, a typical example being the pharmaceutical industry (Guedj and Scharfstein (2004), Danzon et al (2005)). The objective of this paper is to study how these types of projects are financed, in the presence of capital market imperfections.

The real option theory provides a comprehensive framework to think about information acquisition and investment<sup>1</sup>. In a seminal paper, McDonald and Siegel (1986) consider a firm with a project requiring a sunk cost, and show that there exists a “value of waiting to invest”, corresponding to the benefit of learning about the expected value of that project before making an irreversible investment decision. Learning occurs through the observation of a signal that continuously delivers information about the value of future cash-flows. The key question is then to set an optimal investment rule that reflects the trade-off between waiting for more precise information and the cost of delaying cash-flows.

Implicitly, this model, as most real option models, assumes that the option holder has some cash available to cover investment costs. Separating the owner of the option, say an entrepreneur, from the investor who provides funding would not affect the analysis as long as they share the same information. However, the problem becomes more complex as soon as informational asymmetries arise, which seems a reasonable assumption in the context of corporate financing. In particular, the hypothesis that the entrepreneur has some private information about the profitability of his project is central in a stream of theoretical and empirical literature, following the seminal contributions of Stiglitz and Weiss (1981), Myers and Majluf (1984), Greenwald, Stiglitz and Weiss (1984), and De Meza and Webb (1987)<sup>2</sup>. Furthermore informational asymmetries are more likely when projects are innovative and public information or benchmarks are scarce, which corresponds also to a situation where investing in information is particularly valuable. Because informational asymmetries may create adverse selection problems, a natural question is then whether this market imperfection has an impact on learning and investment decisions.

To answer this question we extend a simple model of real option to allow for information asymmetries, and study a financing game. We consider an entrepreneur endowed with a project that requires an irreversible investment and generates a stream of cash-flows. The expected profitability of this productive investment is however uncertain, and may be negative if the project is of bad quality. This creates a value for learning before deciding whether to invest. Information acquisition requires an initial investment giving access to a public signal that takes the form of a poisson process of strictly positive parameter conditional on the project being

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<sup>1</sup>Dixit and Pindyck (1994) give a general presentation on real options.

<sup>2</sup>A comprehensive presentation of adverse selection problems in corporate finance can be found in Tirole (2006), chapter 6.

bad. This investment is a pure informational cost, which initiates a learning period that can be interpreted as a development phase. The time at which the productive investment takes place, if it takes place at all, depends then on what is learned following the initial investment.

The entrepreneur is cash-constrained and needs therefore an outside investor to finance the initial investment and possibly the productive investment. An adverse selection problem arises as a consequence of two related assumptions. First, we assume that in the development period which follows the initial investment, the entrepreneur receives some private information about the probability of failure of the project, that cannot be communicated in a verifiable way to the investor. Even an active investor, such as a venture capitalist, cannot perfectly monitor the entrepreneur and directly verify that all information is truthfully transmitted (Gompers 1995). Furthermore, even if he could obtain that information, the financier may lack the expertise required to correctly interpret it.

Second, the entrepreneur has a vested interest in the project, in that he derives some private utility from operating the project at full-scale once the development phase is completed and the productive investment is made. This can be interpreted as private benefits, as for instance in Stein (1997)<sup>3</sup>, or as an agency rent from an ex-post moral hazard problem, due to contract incompleteness as in Hart and Moore (1994), or to hidden effort as in Holmström and Tirole (1997). This assumption is also consistent with the idea that an entrepreneur is eager to see his project going through the successive development and financing stages (Hsu 2002). The entrepreneur has therefore an intrinsic preference for investing early, which gives him incentives to distort his private signal by always reporting good news in order to hurry investment. Solving the financing game consists then in deriving a contract between the entrepreneur and the investor which induces truthful reporting.

The analysis delivers rich set of insights. First, it is optimal to include in the financial contract, on top of the cash-flow rights of both parties, the conditions under which the productive investment takes place as a function of the flow of information generated by the initial investment. Because the entrepreneur derives private benefits from the moment where the productive investment is sunk, investment timing can indeed be used to elicit information. This is in line with common practices in venture capital or private equity where term sheets often contain covenants making the future release of funds contingent on measures of performance (Kaplan and Stromberg (2003) and (2004)). Second, because the investment rule is used as an instrument to relax the adverse selection problem, investment timing may be delayed relative to the first-best. Delaying the investment of an entrepreneur who reports good news decreases indeed his incentive to lie when he receives bad news, and therefore the cost of inducing truthful reporting. However, investment may occur earlier or later than what would maximize the financial value of the firm. Furthermore entrepreneurs with more cash tend to invest earlier. Intuitively, distortions occur when the pledgeable income available to induce truthful reporting is too low, a constraint which is relaxed by any personal investment the entrepreneur can make. These findings match the results of the empirical study by Guedj and Scharfstein (2004) on drug development. Third, the adverse selection problem may even lead to a complete market breakdown where the initial investment cannot be financed. This happens when private information is a critical input to set the optimal timing of the productive investment, but the maximal pledgeable income available to induce the entrepreneur to reveal information is too low. Fourth, we show that it may be optimal to reward the entrepreneur following a bad performance. The fundamental problem in our model is indeed the reluctance of the entrepreneur to reveal bad

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<sup>3</sup>In a real option setting, Morellec (2004) and Barclay et al. (2006) also assume that private benefits from investment drive a wedge between shareholders and the manager.

news, because it delays investment and reduces the expected value of private benefits. One way to overcome this agency problem is to compensate the manager when he acknowledges lower profit expectations. This gives a rationale for managerial incentives seemingly unrelated to performance, or even negatively correlated to performance, such as stock options repricing or severance payments. Interestingly, these compensation schemes do not emerge here as the result of managerial discretion over compensation setting, but as part of an optimal contract between the entrepreneur and investors. This in line with empirical studies by Yermack (2006), Chidambaran and Prabhala (2003) and Brenner et al (2000). We also show that executive packages announcements may convey information on operational prospects and therefore generate a stock price reaction. Finally, we suggest that firms with a better corporate governance are more reactive to information, while firms with a poorer governance exhibit inertia in investment decisions.

In the final part of this article, we look at the case where the entrepreneur has private information from the beginning (i.e. before contracting with the investor). This can be seen as direct extension of standard models of adverse selection in finance (Stiglitz and Weiss 1981) to the case where investment generates information instead of cash-flows. We show that the distortion in investment timing is then even higher than under the previous specification. We derive stock options with a vesting period as part of the optimal incentive scheme, a type of incentive often used in venture capital (Kaplan and Stromberg (2003) and (2004)). We obtain additional empirical predictions on the link between the type of compensation given to the entrepreneur and the expected profitability of the firm, and the impact of the initial reputation of the entrepreneur on the duration of the development phase.

The remainder of the paper is structured as follows. Section 2 discusses how the paper relates to the existing literature. Section 3 introduces the model and derives a symmetric information benchmark. Section 4 solves the model under asymmetric information, section 5 discusses the results and the predictions of the model, section 6 looks at the case where the entrepreneur has private information before contracting, section 7 concludes. All proofs are in the appendix.

## 2 Literature

This article is related to the literature on real options and agency. A first series of papers looks at investment timing, when outside financing is limited. Boyle and Guthrie (2003) show that credit constraints may either hurry or delay investment. Distortions are the consequence of the uncertainty created by the random fluctuation of cash reserves. Investment may be delayed because external financing is restricted, and the internal financing capacity of the firm is too low at the moment where investment would be optimal. It may be hurried when internal financing is sufficiently high, and the firm is worried that a negative shock on cash reserves could constrain investment if it were to wait some more. Belhaj and Djembissi (2007) also assume that firms have a limited debt capacity, but focus on the impact of the financial structure on investment timing. They show in particular that relaxing credit constraints and allowing for a higher leverage may accelerate or delay investment, depending on the relative weight of the tax shield and the default costs in the investment decision. However, these papers do not model explicitly the financing constraint which remains an exogenous borrowing limit. In contrast, we propose a setting where the external financing capacity of the entrepreneur is endogenously determined by a fundamental adverse selection problem, and focus on the form of the optimal financial contract.

In a second stream of papers, the agency conflict stems from managerial discretion over the option exercise. Mauer and Sarkar (2005) look at an option to invest, and assume that a firm

and a bank agree in advance on the terms of a revolving credit line on which the firm can draw at the time where investment takes place. The presence of debt induces an equity-maximizing manager to hurry investment ex post which reduces ex-ante the value of the bondholders' claim and generates an agency cost of debt. In a more general model allowing for both asset expansion and substitution, Childs et al. (2005) derive also an agency cost of debt resulting from the disagreement between shareholders and bondholders on the optimal exercise policy. They argue however that by allowing for a more frequent repricing, short-term debt eliminates the agency conflict. In a similar vein, Décamps and Faure-Grimaud (2002) show that an indebted entrepreneur has less incentives to exert his option to default, and leveraged firms are therefore prone to excessive continuation. Finally, Morellec (2004) and Barclay et al. (2006) look at the decision to exert a growth option when the manager and the shareholders disagree on the optimal investment policy. They show that debt as well as the external market for corporate control are (costly) disciplining instruments for the manager. A key assumption in this series of papers is that the option exercise is not contractible, creating a scope for managerial opportunism. In contrast, we allow for the investment timing to be part of the contract between the entrepreneur and the investor. Whereas this would solve the agency problem in the previous settings, it may create inefficiencies when information asymmetries are introduced<sup>4</sup>. The entrepreneur may indeed propose a contract which entails investment distortion and reduces the global value of the venture, in order to be able to raise funds.

Closer to our model, Grenadier and Wang (2005), and Mæland (forthcoming) propose a setting where investment timing is contractible, and the optimal contract under adverse selection implies a distortion towards late investment<sup>5</sup>. Our analysis differs however from their approach in two respects. First, they are interested in a delegation problem, where a principal with no cash constraints owns an option and the investment decision is left to an agent with private information. Our focus is rather on a financing problem where a cash-poor agent owns an option and needs financing from an uninformed party. This allows to relate investment distortions to financial constraints. Second, the nature of the agency problem is different. Grenadier and Wang, and Mæland build on a cash-diverting setting where the agent has some private information on the productive investment cost (or equivalently on an additional payoff that this investment could generate), and may be tempted to overstate it in order to pocket the difference between the announced cost and the real cost. The agent has then an intrinsic preference for late investment, while the entrepreneur has an intrinsic preference for early investment in our model<sup>6</sup>. More importantly, we assume that the information asymmetry affects the interpretation of the stochastic process through which both the entrepreneur and the investor learn. This has consequences on the form of the optimal incentive scheme, and in particular on the existence of a vesting period in the case where the entrepreneur has private information at the contracting date. Furthermore, we obtain different distortion patterns, in particular cases of bunching where private information is not revealed in equilibrium, and cases where the order in which types of entrepreneurs invest is reversed compared to the symmetric information benchmark.

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<sup>4</sup>Barclay, Smith and Morellec (2006) suggest that the manager discretion over the decision to invest is due to his superior information on investments profitability. However, they do not study a mechanism that could elicit private information.

<sup>5</sup>Grenadier and Wang's setting features also a moral hazard problem.

<sup>6</sup>Equivalently, an entrepreneur with a good project has an incentive to pretend to hold a bad project in Grenadier and Wang, and Mæland, while an entrepreneur with a bad project has an incentive to pretend to hold a good project in our setting.

### 3 Description of the model

#### 3.1 The project

Time is continuous, indexed by  $t$ ,  $r > 0$  denotes the players' common discount rate. The project can be launched at any point in time, and has the following characteristics. It requires an investment  $I$  and generates a profit  $\mu + b$  per unit of time until a final date, determined by the first jump of a poisson process<sup>7</sup>. There is some uncertainty about the parameter of this process, it may be equal to 0, in which case the project is good and never stops, or to  $\lambda > 0$ , in which case the project is bad and may stop at each date with an instantaneous probability equal to  $\lambda$ . At the time where the project is launched, the expected profit is therefore  $(\mu + b)/(r + \lambda)$  if the project is bad,  $(\mu + b)/r$  otherwise.

We assume that only good projects are profitable in expectation:

$$(A1) \quad \frac{\mu + b}{r} > I$$

and

$$(A2) \quad \frac{\mu + b}{r + \lambda} < I.$$

The option feature of the model lies in the possibility to learn about the value of the project before investing  $I$ . By making an initial investment  $I_0$  at  $t = 0$ , agents can indeed observe the realization of the poisson process through time without launching the project. Investing  $I_0$  does not generate any cash flow, but initiates a learning phase, where agents continuously update their beliefs about the probability of holding a good project.

Learning occurs in the following way. Agents start with a prior belief  $p_0$  that the project is good, and we denote  $p_t$  the belief that the project is good given the realization of the process from 0 to  $t$ . As soon as a jump occurs,  $p_t$  goes down to 0, and it is known for sure that the project is bad. If this happens before  $I$  was sunk, it prevents an unprofitable investment. On the contrary, as long as no jump occurs,  $p_t$  increases deterministically and agents become more and more optimistic about the profitability of the project.

Formally, conditional on no jump in a time interval  $dt$ ,  $p_t$  evolves as follows:

$$p_t + dp_t = \frac{p_t}{p_t + (1 - p_t)(1 - \lambda dt)}. \quad (1)$$

A natural interpretation of this setup is as follows. An entrepreneur designed the plans of a new appliance, but is worried that his innovation may be flawed by some unforeseen imperfection. If that was the case, the defect would become apparent at some (random) date in the future and all potential profits would vanish from this point on.  $I_0$  is then the cost of building a prototype in order to run tests. If they detect some major problem, the project can be stopped and  $I$  is saved. On the other hand, if tests have shown no evidence of an imperfection after some time, the probability that a failure will occur in the future becomes sufficiently low to invest  $I$  and go to mass-production<sup>8</sup>.

From assumption (A2), there is value of waiting to invest when  $p_t$  is low, because it decreases the probability of investing in an unprofitable project. However, because agents have a strictly

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<sup>7</sup>For clarity, we postpone the explanation of the instantaneous profit decomposition into two components  $\mu$  and  $b$  to subsection 3.2 below.

<sup>8</sup>Décamps and Mariotti (2004) use a similar learning technology.

positive discount factor, waiting to invest is also costly as it delays revenues. The optimal investment rule should therefore reflect the tradeoff between the costs and the benefits of waiting.

### 3.2 Players and information structure

There are two risk-neutral players, an entrepreneur who owns the project, and an investor. The entrepreneur has limited liability and no cash<sup>9</sup>. He can therefore finance neither  $I_0$  nor  $I$ . The investor is willing to finance  $I_0$  and potentially  $I$  as long as he breaks even in expectation.

The informational asymmetry between the investor and the entrepreneur is driven by the following hypothesis. Once the initial investment  $I_0$  is made, the entrepreneur receives an imperfect signal  $\theta$ , that can be high ( $\theta = h$ ) or low ( $\theta = l$ ). To simplify the exposition, I assume that the signal is received at  $t = 0$ , just after  $I_0$  is sunk, but it could be received later, or even at a random date unknown to the investor, without affecting the analysis<sup>10</sup>. What matters is that at some point after the initial investment, the entrepreneur learns some private information. This specification is meant to capture in the most simple way the idea that part of the information which is learned in the development phase cannot be controlled by the outside investor.

Receiving a high signal is good news, as the probability  $\alpha$  that  $\theta = h$  when the project is good is strictly greater than the probability  $\beta$  of that same event conditional the project being bad. Furthermore, conditional on the project being good or bad,  $\theta$  and the poisson process are independant.

We denote  $p_h$  (respectively  $p_l$ ) the posterior belief of the agent at date 0 given that  $\theta = h$  (respectively  $\theta = l$ ),

$$p_h = \frac{p_0\alpha}{p_0\alpha + (1-p_0)\beta} \text{ and } p_l = \frac{p_0(1-\alpha)}{p_0(1-\alpha) + (1-p_0)(1-\beta)}.$$

We will refer to “high” and “low” as the type of the entrepreneur, and denote  $q = p_0\alpha + (1-p_0)\beta$ , the probability that the agent receives a high signal (and is therefore a high type).

It is worth stressing that  $\theta$  is the only private information of the agent, everything else being public information. In particular, the realization of the poisson process remains observable to both players.

Because our objective is to study the impact of private information on investment timing, we want to ensure that it is still efficient to learn, even following a high signal. This motivates the following assumption,

$$\text{(A3)} \quad p_h \frac{\mu+b}{r} + (1-p_h) \frac{\mu+b}{r+\lambda} - I < 0 \iff p_h < (I - \frac{\mu+b}{r+\lambda}) / (\frac{\mu+b}{r} - \frac{\mu+b}{r+\lambda}).$$

(A3) implies also that investing  $I_0$  is a necessary step, as directly investing  $I$  would yield negative profits in expectation.

Finally, a key assumption of our model is that only part of the surplus generated by the project can be pledged to the investor. More precisely,  $\mu$  can be taken out of the instantaneous profit to repay the investor, but the other part,  $b$ , goes to the entrepreneur. As discussed in the introduction,  $b$  can be interpreted as a private benefit from operating the project, or as a agency rent from an ex-post moral hazard problem.

<sup>9</sup>This last assumption is made for simplicity only. The analysis of the case where the entrepreneur can finance part of the project is similar. This point is discussed in section 5.2.

<sup>10</sup>A more precise discussion of this point is in the appendix, in the subsection called “Private signal timing”.

### 3.3 Optimal investment timing under symmetric information

The first-best investment rule maximizes the total value of the project. Its derivation is as in Décamps and Mariotti (2004). Paying  $I_0$  is equivalent to buying a call option with an infinite maturity, where the value of the underlying asset is driven by  $p_t$ . The optimal strategy is then to invest when  $p_t$  reaches a threshold  $p^*$ . It is more convenient to formulate the optimal investment rule in terms of belief  $p_t$  rather than time. This is equivalent in this setting since beliefs increase strictly and deterministically through time as long as no jump occurs.

Denote  $V(p_t)$  the value of the option at  $t$  for an arbitrary investment threshold  $\hat{p}$ . As long as  $p_t < \hat{p}$ , the expected value of the option at  $t + dt$  is

$$V(p_t) + dV(p_t) = \left[ V(p_t) + \dot{V}(p_t)dp_t \right] [p_t + (1 - p_t)(1 - \lambda dt)].$$

Because players are risk-neutral and have a common discount rate  $r$ , this last expression must be equal to  $V(p_t) + V(p_t)r dt$ . This results in a Bellman equation which, using (1), yields the following differential equation:

$$\dot{V}(p_t)p_t(1 - p_t)\lambda = V(p_t)[r + (1 - p_t)\lambda]. \quad (2)$$

Guess that the general solution to (2) has the following form:

$$V(p_t) = \gamma p_t \left( \frac{p_t}{1 - p_t} \right)^{\frac{r}{\lambda}}.$$

The investment rule prescribes to invest when  $p_t$  reaches  $\hat{p}$ .  $V$  satisfies therefore the following boundary condition:

$$V(\hat{p}) = \hat{p} \frac{\mu + b}{r} + (1 - \hat{p}) \frac{\mu + b}{r + \lambda} - I.$$

This yields

$$\gamma = \frac{1}{\hat{p}} \left( \frac{1 - \hat{p}}{\hat{p}} \right)^{\frac{r}{\lambda}} \left[ \hat{p} \frac{\pi}{r} + (1 - \hat{p}) \frac{\pi}{r + \lambda} - I \right].$$

Optimizing  $V(p_t)$  with respect to  $\hat{p}$  gives then the value of the optimal investment threshold,

$$p^* = \left( 1 + \frac{r}{\lambda} \right) \frac{I - \frac{\mu + b}{r + \lambda}}{I}.$$

The following notation is useful and has an intuitive interpretation,

$$D(p, \hat{p}) = \frac{p}{\hat{p}} \left( \frac{p}{1 - p} \right)^{\frac{r}{\lambda}} \left( \frac{1 - \hat{p}}{\hat{p}} \right)^{\frac{r}{\lambda}}. \quad (3)$$

$D(p, \hat{p})$  is the expected present value of a claim to 1 euro at the first date where  $p_t$  reaches  $\hat{p}$  starting from  $p$ , if this ever happens<sup>11</sup>. The expected value  $\pi(p, \hat{p})$ , created by the initial investment  $I_0$ , given a prior belief  $p$  and an investment rule  $\hat{p}$  is then

$$\pi(p, \hat{p}) = D(p, \hat{p}) \left[ \hat{p} \frac{\mu + b}{r} + (1 - \hat{p}) \frac{\mu + b}{r + \lambda} - I \right].$$

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<sup>11</sup>To see this, it suffices to do the same exercise using the boundary condition by  $V(\hat{p}) = 1$ .

To simplify notation further, we introduce two complementary reduced forms.  $\pi^f(p, \hat{p})$  corresponds to the financial value created by that same investment given a prior belief  $p$  and an investment rule  $\hat{p}$ ,

$$\pi^f(p, \hat{p}) = D(p, \hat{p}) \left[ \hat{p} \frac{\mu}{r} + (1 - \hat{p}) \frac{\mu}{r + \lambda} - I \right].$$

$\pi^b(p, \hat{p})$  corresponds to the expected value of private benefits for the entrepreneur given a prior belief  $p$  and an investment rule  $\hat{p}$ ,

$$\pi^b(p, \hat{p}) = D(p, \hat{p}) \left[ \hat{p} \frac{b}{r} + (1 - \hat{p}) \frac{b}{r + \lambda} \right].$$

Obviously, the following relationship holds,  $\pi(p, \hat{p}) = \pi^f(p, \hat{p}) + \pi^b(p, \hat{p})$ . Notice also that  $\pi^b(p, \hat{p})$  is strictly decreasing in  $\hat{p}$ , reflecting that the entrepreneur has a preference for early investment.

The next assumption simply states that the project can be financed under the first-best investment rule when information is symmetric:

$$\text{(A4)} \quad q\pi^f(p_h, p^*) + (1 - q)\pi^f(p_l, p^*) \geq I_0.$$

This ensures that any distortion arising under asymmetric information is a consequence of the adverse selection problem.

### 3.4 Contracting

Before  $t = 0$ , the entrepreneur makes a take-it-or-leave-it offer to the investor. Because the entrepreneur will receive private information after the initial investment  $I_0$ , he has to be induced to reveal his signal. The entrepreneur and the investor should therefore agree on a menu of contracts from which the entrepreneur will choose once he learns his private signal  $\theta$ . From the revelation principle, we can restrict attention to a menu of two contracts indexed by  $\tilde{\theta} \in \{h, l\}$ , which are such that the entrepreneur picks the contract that corresponds to his private signal. Contracts have the following form: they specify a lump-sum transfer  $w_{\tilde{\theta}}$  to the entrepreneur at the date where he picks one contract, and a date  $T_{\tilde{\theta}}$  at which the productive investment takes place if no jump was observed before. In principle, payments could be made contingent on the realization of the poisson process. However, in this case where private information is learnt after contracting, this would not allow to increase the expected utility of the entrepreneur<sup>12</sup>. We restrict attention to this form of contract, in order to simplify the exposition (a more complete analysis is in the appendix).

If the investor accepts the contract,  $I_0$  is sunk, the agent learns  $\theta$ , chooses one contract, and the learning process starts. If a jump occurs before  $T_{\tilde{\theta}}$  the project is abandoned. If not, the productive investment takes place, the project is launched, and possibly stops if a jump occurs later on.

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<sup>12</sup>This is not true in the case where the entrepreneur has private information before signing the contract, as will be seen in section 6.

## 4 Equilibrium analysis

### 4.1 Incentive compatibility constraints

The contractual investment rule impacts the expected payoff of the entrepreneur through the private benefit component  $b$ : the longer the development period  $T_{\hat{\theta}}$ , the lower the expected value of that rent for the entrepreneur. In order to write incentive compatibility constraints which ensure that each type will indeed pick the contract designed for him, we need to derive the utility from private benefits that an entrepreneur could expect if he was to deviate and pick the contract designed for the other type. As already argued, beliefs increase strictly and deterministically as long as no jump occurs. Given an initial belief  $p_{\theta}$ , there exists therefore a one-to-one mapping between any investment timing  $T_{\hat{\theta}}$  specified in a contract, and a threshold  $\hat{p}_{\theta}(T_{\hat{\theta}})$  at which investment takes place, and  $\hat{p}_{\theta}(\cdot)$  is strictly increasing.

Formally,

$$\hat{p}_{\theta}(T_{\hat{\theta}}) = \frac{p_{\theta}}{p_{\theta} + (1 - p_{\theta})e^{-\lambda T_{\hat{\theta}}}}. \quad (4)$$

Denote  $T_h^*$  and  $T_l^*$  the investment timings which implement the first-best investment rule for each type. From subsection 3.3 above, they are defined by the following equations,  $\hat{p}_h(T_h^*) = \hat{p}_l(T_l^*) = p^*$ . Because the initial belief of a low type  $p_l$  is strictly lower than the initial belief of a high type  $p_h$ ,  $T_l^* > T_h^*$ . This last inequality gives the essence of the adverse selection problem: efficiency would require for a low type to wait some more as he received bad news, but since the entrepreneur strictly prefers early investment, all other things being equal, he has an incentive to always report a high signal in order to shorten the development period.

Equipped with this notation, the incentive compatibility constraint of a low type writes:

$$\pi^b[p_l, \hat{p}_l(T_l)] + w_l \geq \pi^b[p_l, \hat{p}_l(T_h)] + w_h. \quad (5)$$

Conversely, the IC constraint of the high type is  $\pi^b[p_h, \hat{p}_h(T_h)] + w_h \geq \pi^b[p_h, \hat{p}_h(T_l)] + w_l$ .

### 4.2 The optimization program

An optimal contract should maximize the expected utility of the entrepreneur, subject to incentive compatibility constraints and the participation constraint of the investor. It solves therefore the following optimization program.

$$(P1) : \max_{T_h, T_l, w_h, w_l} q \{ \pi^b[p_h, \hat{p}_h(T_h)] + w_h \} + (1 - q) \{ \pi^b[p_l, \hat{p}_l(T_l)] + w_l \},$$

$$s.t. \quad \pi^b[p_h, \hat{p}_h(T_h)] + w_h \geq \pi^b[p_h, \hat{p}_h(T_l)] + w_l, \quad (6)$$

$$\pi^b[p_l, \hat{p}_l(T_l)] + w_l \geq \pi^b[p_l, \hat{p}_l(T_h)] + w_h, \quad (7)$$

$$q \{ \pi^f[p_h, \hat{p}_h(T_h)] - w_h \} + (1 - q) \{ \pi^f[p_l, \hat{p}_l(T_l)] - w_l \} \geq I_0, \quad (8)$$

$$w_l \geq 0, w_h \geq 0. \quad (9)$$

(6) and (7) are incentive compatibility constraints respectively for the high type and the low type, (8) ensures that the investor makes non-negative profits, and (9) is the limited liability constraint of the entrepreneur.

Notice that, in spite of the dynamic created by the learning process, the optimization problem is static since everything can be contracted upon at the initial date. The resolution proceeds therefore as usual in models of adverse selection. First, it is easy to see that the participation constraint (8) of the investor is binding at optimum. If it was not, increasing both  $w_h$  and  $w_l$  by some  $\epsilon$  would increase the objective function without affecting the other constraints. A direct consequence is that the entrepreneur simply seeks to maximize the social surplus generated by the project,  $q\pi[p_h, \hat{p}_h(T_h)] + (1-q)\pi[p_l, \hat{p}_l(T_l)] - I_0$ . Second, the incentive compatibility constraint of the high type (6) can be ignored. If a solution to the optimization program exists, then it is always possible to leave the low type with just enough utility to make him indifferent between the two contracts, in which case the high type strictly prefers the contract designed for him<sup>13</sup>. Investment distortions are therefore driven by the combination of the incentive compatibility constraint of the low type (7) and the limited liability constraints (9).

Building on these intuitions, a simplified program obtains

**Lemma 1** *The optimization program (P1) is equivalent to*

$$\max_{T_h, T_l} q\pi[p_h, \hat{p}_h(T_h)] + (1-q)\pi[p_l, \hat{p}_l(T_l)],$$

$$s.t. \quad q\pi^f[p_h, \hat{p}_h(T_h)] + (1-q)\pi^f[p_l, \hat{p}_l(T_l)] - I_0 \geq (1-q) \{ \pi^b[p_l, \hat{p}_l(T_h)] - \pi^b[p_l, \hat{p}_l(T_l)] \}, \quad (10)$$

$$T_h \leq T_l. \quad (11)$$

**Proof** In the appendix.

Condition (10) has a natural interpretation. The LHS of the inequality is the expected value of the financial income generated by the project net of repayments to the investor. The RHS is the increase in utility from private benefits that a low type could enjoy by investing at the same time as a high type, rather than waiting until  $T_l$ , multiplied by the probability of a low signal. What (10) imposes is therefore that the expected value of financial revenues available for the entrepreneur should be sufficiently high to cover the payment necessary to induce truth-telling when he gets a low signal.

From (11), it is always the case that a high type invests before a low type. Although the adverse selection problem may create investment timing distortions, the order in which types invest remains as in the symmetric information benchmark<sup>14</sup>. The conditions under which those distortions may occur are the object of the next subsection.

### 4.3 Optimal investment timing under asymmetric information

We denote  $T_h^{sb}$  and  $T_l^{sb}$  the two investment rules that solve (P1') when a solution exists. One of the key idea of this paper is that the entrepreneur may optimally propose a contract where

<sup>13</sup>Formally, the Spence-Mirrlees condition holds.

<sup>14</sup>The order in which types of entrepreneur invest can be reversed when the entrepreneur has private information at the contracting date. This is the object of section 6.

investment timing is distorted compared to the first-best, in order to be able to raise funds.

Distortions are however not systematic. From the optimization program ( $P1'$ ), the investment timings that maximize the unconstrained objective function are  $T_h^*$  and  $T_l^*$ , in which case investment takes place for both types with the belief  $p^*$ . Furthermore they satisfy (11). The condition under which the project can be financed without distortions is therefore that (10) should be satisfied when both types follow the first-best investment rule,

$$q\pi^f(p_h, p^*) + (1 - q)\pi^f(p_l, p^*) - I_0 \geq (1 - q) \left\{ \pi^b[p_l, \hat{p}_l(T_h^*)] - \pi^b[p_l, \hat{p}_l(T_l^*)] \right\} \quad (12)$$

Hence our first result:

**Proposition 1** *If (12) holds then investment timing is as in the first best,  $T_h^{sb} = T_h^*$  and  $T_l^{sb} = T_l^*$ , or equivalently  $p_h(T_h^{sb}) = p_l(T_l^{sb}) = p^*$*

The absence of a distortion in that case is directly linked to the size of the pledgeable income, that is the part the income generated by the project that can be pledged to the outside investor. Under symmetric information, assumption (A4) ensures that its expected value is high enough to allow financing at the first-best investment triggers. However, under asymmetric information, the pledgeable income decreases by an amount which corresponds exactly to the RHS of (12). Because the entrepreneur has limited liability, the only way to ensure information revelation is indeed to give a higher payment to the low type in order to compensate him for investing later. This increases the amount of cash which needs to be left to the entrepreneur, and explains why condition (12) which ensures first-best investment timing is more restrictive than (A4). We turn now to the case where this condition is not verified.

Suppose that (12) does not hold, then (10) must be binding. The incentive cost of implementing the first-best investment rule is now too high compared to the financial income available for the entrepreneur. Then the only way the project can be financed is by decreasing the RHS of (10), and therefore the amount that needs to be paid to the low type. This implies however investment timing distortion.

Several configurations are possible. We start with the case where only the high type timing is affected.

Define a threshold for the high type,

$$p_h^f = \left(1 + \frac{r}{\lambda}\right) \frac{q \left( I - \frac{\mu}{r + \lambda} \right) + (1 - q) \frac{1 - p_l}{1 - p_h} \frac{b}{r + \lambda}}{qI + (1 - q) \frac{p_h - p_l}{p_h(1 - p_h)} \frac{b}{\lambda}},$$

and the corresponding investment timing,  $T_h^f = p_h^{-1}(p_h^f)$ .

**Proposition 2** *If (12) does not hold and  $T_h^f < T_l^*$ , investment occurs as in the first best for the low type,  $T_l^{sb} = T_l^*$ , and later than in the first best for the high type,  $T_h^* < T_h^{sb} \leq T_h^f$ , anytime  $I_0$  can be financed.*

**Proof** In the appendix.

Delaying investment for the high type, while leaving the low type invest at the first-best investment trigger is the more efficient way to boost the pledgeable income. To provide more intuition on this result, we introduce a second benchmark:

$$p^f = \left(1 + \frac{r}{\lambda}\right) \frac{I - \frac{\mu}{r + \lambda}}{I}.$$

$p^f$  is the investment trigger that maximizes the expected value of the financial income generated by the project, ignoring the private benefits of the entrepreneur. In other terms,  $p^f$  maximizes the pledgeable income under symmetric information. Because the expected profit from investing  $I$  is lower when only  $\mu$  rather than total surplus  $\mu + b$  is considered,  $p^f > p^*$ . It is also easy to check that  $p_h^f > p^f$ .

The intuition for proposition 2 is then as follows. The fundamental problem of the entrepreneur is to increase the pledgeable income by a sufficient amount to be able to raise funds. Starting from  $T_h^*$ , delaying  $T_h$  has two effects. First, it boosts the financial income by bringing  $\hat{p}_h(T_h)$  closer to the financial optimum  $p^f$ . Second, it relaxes the incentive constraint of the low type by decreasing his gains in private benefits, when he deviates and invests in the same timing as a high type. There is therefore a double benefit in terms of pledgeable income in postponing investment for the high type: it increases the financial income and decreases the amount of cash needed to provide incentives. This is true as long as  $\hat{p}_h(T_h) \leq p^f$ . Once the financial optimum is overcome, effects start working in opposite direction: delaying investment is still good for incentives, but now decreases the financial income. The two marginal effects exactly cancel out when  $\hat{p}_h(T_h)$  reaches  $p_h^f$ .  $T_h^f$  maximizes the pledgeable income under asymmetric information, and distorting  $T_h$  further would therefore be useless. The reason why the threshold  $p_h^f$  that maximizes the pledgeable income under asymmetric information is higher than the threshold  $p^f$  that maximizes the pledgeable income under symmetric information, is that the former takes into account the additional amount needed to provide incentives to the entrepreneur to reveal information.

Because  $T_h^f < T_l^*$ , and  $T_h \leq T_h^f$ , a high type invests strictly before a low type (if the project can be financed). Types are therefore always separated in that case. Intuitively, in the maximization of the pledgeable income, the benefit in terms of investment efficiency in having the two types invest at different times is greater than the cost of inducing this separation. This is not always the case as will be seen in the next proposition.

**Proposition 3** *If (12) does not hold and  $T_h^f \geq T_l^*$ , then*

- (i) *if  $q\pi^f[p_h, \hat{p}_h(T_l^*)] + (1 - q)\pi^f[p_l, p^*] \geq I_0$ , investment occurs as in the first best for the low type,  $T_l^{sb} = T_l^*$ , and later than in the first best for the high type,  $T_h^* < T_h^{sb} \leq T_l^*$ ,*
- (ii) *if  $q\pi^F[p_h, \hat{p}_h(T_l^*)] + (1 - q)\pi^F[p_l, p^*] < I_0$ ,  $T_h^{sb} = T_l^{sb}$  (both types invest at the same time), and investment occurs later than in the first best for both types, anytime the project can be financed,  $T_h^{sb} > T_h^*$  and  $T_l^{sb} > T_l^*$ .*

**Proof** In the appendix.

$T_h^{sb}$  is distorted at optimum for the same reasons as above, the quest for pledgeable income. The difference with the previous case is that private information being now less valuable in the maximization of the pledgeable income, it might be more efficient to save on incentive costs by

bunching types and therefore ignore their private signals. This does not happen systematically, even when  $T_h^f > T_l^*$ , because in some cases, the need for outside funding is lower and a slight increase in the investment threshold of the high type is sufficient to raise the pledgeable income to the level required to finance the project, while maintaining private information revelation in equilibrium. However, as more outside financing is required, there may exist optimal contracts with no information revelation.

Finally, distorting investment thresholds in order to boost the pledgeable income might not be sufficient to have the initial investment financed. In certain cases,  $I_0$  cannot be financed under asymmetric information, although it can be financed with first-best investment triggers under symmetric information. Consider the following condition,

$$q\pi^f(p_h, p_h^f) + (1 - q)\pi^f(p_l, p^*) - I_0 \geq (1 - q) \left\{ \pi^b[p_l, \hat{p}_l(T_h^f)] - \pi^b(p_l, p^*) \right\} \quad (13)$$

**Proposition 4a** *Suppose that  $T_h^f \leq T_l^*$ , then there exist values of the parameters, such that (13) does not hold. As a result,  $I_0$  cannot be financed under asymmetric information.*

Proposition 4 corresponds to a complete market breakdown where the intensity of the adverse selection problem does not even leave the possibility to launch the development phase. In that case, even the best compromise  $T_h^f$  between reducing the rent of the low type while not decreasing too much the financial revenue generated by the project, does not leave enough free cash-flow to cover in expectation the initial investment  $I_0$ . What prevents the financing of the project is simply the impossibility of exploiting in an efficient way the private information of the entrepreneur.

Similarly, bunching the two types reduces the rent of the low type to zero but distorts investment from the financial optimum and may eventually prevent initial financing. We obtain then a similar result.

**Proposition 4b** *Suppose that  $T_h^f > T_l^*$ , then there exist values of the parameters, such that*

$$\max_T q\pi[p_h, \hat{p}_h(T)] + (1 - q)\pi[p_l, \hat{p}_l(T)] < I_0.$$

*As a result,  $I_0$  cannot be financed under asymmetric information.*

## 5 Implications of the model

### 5.1 Financial contracts

A first interesting result is that it might be useful to include in a financial contract, on top of cash-flow rights, a clause that makes future financing contingent on some measure of performance through time. Because private benefits are attached to the full completion of the project, investment timing may indeed be used as an instrument to mitigate the adverse selection problem, and relax the financing constraint. One key condition here is the ability of the investor to commit ex ante in a contract to release funds when the investment threshold is reached. Kaplan and Stromberg (2003) document the existence of such provisions in venture capital contracts, where future financing is made explicitly contingent on non-financial measures of performance (“milestones”), such as the completion of clinical tests. Interestingly, in Kaplan

and Stomberg (2004), the existence of ex-ante staging is related to the degree of internal risk, which includes the difficulty for the investor to value the project and monitor the entrepreneur, and is a measure of the likelihood of agency problems between the venture capitalist and the management. Our model formalizes the idea that contractual staging might be a way to elicit information ex ante, but shows that it might result in an inefficient investment rule ex-post. Furthermore, monetary payments to the entrepreneur and investment covenants are related, they are two instruments which can be jointly used to induce information revelation. This is also consistent with evidence on private equity contracts, in which a wide range of rewards and controls is usually included. Payments and investment distortions are substitute in this model, as investment distortion is used when the financial income available to provide incentives to the entrepreneur is too low. Conversely, the difference in monetary payments between the low type and the high type increases with the difference in investment timing.

Renegotiation might be an issue in this model as investment timing is ex-post socially inefficient. Once the entrepreneur has revealed his type by picking a contract, parties could try and improve upon the investment rule and share the additional surplus. What we implicitly assumed in the exposition of the model is that parties have the ability to commit not to renegotiate. A first way of addressing this question, is by giving some arguments in favor of this assumption. It might be difficult and costly to renegotiate when the number of investors increases. This might indeed imply gathering a large number of small shareholders, feeding them with appropriate information and being able to enforce an agreement among them even before starting discussions with the entrepreneur. Furthermore, there might be technical constraints which make investment timing less flexible once it is decided. Suppose for instance that investing is subject to an administrative permit allocated for a certain period of time, then changing the date at which investment is allowed to take place might simply not be feasible.

There is an even more fundamental way of tackling this issue. It turns out indeed that in a range of situations, contracts implying investment distortions are renegotiation-proof. More precisely, this is true in cases where investment takes place before the financial optimum ( $\hat{p}_h(T_h^{sb}) \leq p^f$ ). The intuition is follows. When  $p^* < \hat{p}_h(T_h^{sb}) \leq p^f$ , the entrepreneur and the investor have opposite preferences over the investment timing. The former would like to invest earlier, which would indeed increase the total surplus, but the latter would prefer to invest at  $p^f$  which maximizes the financial value of the firm. As a result, renegotiation can occur only if the entrepreneur can transfer to the investor part of the increase in utility from private benefits which he would enjoy by investing earlier. But this is impossible since private benefits cannot be transferred, and the limited liability constraint of the high type is binding any time a distortion occurs. There is therefore no mean for him to compensate the investor. Investment timing distortions would therefore still exist, even if renegotiation was allowed. Finally, notice that proposition 4 would a fortiori be true if parties lacked the ability to commit not to renegotiate. Therefore the constraint created by asymmetric information on financing, would be even more severe if we allowed for renegotiation.

## 5.2 The impact of financing constraints

Another insight from this analysis is that financing frictions created by agency problems might have delayed effect. As already mentioned, the agency problem here can be analyzed as one where the entrepreneur tries to increase the pledgeable income in order to attract an investor. In Homlström and Tirole (1997), this can be done by scaling down the project and therefore reducing current investment. Our analysis suggests that achieving this goal might also imply reducing investment in the future, late investment being indeed a form of underinvestment.

This has implications for the measure of financing constraints and their effects on investment. A certain number of studies have looked empirically at the relationship between financing constraints and the sensitivity of investment to cash-flow (Fazzari et al (1988), Kaplan and Zingales (1997), Cleary (1999)). The agency theory predicts indeed that financially constrained firms should exhibit a higher sensitivity of investment to cash-flow since internal cash relaxes financial constraints and makes therefore investment easier. Internal financing can be easily introduced in our model, by simply assuming that  $I_0$  is the difference between the total investment cost required to launch development and the cash held by the entrepreneur. Then our model predicts that when cash reserves are low, a cash inflow may indeed have an immediate impact by decreasing  $I_0$ , and allowing to switch from a regime where financing is impossible to a regime where the initial investment takes place (for instance from a regime where (13) does not hold to a regime where it is satisfied). However, for an intermediate range of cash reserves where  $I_0$  can be financed but the financial contract implies investment timing distortions, a cash inflow would have no effect on current investment, but an effect on future investment, by allowing a contract where investment triggers are closer to the first-best investment rule (formally, by relaxing the constraint (10) in the optimization program (P1')). This suggests an empirical approach where investment in subsequent periods is included in the measure of investment to cash-flow sensitivity.

A related implication is that firms with higher cash reserves should be more reactive, by investing earlier on average, and therefore have a higher probability of failing once the productive investment is made, since the expected probability of holding a good project at the time where the second investment is made is lower. This is consistent with the empirical findings of Guedj and Scharfstein (2004) on drug development. They find indeed, that among early-stage firms, which are presumably financially constrained, those who hold higher cash reserves are more likely to move from the early development phase (phase I) to a more capital-intensive phase (phase II). Furthermore those firms have a significantly higher rate of failure in phase II. Guedj and Scharfstein also suggest that this distortion might be caused by the existence of private benefits. They don't explicitly consider however the impact of information asymmetries.

### 5.3 Pay and performance

A direct consequence of the high type always investing earlier than the low type is that the payment to the low type is always larger ( $w_l \geq w_h$ ). If it was not the case, the low type would always prefer to lie, and incentive compatibility would be violated. This has several consequences on the relationship between pay and performance. First, entrepreneurs with a higher pay have a higher probability to fail independently of the occurrence of the second investment. That is simply due to the fact they receive bad news on the probability of holding a good project. However, under symmetric information, the difference between high types and low types, in terms of probability to fail, vanishes when only fully completed projects are considered. At the time where the second investment is made, they have indeed the same belief  $p^*$ , and the fact that a low type initially had a lower chance of holding a good project is compensated by a longer learning phase. This is not necessarily true under asymmetric information. When an investment distortion occurs, a low type invests indeed at a lower threshold than a high type. As a result, entrepreneurs with a higher pay also have a relatively higher probability to fail, even when considering only projects which are fully implemented.

A related observation is that the stock price should react to the announcement of the entrepreneur compensation scheme. The equity value of the firm should drop when a higher payment to the entrepreneur is announced, by an amount which reflects both the cost of a

higher executive package and the signal that the firm has lower operational prospects. This happens even though this transfer is part of an optimal contract.

These results contribute to the debate on executive compensation and managerial power initiated by Bebchuk and Fried (2004), in an influential book. They argue that many common features of executive compensation schemes are inconsistent with the assumption that contracts are negotiated at arm's length between shareholders and managers. In particular, the decoupling of pay from performance, through the use of stock option repricing or severance packages, suggests rather a situation where managers use their discretionary power to divert resources from the company at the expense of shareholders. In many cases, managers seem to be even rewarded for destroying shareholders' wealth. We show here that such features can be part of an optimal contract, because inducing managers to reveal bad news requires a compensation. In our setting, an optimal contract could very well be implemented by promising a severance package to a low type if a failure occurs. This would generate a negative correlation between the existence of a severance package and performance, a positive correlation between the existence of a severance package and risk-taking (because low types investment thresholds are lower) and a negative stock price reaction to severance packages announcements. Yet, these effects, as well as the possibility left to the entrepreneur to choose this type of compensation would still be the result of optimal contracting at arm's length.

#### 5.4 Corporate governance

There is a sense however in which the payment structures we just discussed and distortions in investment timing can be related to the efficiency of corporate governance, in line with the claim of Bebchuk and Fried. The ratio of private benefits  $b$  over the total profit  $\mu + b$  can indeed be interpreted as measure of the part of the surplus generated by the project that cannot be credibly promised to investors in the absence of effective control mechanisms. This suggests two types of predictions.

First, the minimum difference between the payment  $w_l$  to the low type and the payment  $w_h$  to the high type is an increasing function of  $b$ . Holding  $\mu + b$  constant, increasing  $b$  pushes indeed  $w_h$  down, possibly to zero, and  $w_l$  up. As a result, firms with less or less efficient corporate governance should also exhibit more extreme incentive structures. In particular, payments apparently unrelated to performance, such as extraordinary bonuses or severance packages, should be higher in those firms. Second, increasing this ratio makes (12) more difficult to satisfy, and thus investment distortions more likely. Another prediction is therefore that firms with a more efficient corporate governance should also have shorter development period and be prompter to invest, while firms with corporate governance problems should exhibit more inertia. This might be another reason why specialized active investors, such as venture capitalists add value to innovative projects, where learning plays a crucial role. By being able to monitor more closely entrepreneurs once the productive investment is made, and by implementing more efficient corporate governance rules, they might be able to reduce investment timing distortions and increase the ex-ante value of the project for the entrepreneur (Gorman and Sahlman (1989), Sahlman (1990), Casamatta (2003)).

## 6 Contracting with private information

By slightly modifying the timeline of the model, we can allow for private information to be learned by the entrepreneur before he seeks financing from an outside investor. Under this

specification, the model can be seen as a direct extension of standard models of adverse selection in finance (Stiglitz and Weiss 1981), to the case where investment generates information rather than cash-flows. We find that distortions are systematic and larger in this case. Intuitively, a high type has an additional motive for distorting investment. He wants now to signal that his project has a higher financial value in order to obtain a higher transfer from the investor in expectation. This may even create situations where the order in which types invest is reversed compared to the first-best. Second, we find that delaying the reward to the high type as much as possible is part of any optimal contract, which gives a rationale for long-term incentive schemes (e.g. stock-option with a vesting period).

## 6.1 Setting

We simply suppose here that there are two types of entrepreneur, a high type who has a probability  $p_h$  of holding a good project, and a low type who owns a profitable project with probability  $p_l$  only ( $p_l < p_h$ ). Types are private information of the entrepreneur and may reflect either his personal talent or information about the quality of the project. We assume that it cannot be transmitted to the investor in a verifiable way.  $q$  denote the prior probability that the entrepreneur is a high type, it can be seen as his reputation. The entrepreneur knows his type from the beginning.

We strengthen assumption (A4), by allowing both types of entrepreneur to be financed under the first-best investment rule when information is symmetric. This is the most interesting case for two reasons. First, it results in both types being financed in equilibrium (under asymmetric information) and therefore captures the differences in incentive structures and investment timing between the two types. Second, it ensures that the symmetric information benchmark coincides with the first-best investment rule.

$$\text{(A4')} \quad \pi^f(p_l, p^*) - I_0 \geq 0$$

Unlike the previous case, the entrepreneur has now some private information at the contracting date, and contracts he proposes to the investor may therefore convey information. In line with Ueda (2004) or Hennessy et al. (2008), we focus on menus of contracts which are incentive compatible, and maximize the expected payoff of the more profitable type provided the investor breaks even and the less profitable type gets at least his reservation payoff  $\pi(p_l, p^*) - I_0$ . The first-best value of the low type's project should indeed be his reservation value, since from (A4'), he can always be financed under the first-best investment rule, and extract all the surplus. This is consistent with the informed principal approach in Maskin and Tirole (1992), where equilibrium allocations are those that weakly pareto-dominate the so-called Rotschild-Stiglitz-Wilson allocation<sup>15</sup>, which is precisely the type of allocation we will obtain.

We consider a more general form of contract than in the previous case, as it may now be optimal to delay payment. Specifically, a contract is a quadruplet  $(v_{\bar{\theta}}, t_{\bar{\theta}}^v, s_{\bar{\theta}}, T_{\bar{\theta}})$ .  $v_{\bar{\theta}}$  is a transfer taking place at  $t_{\bar{\theta}}^v$  if no jump occurred up to that date.  $s_{\bar{\theta}}$  is an unconditional payment. Finally,  $T_{\bar{\theta}}$  is the point in time at which investment occurs if no jump was observed until then<sup>16</sup>.

The only contractual restriction we introduce is a time limit. We impose that any payment that could take place according to the contract must be realized before some date  $\tau$ . Absent this

<sup>15</sup>The RSW allocation, also called low information intensity optimum in Tirole (2006), is an incentive compatible allocation, that maximizes the payoff of each type, and is such that the investor breaks even for each type. It is easy to check that the RSW allocation in our case is such that the low type gets  $\pi(p_l, p^*) - I_0$ .

<sup>16</sup>It can be shown that more complete contracts would not be useful.

limit, it could indeed be optimal to promise an infinite amount of money to the entrepreneur in an infinite amount of time. This exogenous constraint could be justified by some liquidity needs of the entrepreneur, which bound the extent to which monetary payments can be delayed. However, this assumption is not critical as taking  $\tau$  arbitrarily large will not make investment distortions vanish. For the moment we just assume that  $\tau$  is large enough. In particular, it is set after the date at which investment takes place for the high type under the first-best investment rule ( $\tau > T_h^*$ ).

## 6.2 Equilibrium analysis

The preliminary steps are as in the proof of lemma 1. First, it is easy to show that attention can be restricted to contracts where  $s_h = 0$  and  $v_l = 0$ . To simplify notation, we simply denote  $v$  the expected value for the high type of the future contingent payment  $v_h$ . Formally,

$$v = \left[ p_h e^{-rt_h^v} + (1 - p_h) e^{-(r+\lambda)t_h^v} \right] v_h.$$

Then the expected value for a low type of the contingent payment  $v_h$ , is  $\delta(t_h^v)v$ , where 
$$\delta(t_h^v) = \frac{p_l + (1 - p_l)e^{-t_h^v\lambda}}{p_h + (1 - p_h)e^{-t_h^v\lambda}}.$$

Finally, to simplify further the exposition, we drop the incentive constraint of the high type, which as in the previous case, never binds at optimum.

The optimization program is then as follows,

$$(P2) : \max_{T_h, T_l, v, t_h^v \in [0, \tau], s} \pi^b[p_h, \hat{p}_h(T_h)] + v,$$

$$\pi^b[p_l, \hat{p}_l(T_l)] + s \geq \pi^b[p_l, \hat{p}_l(T_h)] + \delta(t_h^v)v, \quad (14)$$

$$q \{ \pi^f[p_h, \hat{p}_h(T_h)] - v \} + (1 - q) \{ \pi^f[p_l, \hat{p}_l(T_l)] - s \} \geq I_0, \quad (15)$$

$$\pi^b[p_l, \hat{p}_l(T_l)] + s \geq \pi(p_l, p^*) - I_0, \quad (16)$$

$$v \geq 0, s \geq 0. \quad (17)$$

There are similarities between this program and the program (P1) we derived in the previous section. As before, (14) is an incentive compatibility constraint ensuring that a low type prefers to invest under the terms designed for him, (15) is the participation constraint of the investor, and (17) are limited liability constraints of the entrepreneur. However, when the entrepreneur was uninformed at the contracting date, he maximized an expectation over types of his profit. Which type received higher transfers was therefore indifferent as long as profit remained constant in expectation. This changes dramatically here as an entrepreneur who knows that he is a high type when he contracts, is now willing to maximize his own profit, while leaving the low type with the lowest possible rent. The objective function is then the expected utility of the high type only, and (16) ensures that the low type gets, at least, his reservation utility and is therefore willing to accept the offer.

A first consequence of this difference is that the reward  $v$  to the high type can be strictly positive at optimum with the incentive compatibility constraint of the low type (14) binding. This could not happen in the previous specification, as the limited liability constraint of the high type was binding anytime the incentive compatibility constraint of the low type (7) was binding. Then, since  $\delta(\cdot)$  is decreasing, setting  $t_h^v$  as large as possible, that is equal to  $\tau$ , relaxes (14) and is therefore optimal. Intuitively, delaying the reward to the high type as much as possible is optimal as waiting generates information on the type of the entrepreneur. We formalize this in the following lemma.

**Lemma 2** *There always exists an optimal contract where  $t_h^v = \tau$ , furthermore if  $v > 0$  at optimum then  $t_h^v = \tau$  in any optimal contract.*

**Proof** In the appendix

A second consequence is that the incentive compatibility constraint of the low type (14) is always binding at optimum. This was not the case in the previous specification as it was possible to reallocate payments from the high type to the low type without affecting the objective function. This is not true anymore in this setting, and as a result, investment is systematically distorted.

**Proposition 5** *The optimal investment rules are such that the low type invests at the first-best investment threshold  $T_l^{sb} = T_l^*$  and the high type always invests later than the first-best investment threshold,  $T_h^{sb} > T_h^*$ .*

**Proof** In the appendix

For the same reason as above, distortions in investment timing are larger. In particular, there can be cases where a high type invests not only at a higher threshold ( $\hat{p}_h(T_h^{sb}) > \hat{p}_l(T_l^{sb}) = p^*$ ), but also later than a low type ( $T_h^{sb} > T_l^{sb} = T_l^*$ ). Intuitively, a high type wants to be separated from a low type not only to enjoy earlier private benefits, but also because his project has a higher financial value. By delaying investment, he loses on private benefits, but manages to maintain a high level of financial revenues, while preserving incentive compatibility.

**Corollary 1** *For  $\tau$  sufficiently large, there exists  $q_1$  such that  $T_h^{sb} > T_l^{sb} = T_l^*$  if  $q < q_1$ .*

Regarding the low type, (16) ensures that he gets at least his first-best payoff, but he may obtain more in certain cases, at the expense of the high type. This happens when the prior probability  $q$  of a high type is high.

**Corollary 2** *For  $\tau$  sufficiently large, there exists  $q_2 \in (0, 1]$  such that,*

- *as a function of  $q$ ,  $T_h^{sb}$  is constant on  $(0, q_2]$ , and strictly decreasing on  $(q_2, 1)$ ,*
- *the expected surplus of the low type is strictly greater than  $\pi(p_l, p^*)$  if  $q > q_2$  (cross-subsidiarization), and equal to  $\pi(p_l, p^*)$  if  $q \leq q_2$  (type-by-type profitable allocation).*

Remembering that the first-best investment timing  $T_h^*$  of the high type is independent of  $q$ , corollary 2 shows that the distortion in the investment timing of the high type decreases with  $q$ . Then,  $q_2$  establishes a separation between two regimes. The situation where  $q > q_2$  corresponds to the case where (16) is not binding, and the optimal investment rule for the high type results then from a rent-efficiency tradeoff. The rent that the high type needs to leave to the low type in order to deter a deviation, is fixed by the incentive compatibility constraint (14), which does not depend on  $q$ . The cost of this rent for the high type is however weighted by  $1 - q$  through the participation constraint (15) of the investor. When  $q$  is high, the weight of the low type is low, and the high type is therefore ready to leave the low type with a high rent (through  $s$ ) in order to come closer to the first-best investment rule. As a result,  $T_h^{sb}$  decreases when  $q$  increases and tends to  $T_h^*$  as  $q$  tends to 1. Because the monetary transfer  $s$  that has to be left to the low type in order to maintain incentive compatibility is high, the low type is “cross-subsidiarized” in this regime. He gets indeed a payoff which is strictly higher than his first-best payoff. The investor makes then a strictly negative profit in expectation with a low type compensated by a strictly positive profit with the high type.

As  $q$  decreases, the weight of the low type’s rent increases. In order to maximize the high type payoff,  $s$  must then decrease, which requires to distort  $T_h$  towards late investment to maintain incentive compatibility. At some point however, (16) may start to bind and prevent from pushing  $s$  further down and  $T_h$  further up. The allocation is then type-by-type profitable in the sense that the investor breaks even whatever the type of entrepreneur he faces, and therefore the probability of a high type does not play a role anymore.

Finally, it is easy to show that taking  $\tau$  arbitrarily large does not make the distortion vanish. This is due to the imperfection of the entrepreneur’s private information. Even in the case where the quality of the project can be learned almost perfectly in the end, there is still some uncertainty about the initial information of the entrepreneur, and therefore a scope for adverse selection.

### 6.3 Discussion

This extension gives a rationale for the use of long-term performance plans for entrepreneurs, such as stock-options with a vesting period. While it has already been argued that this form of incentive could be an instrument to mitigate agency problems, the underlying reason was usually different. The traditional idea, following early papers on agency in finance (Jensen and Meckling 1976), is that adopting compensation plans sensitive to long-term performance provides incentives to the entrepreneur to choose future actions that maximize shareholders value. For instance, in a dynamic moral hazard framework, Biais et al (2007) show that it is optimal to reward the entrepreneur only if he accumulates a sufficient number of good performances. In our case, the vesting period is used as a way to elicit information ex ante. Furthermore, increasing the vesting period is not meant to induce choices which persistently improve performance, but to relax the initial adverse selection problem by increasing the precision of the information on the entrepreneur’s type. In Kaplan and Stromberg (2004), the existence of vesting provisions for the manager is correlated with execution risk, which covers essentially the uncertainty about the viability of project due to the lack of reliable information and the fact that development is still at an early stage. This is consistent with our claim that vesting is a way to make the reward of the entrepreneur more informative about his private signal.

Empirical predictions can be derived on the correlation between the form of incentive schemes and performance. Projects where entrepreneurs receive a high amount of vested options should indeed have higher operational performances, which is in line with empirical evidence (Hanlon,

Rajgopal and Shevlin 2003). This is not related to a higher effort provision induced by the incentive structure, but to a self-selection process where better entrepreneurs choose this type of contract. Second, entrepreneurs who adopt long-term performance plans take less risk, as they invest at a higher threshold, and therefore reduce the probability of a failure taking place after the productive investment is made. Notice however that this is a distortion from the first-best investment rule created by the adverse selection problem.

Finally, interpreting  $q$  as the market belief on the entrepreneur's talent, we also predict that entrepreneurs with a higher reputation should have a shorter development phase on average, both because they are more likely to be high types and because  $T_h^{sb}$  decreases with  $q$  (from corollary 2), but also a higher probability of failure  $1 - p_h(T_h^{sb})$  for projects which are fully implemented.

## 7 Conclusion

We study the financing of a real option, when there exist information asymmetries between the entrepreneur who owns the project, and the investor who may provide funding. We model a sequential investment problem in which an initial investment gives access to a public signal on the profitability of a subsequent investment and the entrepreneur receives some private information in the learning period. We show that the adverse selection problem decreases the income which can be pledged to an outside investor. The entrepreneur may then be forced to propose a contract where future investment is delayed, which suggests that financial constraints create inertia. In certain cases, the entrepreneur may even fail to raise funds to finance the initial investment. We also show the existence of a negative correlation between monetary incentives of the entrepreneur and the operational performance of the firm. To ensure truthful information transmission, the entrepreneur needs indeed to be compensated when he reveals bad news. The decoupling between pay and performance, often put forward as a proof of excessive managerial discretion in setting compensations, might therefore be the outcome of an optimal contract. We show however that this correlation as well as distortions in investment timing are reduced when the share of the entrepreneur's private benefits in the total surplus generated by the investment decreases. This suggests in particular that efficient corporate governance improves firms reactivity. When private information is learned by the entrepreneur before contracting, distortions in investment timing are systematic and larger than in the initial case. We derive then stock options with a vesting period as part of the optimal incentive scheme, and find that rewards based on long-term performance should be associated with a higher operational performance, in line with empirical evidence. We provide a rationale for this relationship which differs from the standard moral hazard explanation.

We plan to extend this research in different ways. First, we could allow for the productive investment size to be flexible, which would provide another instrument to extract information and would generate additional predictions on the impact of market frictions on firms development. Second, we could allow for the intensity of the learning process to differ before and after the investment. Because the public signal is correlated with the private information of the entrepreneur, this would give another reason for using investment timing to extract information.

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## 8 Appendix

### 8.1 Proofs of section 4

#### Proof of Lemma 1

We do two things at the same time in this section. We prove Lemma 1, while extending the analysis of the problem by allowing more general payments schemes. This is to ensure that there exists indeed an optimal contract with lump-sum payments even when the contract space is not restricted.

We consider a very general contract. For each  $\tilde{\theta} \in \{h, l\}$ , a contract is a quadruplet  $(v_{\tilde{\theta}}, t_{\tilde{\theta}}^v, s_{\tilde{\theta}}, T_{\tilde{\theta}})$ .  $v_{\tilde{\theta}}$  is a payment that takes place at date  $t_{\tilde{\theta}}^v$  if no jump occurred until that date (notice that  $t_{\tilde{\theta}}^v$  can be set before or after the investment date).  $s_{\tilde{\theta}}$  is a lump-sum payment taking place the first time a jump occurs. Finally,  $T_{\tilde{\theta}}$  is the point in time at which investment occurs if no jump was observed until then.

The expected value for an entrepreneur of type  $\theta$  of payments under the contract  $\tilde{\theta}$  is therefore

$$F(\theta, \tilde{\theta}) = \left[ p_{\theta} e^{-rt_{\tilde{\theta}}^v} + (1 - p_{\theta}) e^{-(r+\lambda)t_{\tilde{\theta}}^v} \right] v_{\tilde{\theta}} + \int_0^{+\infty} (1 - p_{\theta}) \lambda e^{-(r+\lambda)t} s_{\tilde{\theta}} dt.$$

To spare the reader long formulas, we skip the general form of the optimization program, which can be greatly simplified by showing that attention can be restricted to contracts where  $s_h = 0$ , and  $v_l = 0$ . A sketch of the proof is as follows. Suppose that  $s_h > 0$ , then decreasing  $s_h$  while increasing  $v_h$  in order to keep  $F(h, h)$  constant does affect neither the objective function, nor the participation constraint of the investor, nor the incentive constraint of the high type, and relaxes the incentive compatibility constraint of the low type. Similarly, if  $v_l > 0$ , decreasing  $v_l$  and increasing  $s_l$  so as to keep  $F(l, l)$  constant relaxes the incentive compatibility constraint of the low type without affecting any of the other constraints.

To simplify further notation, we make the following change of variable,

$$s = \int_0^{+\infty} (1 - p_l) \lambda e^{-(r+\lambda)t} s_l dt \quad \text{and} \quad v = \left[ p_h e^{-rt_h^v} + (1 - p_h) e^{-(r+\lambda)t_h^v} \right] v_h.$$

$$\text{Define finally } \delta(t_h^v) = \frac{p_l + (1 - p_l) e^{-t_h^v \lambda}}{p_h + (1 - p_h) e^{-t_h^v \lambda}}.$$

The optimization program is then as follows,

$$(P1) : \max_{T_h, T_l, v, t_h^v, s} q \{ \pi^b[p_h, \hat{p}_h(T_h)] + v \} + (1 - q) \{ \pi^b[p_l, \hat{p}_l(T_l)] + s \},$$

$$s.t. \quad \pi^b[p_h, \hat{p}_h(T_h)] + v \geq \pi^b[p_h, \hat{p}_h(T_l)] + \frac{1 - p_h}{1 - p_l} s, \quad (18)$$

$$\pi^b[p_l, \hat{p}_l(T_l)] + s \geq \pi^b[p_l, \hat{p}_l(T_h)] + \delta(t_h^v) v, \quad (19)$$

$$q \{ \pi^f[p_h, \hat{p}_h(T_h)] - v \} + (1 - q) \{ \pi^f[p_l, \hat{p}_l(T_l)] - s \} \geq I_0, \quad (20)$$

$$v \geq 0, s \geq 0. \quad (21)$$

We ignore (18), we will check ex post that it is not binding. (20) must be binding at optimum, therefore the objective function becomes simply the total surplus

$$q\pi[p_h, \hat{p}_h(T_h)] + (1 - q)\pi[p_l, \hat{p}_l(T_l)],$$

which is independent from  $s$  and  $v$ . We use then the fact that (20) holds with equality at optimum to write  $s$  as the function of the other parameters. (19) and (21) become then

$$q\pi^f[p_h, \hat{p}_h(T_h)] + (1 - q)\pi^f[p_l, \hat{p}_l(T_l)] - I_0 \geq (1 - q) \{ \pi^b[p_l, \hat{p}_l(T_h)] - \pi^b[p_l, \hat{p}_l(T_l)] \} + qv + (1 - q)\delta(t_h^v)v, \quad (22)$$

$$q\pi^f[p_h, \hat{p}_h(T_h)] + (1 - q)\pi^f[p_l, \hat{p}_l(T_l)] \geq I_0 + qv, \quad (23)$$

$$v \geq 0. \quad (24)$$

It is then immediate that (22) and (24) imply (10). Similarly (23) and (24) imply

$$q\pi^f[p_h, \hat{p}_h(T_h)] + (1 - q)\pi^f[p_l, \hat{p}_l(T_l)] \geq I_0. \quad (25)$$

Conversely, if (10) and (25) are true, there exists  $v$  positive that satisfies (22) and (23). Therefore (22), (23) and (24) can be replaced by (10) and (25). Notice then that if (10) does not bind at optimum, then the optimal investment rules for each type under asymmetric information are the first-best investment rules  $T_h^*$  and  $T_l^*$  since they maximize the objective function and satisfies (25) (from assumption A4). In that case, since  $T_h^* < T_l^*$ , (11) is satisfied. If (10) does bind at optimum, then the RHS of (10) must be non-negative otherwise (25) would be violated. This implies also that (11) is true. Conversely (11) implies that the RHS of (10) is positive, which implies (25).

Finally, notice that we did not make any assumption on  $t_h^v$ , and we can therefore as well take  $t_h^v = 0$ , which is equivalent to giving a lump-sum payment to the high type when he picks the contract. Doing the same with the low type, instead of compensating him when the project fails, only affects the incentive constraint of the high type, which we ignored. A last check is therefore whether this constraint is satisfied at optimum, in the original program (P1) (it will be a fortiori satisfied in the more general program we considered in this proof).

We can always take the incentive compatibility constraint of the low type (7) holding with equality. Simple algebra shows

$$\hat{p}_\theta(T_\theta) = \frac{p_\theta(1 - p_{\hat{\theta}})[\hat{p}_{\hat{\theta}}(T_{\hat{\theta}})]}{p_\theta[\hat{p}_{\hat{\theta}}(T_\theta) - p_{\hat{\theta}}] + p_{\hat{\theta}}[1 - \hat{p}_{\hat{\theta}}(T_\theta)]}. \quad (26)$$

Using this relation, (6) is equivalent to

$$D[p_h, \hat{p}_h(T_h)] \left[ \frac{\hat{p}_h(T_h)}{p_h} \frac{1}{r} - \frac{1 - \hat{p}_h(T_h)}{1 - p_h} \frac{1}{r + \lambda} \right] \geq D[p_l, \hat{p}_l(T_l)] \left[ \frac{\hat{p}_l(T_l)}{p_l} \frac{1}{r} - \frac{1 - \hat{p}_l(T_l)}{1 - p_l} \frac{1}{r + \lambda} \right]. \quad (27)$$

The LHS of (27) is always positive and decreasing in  $T_h$ , and (27) holds with equality when  $T_h = T_l$ . Therefore (27) is always true at optimum.  $\square$

### Proof of Proposition 2

Suppose that  $T_h^f \leq T_l^*$ .

Ignore (11). Then  $T_l^* = T^*$ , because it maximizes the objective function and relaxes (10). Consider the following function.

$$G(\hat{p}_h, \hat{p}_l) = q\pi^f(p_h, \hat{p}_h) + (1-q)\pi^f(p_l, \hat{p}_l) + (1-q)D(p_l, \hat{p}_l) \left[ \hat{p}_l \frac{b}{r} + (1-\hat{p}_l) \frac{b}{r+\lambda} \right] \\ - (1-q)D(p_h, \hat{p}_h) \left[ \frac{p_l}{p_h} \hat{p}_h \frac{b}{r} + \frac{1-p_l}{1-p_h} (1-\hat{p}_h) \frac{b}{r+\lambda} \right].$$

Using (26), (10) is simply  $G[\hat{p}_h(T_h), \hat{p}_l(T_l)] \geq I_0$ . Differentiating, we have

$$\frac{\partial G}{\partial \hat{p}_h} = \varphi(\hat{p}_h) \left\{ \left(1 + \frac{r}{\lambda}\right) \left[ q \left( I - \frac{\mu}{r+\lambda} \right) + (1-q) \frac{1-p_l}{1-p_h} \frac{b}{r+\lambda} \right] - \hat{p}_h \left[ qI + (1-q) \frac{p_h - p_l}{p_h(1-p_h)} \frac{b}{\lambda} \right] \right\},$$

where  $\varphi(\hat{p}_h)$  is a strictly positive function on  $[0, 1)$ .

$T_h^f \leq T_l^*$  implies  $p_h^f < \hat{p}_h(T_l^*)$ , therefore  $p_h^f < 1$  and  $p_h^f$  is then the value that maximizes  $G(., p^*)$  on  $[0, 1)$ . There are then two possibilities: either  $G(p_h^f, p^*) < I_0$  and the problem does not have any solution as the feasible set is then empty, or  $G(p_h^f, p^*) \geq I_0$ . The first case is the object of proposition 4, we consider here the second case.

Notice that  $G(., p^*)$  is strictly increasing on  $[0, p_h^f]$ . Since  $G(p^*, p^*) < I_0$ ,  $T_h$  must be in the interval  $(T_h^*, T_h^f]$ , to belong to the feasible set. Since the objective function is strictly decreasing on that interval, the optimal investment trigger  $T_h^{sb}$  is the unique solution to the equation  $G[\hat{p}_h(T_h), p^*] = I_0$  in the interval  $(T_h^*, T_h^f]$ . Finally,  $T_h^f \leq T_l^*$  implies  $T_h^{sb} < T_l^* = T_l^{sb}$  at optimum, and (11) is verified.

### Proof of Proposition 3

Turn to the case where  $T_h^f > T_l^*$ .

Suppose that  $q\pi^f[p_h, \hat{p}_h(T_l^*)] + (1-q)\pi^f(p_l, p^*) \geq I_0$ , which is equivalent to  $G[\hat{p}_h(T_l^*), p^*] \geq I_0$ .

Ignore (11).

Then as in the proof of proposition 2, it must be that  $T_l^{sb} = T_l^*$ , and  $T_h^{sb} > T_h^*$ . We should also have  $T_h^{sb} \leq T_l^* = T_l^{sb}$ , since increasing  $T_h$  above  $T_l^*$  decreases the objective function whereas (10) already holds. Therefore (11) holds indeed.

Suppose now that  $q\pi^f[p_h, \hat{p}_h(T_l^*)] + (1-q)\pi^f(p_l, p^*) < I_0$ , or equivalently  $G[\hat{p}_h(T_l^*), p^*] < I_0$ . Since  $G(., p_l)$  is strictly increasing on  $[p_h, p_h^f]$ ,  $G(p_h, .)$  is maximal at  $p^*$  and  $\frac{\partial G}{\partial \hat{p}_h \partial \hat{p}_l} = 0$ , it must be that  $G[\hat{p}_h(T_h), \hat{p}_l(T_l)] \leq G[\hat{p}_h(T_l^*), p^*]$  for any  $T_l$  and  $T_h \leq T_l^*$ . Therefore if a solution exists, it must be such that  $T_h > T_l^*$ . Then (11) implies  $T_l \geq T_h$ . But this implies that (11) is binding at optimum (otherwise decreasing  $T_l$  would relax (10) and improve the objective function), and therefore  $T_h^{sb} = T_l^{sb} > T_l^*$ .

### Proof of Proposition 4a and 4b

We prove here that for a range of the initial parameters  $q$ ,  $p_l$  and  $I_0$ , the initial investment  $I_0$  cannot be financed, although from (A4), it can be financed at the first-best investment trigger under symmetric information.

From (A4),  $\pi^f(p_h, p^*) > 0$ , and  $\pi^f(p_h, \hat{p})$  tends to 0 when  $\hat{p}$  tends to 1. Besides, using (26),  $p_h(T_l^*)$  tends to 1 when  $p_l$  tends to 0 and  $p^*$  does not depend on  $p_l$ . Therefore there exists  $p_l > 0$  such that  $p^f < \hat{p}_h(T_l^*) < 1$  and  $\pi^f[p_h, \hat{p}_h(T_l^*)] < \pi^f(p_h, p^*)$ .

Notice then that  $p_h^f$  tends to  $p^f$  when  $q$  tends to 1 and to  $p_h(1 - p_l)/(p_h - p_l) > 1$  when  $q$  tends to 0. Since  $p_h(T_l^*)$  does not depend on  $q$ , there exists  $q < 1$  such that  $p_h^f < \hat{p}_h(T_l^*)$  and

$$\pi^f(p_h, p_h^f) < \pi^f(p_h, p^*). \quad (28)$$

If  $p_h^f < \hat{p}_h(T_l^*) \Leftrightarrow T_h^f < T_l^*$ ,  $I_0$  can be financed if and only if (13) holds. Take assumption (A4), and consider the limit case where  $q\pi^F(p_h, p^*) + (1 - q)\pi^F(p_l, p^*) = I_0$ . Then a necessary condition for (13) to hold is  $\pi^f(p_h, p_h^f) > \pi^f(p_h, p^*)$ , which contradicts (28). This proves proposition 4a.

$p_h^f$  is strictly increasing in  $q$  and tends to  $p_h(1 - p_l)/(p_h - p_l) \geq 1$  when  $q$  tends to 0. Therefore for any  $p_l \geq 0$  there exists a unique  $q(p_l) = \left[ \frac{p_l b}{p_h r} \right] / \left[ \frac{\mu}{r} - I + \frac{p_l b}{p_h r} \right]$  such that  $p_h^f = 1$ . Consider the following expression,

$$H(p_l) = q(p_l) \left[ \pi^f(p_h, p^*) - \pi^f(p_h, \hat{p}_h(T_l^*)) \right] + (1 - q(p_l)) \left[ \pi^f(p_l, p^*) - \pi^f(p_l, p^f) \right].$$

$H(0) = 0$ , and  $H'(0) > 0$ . Therefore there exists a neighborhood of 0, where  $H(\cdot)$  is strictly positive. Therefore there exists  $q$  and  $p_l$  strictly positive such that  $p_h^f = 1$ , and

$$q \left[ \pi^f(p_h, p^*) - \pi^f(p_h, \hat{p}_h(T_l^*)) \right] + (1 - q) \left[ \pi^f(p_l, p^*) - \pi^f(p_l, p^f) \right] > 0. \quad (29)$$

Finally, slightly decreasing  $p_l$ , we can find  $q$  and  $p_l$  strictly positive such that  $\hat{p}_h(T_l^*) < p_h^f < 1$  and (29) still holds.

Assume again  $q\pi^f(p_h, p^*) + (1 - q)\pi^f(p_l, p^*) = I_0$ . From (29),  $\pi^f[p_h, \hat{p}_h(T_l^*)] < \pi^f(p_h, p^*)$ , therefore the project cannot be financed for  $T_h = T_l^*$ . Furthermore, because  $G(\cdot, p^*)$  is increasing on  $[p^*, p_h^f)$ , the project cannot be financed either for  $T_h < T_l^*$ . Finally, for any  $T_h > T_l^*$  and any  $T_l > T_l^*$ ,  $q\pi^f[p_h, \hat{p}_h(T_h)] + (1 - q)\pi^f[p_l, \hat{p}_l(T_l)] < q\pi^f[p_h, \hat{p}_h(T_l^*)] + (1 - q)\pi^f[p_l, p^f] < q\pi^f(p_h, p^*) + (1 - q)\pi^f(p_l, p^*) = I_0$ . This proves proposition 4b.  $\square$

### Private signal timing

We show here that the problem where the private signal  $\theta$  is learned by the entrepreneur at  $t = 0$  is equivalent to a problem where  $\theta$  is learned at some ulterior date, possibly random and unobserved by the investor, as long as the posterior belief of the entrepreneur at the time where he learns a high signal is lower than  $p^*$  (there is still a value of waiting to invest following a high signal).

Start with the case where the timing of the signal is public information. Suppose that the signal is learned by the entrepreneur at some  $t > 0$ , and denote  $p_h^t$  and  $p_l^t$  his beliefs at that

date,

$$p_h^t = \frac{p_t \alpha}{p_t \alpha + (1 - p_t) \beta} \text{ and } p_l^t = \frac{p_t (1 - \alpha)}{p_t (1 - \alpha) + (1 - p_t) (1 - \beta)}.$$

Likewise  $q^t = p_t \alpha + (1 - p_t) \beta$ .

Finally,

$$\hat{p}_\theta^t(T_{\bar{\theta}}) = \frac{p_\theta^t}{p_\theta^t + (1 - p_\theta^t) e^{-\lambda T_{\bar{\theta}}}}.$$

Assuming that  $p_h^t \leq p^*$ , the maximization program (P1) becomes

$$(P1)_t : \max_{T_h^t, T_l^t, w_h^t, w_l^t} D(p_0, p_t) q^t \{ \pi^b[p_h^t, \hat{p}_h^t(T_h^t)] + w_l \} + D(p_0, p_t) (1 - q^t) \{ \pi^b[p_l^t, \hat{p}_l^t(T_l^t)] + w_l^t \},$$

$$s.t. \quad \pi^b[p_h^t, \hat{p}_h^t(T_h^t)] + w_h^t \geq \pi^b[p_h^t, \hat{p}_h^t(T_l^t)] + w_l,$$

$$\pi^b[p_l^t, \hat{p}_l^t(T_l^t)] + w_l^t \geq \pi^b[p_l^t, \hat{p}_l^t(T_h^t)] + w_h^t,$$

$$D(p_0, p_t) q^t \{ \pi^f[p_h^t, \hat{p}_h^t(T_h^t)] - w_h^t \} + D(p_0, p_t) (1 - q^t) \{ \pi^f[p_l^t, \hat{p}_l^t(T_l^t)] - w_l^t \} \geq I_0,$$

$$w_h^t \geq 0, w_l^t \geq 0.$$

$T_h^t$  and  $T_l^t$  are investment timings at date  $t$  (i.e. investment takes place at date  $t + T_{\bar{\theta}}^t$ ), and  $w_h^t$  and  $w_l^t$  are lump-sum transfers to the entrepreneur at date  $t$ .

Then from Lemma 1, this optimization program is equivalent to

$$\max_{T_h^t, T_l^t} q^t \pi[p_h^t, \hat{p}_h^t(T_h^t)] + (1 - q^t) \pi[p_l^t, \hat{p}_l^t(T_l^t)],$$

$$s.t. \quad q^t \pi^f[p_h^t, \hat{p}_h^t(T_h^t)] + (1 - q^t) \pi^f[p_l^t, \hat{p}_l^t(T_l^t)] - \frac{I_0}{D(p_0, p_t)} \geq$$

$$(1 - q^t) \{ \pi^b[p_l^t, \hat{p}_l^t(T_l^t)] - \pi^b[p_l^t, \hat{p}_l^t(T_h^t)] \},$$

$$T_h^t \leq T_l^t.$$

Define  $T_{\bar{\theta}} = T_{\bar{\theta}}^t + t$ , and notice that

$$D(p_0, p_t) q^t = q D(p_h, p_h^t) \text{ and } D(p_0, p_t) (1 - q^t) = (1 - q) D(p_l, p_l^t) \quad (30)$$

$$p_\theta^t(T_{\bar{\theta}}^t) = p_\theta(T_{\bar{\theta}}) \quad (31)$$

Multiplying the objective function and the first constraint by  $D(p_0, p_t)$ , and using (30), (31), we obtain (P1').

Suppose now that the timing of the signal  $\theta$  is randomly distributed over an interval  $[0, T]$  and unobserved by the investor. Suppose that (P1) has a solution and denote  $c_t^*$  a menu of contract that solves (P1)<sub>t</sub>. Then at date  $T$ , the belief (conditional on no jump between 0 and  $T$ ) of an entrepreneur who received  $\theta$  is  $p_\theta^T$ , irrespective of the timing of the signal. Therefore  $c_T^*$  allows to reach the optimum of any program (P1)<sub>t</sub> where the timing of the signal is public.  $\square$

## 8.2 Proofs of section 6

### Proof of Lemma 2

To simplify the exposition, I introduce the following notation,

$$\phi(T_h, t_h^v) = \delta(t_h^v) \left[ \pi^f(p_h, \hat{p}_h(T_h)) - I_0 \right] + \pi^b[p_l, \hat{p}_l(T_h)],$$

$$\gamma(T_h) = q \left[ \pi^f(p_h, \hat{p}_h(T_h)) - I_0 \right] - (1 - q)\pi^b[p_l, \hat{p}_l(T_h)].$$

It can be checked ex post (see proof of proposition 5) that the limited liability constraint of the low type in (17) are never binding at optimum. We consider therefore a relaxed optimization program ( $P2'$ ) where these this constraint is ignored. Using the fact that (15) is binding, substitute  $s$  in (14) and (16), to obtain the following program.

$$(P) : \max_{T_h, T_l, v, t_h^v \in [0, \tau]} \pi^b[p_h, \hat{p}_h(T_h)] + v,$$

$$s.t. \quad \gamma(T_h) + (1 - q) [\pi(p_l, \hat{p}_l(T_l)) - I_0] \geq [q + (1 - q)\delta(t_h^v)] v, \quad (32)$$

$$\pi^f[p_h, \hat{p}_h(T_h)] - I_0 + \pi[p_l, \hat{p}_l(T_l)] - \pi(p_l, \hat{p}^*) \geq v, \quad (33)$$

$$v \geq 0. \quad (34)$$

Notice first that  $T_l^{sb} = T_l^*$  since it maximizes  $\pi[p_l, \hat{p}_l(T_l)]$  and relaxes both (32) and (33). Besides one of these two constraints must be binding.

We prove now that (32) always binds.

Suppose that (33) does not bind, then necessarily (32) binds.

Suppose that (33) does bind and ignore (34). Substituting  $v$  in the objective function and (32), ( $P2'$ ) becomes

$$\max_{T_h} \pi[p_h, \hat{p}_h(T_h)],$$

$$s.t. \quad \phi(T_h, t_h^v) \leq \pi(p_l, \hat{p}^*) - I_0. \quad (35)$$

Notice first that  $\pi[p_h, \hat{p}_h(T_h)]$  is strictly increasing on  $[0, T_h^*)$ , and strictly decreasing on  $(T_h^*, +\infty)$ . Moreover, using the fact that  $\delta(t_h^v) \geq p_l/p_h$  and (26), one shows  $\phi(T_h^*, t_h^v) > \pi(p_l, \hat{p}^*) - I_0$ . Therefore  $T_h^*$  does not belong to the feasible set defined by (35), and  $T_h$  must be different from  $T_h^*$  at optimum. Suppose that  $T_h^{sb} < T_h^*$  at optimum, then (35) must be binding, otherwise increasing  $T_h$  would improve the objective function. A similar reasoning applies if  $T_h^{sb} > T_h^*$ . Finally it can be checked ex post that  $v > 0$  when (33) binds at optimum (see proof of proposition 5).

This proves that (32) always binds. Then increasing  $t_h^v$  relaxes (32) without affecting the other constraints or the objective function, therefore setting  $t_h^v = \tau$  is optimal, and if  $v > 0$  at optimum, it is the only optimal value for  $t_h^v$ .

### Proof of Proposition 5

Notice that  $\delta(\tau) = \frac{p_l[p_h(\tau) - p_h] + p_h[1 - p_h(\tau)]}{p_h(1 - p_h)}$ .

Using the fact that (32) always binds, substitute  $v$  in the objective function, (33), and (34). Using (26) again, (P2') become

$$\max_{T_h} \quad q\pi[p_h, \hat{p}_h(T_h)] + (1 - q) \frac{p_h - p_l}{p_h(1 - p_h)} D[p_h, \hat{p}_h(T_h)] \left[ \hat{p}_h(T_h)[1 - p_h(\tau)] \frac{b}{r} - [1 - \hat{p}_h(T_h)]p_h(\tau) \frac{b}{r + \lambda} \right],$$

$$s.t. \quad \phi(T_h, \tau) \geq \pi(p_l, p^*) - I_0, \quad (36)$$

$$\gamma(T_h) + (1 - q) [\pi(p_l, p^*) - I_0] \geq 0. \quad (37)$$

Differentiating the objective function shows that it reaches a maximum when  $\hat{p}_h(T_h)$  is equal to

$$\tilde{p} = \left(1 + \frac{r}{\lambda}\right) \frac{I - \frac{\mu + b}{r + \lambda} + \frac{1 - q}{q} \frac{p_h - p_l}{p_h(1 - p_h)} p_h(\tau) \frac{b}{r + \lambda}}{I + \frac{1 - q}{q} \frac{p_h - p_l}{p_h(1 - p_h)} \frac{b}{\lambda}}.$$

$\tilde{p} > p^*$ , therefore  $\tilde{T} = p_h^{-1}(\tilde{p}) > T_h^*$ . Besides, the objective function is strictly increasing on  $[0, \tilde{T}]$  and strictly decreasing on  $(\tilde{T}, +\infty)$ .

We show now that  $T_h^{sb} \geq T_h^*$ . Suppose  $T_h^{sb} < T_h^*$ , then setting  $T_h = T_h^*$  improves the objective function and relaxes (37). Besides  $\phi(T_h^*, \tau) > \pi(p_l, p^*) - I_0$  and (36) is therefore satisfied.

Furthermore, because  $\lim_{+\infty} \phi(T_h, \tau) = -I_0 < \pi(p_l, p^*) - I_0$ , and  $\phi(\cdot, \tau)$  is a function which is strictly increasing until it reaches a maximum and then strictly decreasing, there exists a unique  $\bar{T} > T_h^*$ , such that  $\phi(\bar{T}, \tau) = \pi(p_l, p^*) - I_0$ . Then (36) can be simply replaced by  $T_h \leq \bar{T}$ .

Consider now the constraint (37). Notice that  $\gamma[T_l^*] + (1 - q) [\pi(p_l, p^*) - I_0] \geq 0$  (i.e. the constraint is satisfied when both types invest in the low type first-best timing). We show that this constraint must be slack if  $T_h^{sb} > T_l^*$ . Suppose it is not, then  $v = 0$ , but the optimal contract is then strictly dominated by a contract where  $s = v = 0$  and  $T_h = T_l = T_l^*$ , since the high type invests earlier, a contradiction. Notice finally that  $\gamma(\cdot)$  is a function which is strictly increasing until it reaches a maximum, and then strictly decreasing. We introduce a threshold  $\underline{T}$  defined as follows. If  $\gamma(T_h^*) + (1 - q) [\pi(p_l, p^*) - I_0] \geq 0$ , then  $\underline{T} = T_h^*$ . If  $\gamma(T_h^*) + (1 - q) [\pi(p_l, p^*) - I_0] < 0$ ,  $\underline{T}$  is the unique solution in  $(T_h^*, T_l^*)$  of the equation  $\gamma(\underline{T}) + (1 - q) [\pi(p_l, p^*) - I_0] = 0$ . Then (37) can be simply replaced by  $T_h \geq \underline{T}$ .

$$\text{Finally, } T_h^{sb} = \begin{cases} \tilde{T} & \text{if } \tilde{T} \in [\underline{T}, \bar{T}], \\ \underline{T} & \text{if } \tilde{T} < \underline{T}, \\ \bar{T} & \text{if } \tilde{T} > \bar{T}. \end{cases}$$

We ignored the limited liability constraint of the low type. From (16) and  $T_l^{sb} = T_l^*$ ,  $s^* \geq 0$ .  $\square$

### Proof of Corollary 1

Notice first that  $\tilde{p}(q)$  is strictly increasing from  $p^*$  to  $p_h(\tau)$ . Notice then that  $\phi(.,.)$  and  $\bar{T}$  do not depend on  $q$ . Notice finally that  $\phi(T_l^*, \tau) > \pi(p_l, p^*) - I_0$ , therefore  $\bar{T} > T_l^*$ .

Suppose that  $p_h(\tau) > p_h(T_l^*) \Leftrightarrow \tau > T_l^*$ , then there exists a unique  $q_1$  such that  $\tilde{p}(q_1) = \hat{p}_h(T_l^*)$  and for any  $q < q_1$ ,  $T_h^{sb} > T_l^{sb} = T_l^*$ .

### Proof of Corollary 2

Suppose that  $p_h(\tau) \geq p_h(\bar{T}) \Leftrightarrow \tau > \bar{T}$ .

There exists a unique  $q_2$  such that  $\tilde{p}(q_2) = p_h(\bar{T})$  and for any  $q > q_2$ ,  $T_h^{sb} = \bar{T}$ , which does not depend on  $q$ . This correspond to the case where (16) binds and the allocation is therefore type-by-type profitable.

Suppose now that  $q < q_2$ , then  $T_h^{sb} = \max(\tilde{T}; \underline{T})$ .  $\tilde{p}$  is strictly decreasing in  $q$ , therefore  $\tilde{T}$  is also strictly decreasing in  $q$ . Using  $\gamma'(\underline{T}) > 0$ , one can show that  $\underline{T}$  is also strictly decreasing in  $q$ . Therefore  $T_h^{sb}$  is also strictly decreasing. In this region, (16) does not hold with equality, therefore the low type gets more than his first-best payoff.