Accounting lighthouse in share market dynamics: a theoretical model of share price formation with dual informational structure

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Abstract

The discovery and processing of firm-specific information is expected to play a role in the making of individual expectations and related financial decisions. The information set available to share market investors is then jointly composed by market and firm-specific (non-market) information. From one side, the accounting system provides collective signals of firm-specific information. From another side, the price system provides collective signals of market information. Both institutional devices are significant for the formation of aggregate share market prices over time. In particular, the accounting system complements the price system by constituting a lighthouse in the amazing dynamics of the share market through hazard, learning and interaction. This theoretical framework applies here to provide an heuristic model of share price formation with such dual informational (and institutional) structure. Implications and recommendations are derived for the concept and occurrence of speculative bubbles, and the cyclical effects of accounting information on share market evolution.

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1 Introduction

Accounting, economics, and finance literature pays increased attention to the link between share market dynamics and alternative accounting structures. In particular, the discovery and processing of accounting information is expected to play a specific role in the making of individual expectations and related decisions, influencing then the formation of aggregate market prices over time. Following the conceptual framework by the US Financial Accounting Standards Board (FASB, CON 2 - par 98), "accounting information cannot avoid affecting behavior, nor should it," for accounting does integrate modes of management, governance, and regulation. This implies that alternative accounting representations cannot be "neutral" with respect to the underlying socio-economic activities, i.e., they cannot rest "without influence on human behavior" (*ibidem*).

Accordingly, the quality of firm-specific information provided by the accounting system under accounting standards (or principles) becomes as much important as share market efficiency in processing such information set that is available to investors. This information set is jointly composed by market-driven and firm-specific information. In particular, the accounting system (and regulation) conveys a specific representation of business affairs that shapes the payments from the business firm to shareholders (influencing share investment pays-off in this way), and communicates collective signals to investors interested in the Share Exchange. Responding to the claim by Khotari (2001) for improved theorizing on accounting information, this concept of a dual information set expands upon that adopted by the semi-strong form of market efficiency developed by Fama (1970), that Fama and French (1992) related to firm-specific information driven by fundamental analysis. Fama (1970) distinguishes three forms of share market efficiency depending on the composition of the information set integrated by investors. Regarding the weak-form, the information set includes only the history of market prices; in the semi-strong form, it reflects all publicly available information; and the strong form tests it against all existing information. Our approach delves into the publicly available information set to disentangle two distinctive subsets: one driven by market (essentially, the history of prices), another comprising the firm-specific information made available to investors by other institutional devices external to the market itself. The financial framework is then featured by two distinctive institutional dimensions: the share market, which endogenously generates a collective information driven by the market dynamics (Phelps 1987; Kirman 1999) – that is, the series of clearing prices fixed by the Share Exchange through time; and a not-market dimension, which generates a collective information external to the market, driven by institutions that are complementary to the market and that facilitate its making (Frydman 1982; Sunder 2002; Biondi 2008).

This institutional approach originates a different perspective on share market dynamics (Shubik 1993; Sunder 1997). According to Fama (1991: 1575-1576),

the main obstacle to inferences about market efficiency (that he relates to collective or individual "rationality") is the joint-hypothesis problem that makes "market efficiency *per se* not testable": "we can only test whether information is properly reflected in prices in the context of a pricing model that defines the meaning of 'properly'." (*ibidem*). There, market efficiency is jointly tested with some equilibrium model that must define the way in which a certain information subset should be reflected by asset-pricing. Fama provisionally suggested that "the market efficiency literature should be judged on how it improves our ability to describe the time-series and cross-section behavior of security returns" (*ibidem*). Our approach maintains the focus on the flow of aggregate market prices through time, but it assumes that this flow is jointly influenced by two distinct sources of information: one driven by the market, another generated by other institutional devices such as established accounting processes of reporting and disclosure. This approach is then concerned with different paths of aggregate market prices depending on alternative institutional configurations, which feature this dual informational structure. The following analysis is expected to "enrich our knowledge on the behavior of returns across securities and through time" (Fama 1991: 1577), and may be extended to the impact of these alternative configurations on creation and allocation of resources among market participants, risk-sharing and bearing, and other featuring aggregate phenomena resulting from market dynamics.

This framework of analysis applies here to provide a theoretical model that captures this special accounting role in the share price formation over time. The accounting system is then supposed to constitute a lighthouse in the amazing dynamics of share market through hazard, learning and interaction. The rest of the paper is organized as follows. The first part presents the basic model, which comprises collective signals based on fundamental financial analysis, the formation of individual expectations and decision-making, and the evolution of the market price by matching aggregate demand and supply of shares through time. The second part provides an extended model and two theoretical simulations. The first simulation investigates the notion and formation of speculative bubbles under the novel framework of analysis. The second simulation investigates the relative impact on market price series of an accounting system that replicates the information provided by the market (so-called fair value accounting model), instead of constituting an autonomous source of firm-specific information (so-called historical cost accounting model). Theoretically informed implications and recommendations may then be derived on the cyclical effects of accounting information on share market dynamics (Boyer 2007; Rochet 2008).

Part I
The basic model

2 The conceptual framework

Consider a share market where only the shares of the business firm j are quoted. The total number of its shares s is normalized to one, and, for each period t, the Share Exchange generates a market price of every share p_t. Furthermore, the business firm distributes dividends per share D_t based on the financial performance and position of the business firm at time $t-1$. No discount nor alternative investment option rate apply.[1]

Risk-neutral investors i interested in this market conjecture about the share price-return relationship drawing upon heterogeneous mindsets:

$$E_{i,t} = E_{i,t}(p_t; D_t \,|\Phi_t) \; \forall i, t. \tag{1}$$

In particular, the individual investor's expectations (and strategies) depend on a composite information set Φ_t that draws upon a monetary (cash flow) and an epistemic (informational) basis. Both bases generate incentive structures that relate to the institutional configuration (regulation) of the share market. Concerning the monetary basis, each investor forms his own expectations on the dividend flow and the equity premium on share market price. The individual investor's financial return (pay-off) depends then on the market price he may obtain by selling his shares (or the market price he should pay for buying the firm's shares), and on the dividend flow that is distributed by the business firm and is established according to accounting institutions, among others. Concerning the epistemic basis, the individual investor's decision-making deals with two information flows provided by distinctive institutional structures. One flow of information is generated by the Share Exchange and is subsumed by the formation of aggregate (collective) pricing through time. Another flow comes from accounting and other regulatory institutions that are complementary to the market, and that facilitate the working of the share market itself (Frydman 1982).

In presence of heterogeneous individual mindsets, the price system and the accounting system complement each other in driving the market price formation trough time. The general *system* (which is no longer an equilibrium)[2] consists in and depends upon the coherence and universal diffusion of relevant and reliable knowledge by means of both a price system and an accounting system publicly

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[1] This is not less restrictive than the widespread hypothesis of a fixed discount rate on the whole time period of analysis.

[2] Our analysis distinguishes system and equilibrium as distinctive concepts.

determined and announced. The current period in-between *ex ante* and *ex post* locates here among future time, submitted to individual guesses and intentions, hopes and fears, and past time, an history of reporting that, in principle, may be partly public, consistent, and conventionally agreed (Shackle 1967). In this context, the financial reporting provided by the accounting system is assumed to be common knowledge (Sunder 2002) that delivers relevant and reliable signals on the financial performance and position generated by the business firm over time. These collective signals will be consistently integrated by individual investor's expectations (or guesses) that drive his strategies. Therefore, individual investors make their financial decisions taking account this composite information set provided by distinctive institutional devices. Each investor decides how to exploit the market-driven information, the firm-specific information, and the revision of its own guesses over time. The basic model assumes that four types of investors exist: S_I, S_O, F_I and F_O, each of them having different ways to form individual expectations, whilst the extended model shall analyze a greater degree of heterogeneity among investors.[3] The class I comprises two types of investors who currently hold shares (shareholders), respectively: investors S_I who speculate on the evolution of the market price and then utilize only the information generated by the Share Exchange; and investors F_I who comprehensively look also at fundamental information provided by the accounting system on the financial performance and position of the business firm over time. The evolution of the share market price will endogenously determine the composition of the shareholding basis, that is, the relative presence s_t of each type of investor "inside the market". Moreover, the class O comprises investors of both types (respectively, speculators S_O and fundamentalists F_O), who rest "outside the market" (that is, they currently do not hold shares). Again, the types of investors comprised in this latter class are expected to form their expectations in distinctive ways, for entering the share market by holding shares arguably changes the individual stake and position respective to the share investment return dynamics.

The Share Exchange is the institutional device that collectively generates the aggregate market price p_t through time. Such price generation may depend on either an exogenous time series (ε_t), or an endogenous response to aggregate demand lack or excess (d_t) - the latter resulting from the mismatching of heterogeneous individual expectations generated by heterogeneous (types of) investors through time:

$$p_t = p(p_{t-1}; d_t; \varepsilon_t) \ \forall t. \tag{2}$$

The market price dynamics fundamentally depends on the matching of individual expectations, which is expected to have greater impact than the exogenous time series that captures a residual shock. In addition, this dynamics also

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[3] This does not change the essential structure of the model, since individual expectations draw upon distinctive provisions of collective information.

depends on the shares' supply that may comprise an exogenous shareholders' quota b_t who must liquidate their position because of budget, liquidity, and credit access constraints, and other individual preferences independent from price expectations.

The following sections will discuss the formation of individual expectations and decision-making; the aggregation of demand and supply through matching pricing; and the resulting market price evolution through time.

3 Disentangling collective signals based upon firm-specific information

Investors form their individual expectations on the basis of a composite set of information. The first subset is generated by the Share Exchange through the "price system", whilst the second subset comes from the accounting institutional framework and is provided by the "accounting system" of the business firm. In sum, the information set available to investors for share investment decision-making comprises two kinds of collective information:

- market-driven information subsumed by market price formation and history;

- firm-specific information subsumed by financial reporting over time.

Whilst the following section treats the individual discovery and processing of market-driven information, this section disentangles the firm-specific collective signal $F_t(\cdot)$ that each investor i integrates in his own mindset through the weight $\varphi_i \geq 0$.

Consider the subset of fundamental information $Y_{t,j}$ that is available on period t and specific to the business firm j. This subset may provide a lighthouse through the share market dynamics by delivering relevant and reliable collective signal $F_t(\cdot)$ that will aid individual investors to form their own expectations:

$$F_t = F_t(\cdot \,|Y_{t,j}) \ \forall t. \tag{3}$$

Analytically, let assume that $F_t(\cdot)$ is composed by three generic methods of evaluation, all based on the same set of information $Y_{t,j}$. Consequently, individual investors may utilize one of these methods, or a linear combination of them:

$$F_t = \lambda F_1\left(\cdot\,|Y_{t,j}\right) + \mu F_2\left(\cdot\,|Y_{t,j}\right) + (1-\lambda-\mu)\,F_3\left(\cdot\,|Y_{t,j}\right) \ \text{ with } 0 \leq \lambda, \mu \leq 1. \tag{4}$$

We define each method according to empirically-based widespread methods of fundamental analysis: the first method is a flow method that relates to price to earnings ratio analysis; the second method is a stock method that relates to the market to book ratio analysis; and the third method is an intelligence) (or expert) method based on qualitative information on the specifics of the ongoing business firm and its changing environment.

Theoretically speaking, this modeling of accounting information analysis to assess market pricing (Demsetz 1995: 93) is in line with *unconstrained* and *constrained* relationships between the market price series and available accounting information. In particular, our design expands upon Lev and Zarowin (1999) who analyze the *unconstrained* relationship between yearly average market prices $p_{j,t}$, accounting measures of residual business income $F_{1_{j,t}}$ and cumulated shareholders equity $F_{2_{j,t}}$, and other relevant information on the business firm $F_{3_{j,t}}$ (independent from $F_{1_{j,t}}$ and $F_{2_{j,t}}$):

$$p_{j,t} = \alpha_0 + \alpha_1 F_{1_{j,t}} + \alpha_2 F_{2_{j,t}} + \alpha_3 F_{3_{j,t}}. \tag{5}$$

Accordingly,[4] concerning the NYSE between 1977-1996, the yearly cross-sectional association between share prices and accounting measures, as assessed by R^2, is above 0.9 during 1977 and 1988, and around 0.6 during 1989-1996 - where R^2 is here a measure of estimation error of p from accounting measures weighted by firm, with weights α obtained by cross-sectional annual regression.

Furthermore, our design corresponds with *constrained* relationships between the market price series and accounting information argued for by fundamental financial analysis literature (Dechow, Hutton, and Sloan 1999; Ohlson 1995; Feltham and Ohlson 1995; Ou and Penman 1989). This literature focuses on persistence of business incomes, based either on time-series behavior or conditioning determinants (Lev and Thiagarajan 1993; Chant 1980; Freeman, Ohlson, and Penman 1982).

3.1   The flow method M_1 of fundamental analysis

The first method M_1 is expected to provide relevant and reliable information to identify over- or under-evaluated shares following a flow method of fundamental analysis. This implies to timely compare accounting information on residual business incomes $f(Y_{t,j})$ to the share pricing p_t over time (Figure 1).

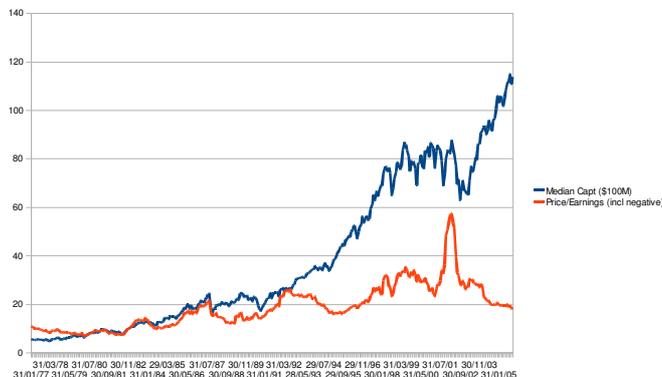[4] See also Nichols and Wahlen (2004).

Figure 1: *Median Market Capitalisation and Price to Earnings ratio (S&P500; 1977-2005).* Descriptive statistics based on S&P/Barra Monthly Indexes available at http://www.barra.com/Research/Description.aspx

For instance, fundamental analysts may try to identify some persistent part of period earnings (so-called "permanent earnings") that should endure trough time. Accordingly, they may derive a relevant and and reliable signal M_1 from fundamental information $Y_{t,j}$, in order to assess the share pricing and confirm the market judgment. Analytically, let denote the result of this analysis as the change of the ratio between the share price and the estimate of permanent earnings through time:

$$\Delta_t(M_1) = \frac{p_t}{E_t(f(Y_{t,j}))} - \frac{p_{t-1}}{f(Y_{t-1,j})} \qquad (6)$$

where $\Delta_t(M_1)$ is the expected variation of the signal M_1, $E_t(f(Y_{t,j}))$ is the expected amount of permanent earnings, and the $f(Y_{t-1,j})$ is the past estimate of permanent earnings. In particular:

- When $\Delta_t(M_1) > 0$, then the change in residual business incomes is lower than the change in share price. In other words, one euro of earnings is more costly than in the past. The signal communicates overvaluation, and individual investors expect that future share prices may decrease;

- When $\Delta_t(M_1) < 0$, then the signal communicates undervaluation, and individual investors expect then that future prices may increase.

Since the actual earnings of the current period are not perfectly known during that period, investors should guess their expected amount starting from the collective information subset $Y_{t,j}$:

$$E_t(f(Y_{t,j})) = f(Y_{t-1,j}) + a_{m1}(f(Y_{t-1,j}) - f(Y_{t-2,j})). \qquad (7)$$

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This generic formulation of *individual* expectations on $f\left(Y_{t,j}\right)$[5] may be avoided by assuming that investors receive this signal by a common expert system or standardized procedure. This formulation implies that the expected amount of permanent earnings depends only on their trend, since the subset of firm-specific collective information $Y_{t,j}$ is supposed to evolve in a sufficiently stable way over time. On the matter, Abarbanell and Bushee (1998, 31-32) note that actual mispricing on the share market (allowing abnormal returns) is corrected only *ex-post* as next year's earnings are revealed.

Furthermore, according to current regulatory framework and business practices, the determination of residual net earnings provided by the accounting system constitutes the upper limit to the distribution of dividends. This implies that the actual dividend flow per share D_t may be rewritten in terms of net earnings per share (actual or permanent), the latter being a function of the accounting information $f(Y_{t,j})$:

$$D_t = \delta f\left(Y_{t-1,j}\right) \text{ with } \delta > 0. \tag{8}$$

Then, the (6) and (7) may be rewritten as follows:

$$\Delta_t(M_1) = \frac{p_t}{E_t(f\left(Y_{t,j}\right))} - \frac{\delta p_{t-1}}{D_{t-1}}, \tag{9}$$

$$E_t(f\left(Y_{t,j}\right)) = \frac{D_{t-1}}{\delta} + a_{m1}\left(\frac{D_{t-1}}{\delta} - \frac{D_{t-2}}{\delta}\right) \tag{10}$$

and, by computing, the (9) becomes

$$\Delta_t(M_1) = \delta\left(\frac{D_{t-1}\left(p_t - p_{t-1}\right) - a_{m1}\left(D_{t-1} - D_{t-2}\right)p_{t-1}}{D_{t-1}\left(D_{t-1} + a_{m1}\left(D_{t-1} - D_{t-2}\right)\right)}\right). \tag{11}$$

In sum, the collective signal based on flow method M_1 implies that:

$$F_1\left(\Delta_t(M_1)\right) = -m_1\Delta_t(M_1) \text{ with } m_1 > 0. \tag{12}$$

3.2 The stock method M_2 of fundamental analysis

The second method M_2 is expected to provide relevant and reliable information to identify over- or under-evaluated shares based on a stock method of fundamental analysis. This implies to timely compare accounting information on net assets $g\left(Y_{t,j}\right)$ to the share pricing p_t over time (Figure 2).

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[5] Whilst the information subset Y_t is collective, its interpretation is different for each (type of) investor i. This heterogeneity is captured by the parameter φ_i applied to F_t.

Figure 2: *Median Market Capitalisation and Price to Book ratio (S&P500; 1977-2005).* Descriptive statistics based on S&P/Barra Monthly Indexes available at http://www.barra.com/Research/Description.aspx

For instance, fundamental analysts may try to identify some consistent part of cumulated net assets (so-called "book value") that is expected to compare with share price (so-called "market value") trough time.[6] Accordingly, they may derive a relevant and and reliable signal M_2 from fundamental information $Y_{t,j}$, in order to assess the share pricing and confirm the market judgment. Analytically, let denote the result of this analysis as the change of the ratio between the share price and the estimate of consistent book value through time:

$$\Delta_t(M_2) = \frac{p_t}{E_t(g\,(Y_{t,j}))} - \frac{p_{t-1}}{g\,(Y_{t-1,j})} \tag{13}$$

where $\Delta_t(M_2)$ is the expected variation of M_2, $E_t(g\,(Y_{t,j}))$ is the expected consistent book value, and $g\,(Y_{t-1,j})$ is the past estimate of the consistent book value. In particular:

- When $\Delta_t(M_2) > 0$, then the change in the consistent book value is lower than the change in the share market price (ideally, these two values should be equal under a perfect share market and a perfect accounting system based on current values). In other words, one euro of consistent shareholders equity is more costly than in the past. The signal communicates overvaluation, and individual investors expect that future prices may decrease; viceversa,

- When $\Delta_t(M_2) < 0$, then the signal communicates undervaluation, and individual investors expect that future prices may increase.

Again, since required accounting data for the period t are not perfectly known during that period, the expectation on their amount depends on their

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[6] In principle, under complete and perfect markets, book value and market value of shares equalise. In actuality, investors may guess about some reasonable relationship over time.

historical trend - assuming a sufficiently stable evolution of the information set $Y_{t,j}$ over time:

$$E_t(g(Y_{t,j})) = g(Y_{t-1,j}) + a_{m2}(g(Y_{t-1,j}) - g(Y_{t-2,j})).$$ (14)

Furthermore, the consistent book value $g(Y_{t,j})$ is, by definition, a proportion γ of the actual book value B_{t-1}:

$$g(Y_{t,j}) = \gamma B_{t-1} \text{ with } \gamma > 0.$$ (15)

Accordingly, the (13) becomes

$$\Delta_t(M_2) = \frac{p_t}{E_t(B_t)} - \frac{\gamma p_{t-1}}{B_{t-1}}$$ (16)

and the (14) becomes

$$E_t(B_t) = \frac{1}{\gamma}(B_{t-1} + a_{m2}(B_{t-1} - B_{t-2})).$$ (17)

Finally, computing the (16) and the (17),

$$\Delta_t(M_2) = \gamma \left(\frac{B_{t-1}(p_t - p_{t-1}) - a_{m2}(B_{t-1} - B_{t-2})p_{t-1}}{B_{t-1}(B_{t-1} + a_{m2}(B_{t-1} - B_{t-2}))} \right).$$ (18)

In sum, the collective signal based on stock method M_2 implies that:

$$F_2(\Delta_t(M_2)) = -m_2 \Delta_t(M_2) \text{ with } m_2 > 0.$$ (19)

3.3 The qualitative method M_3 of fundamental analysis

The third method M_3 is expected to provide relevant and reliable information to identify over- or under-evaluated shares according to the timely analysis of qualitative information included in the information set $Y_{t,j}$, concerned with the perspectives of the business firm j relative to its future and environment. Denoted by the function $h_t(M_3)$, this analysis provides a collective message on positive, negative, or uncertain perspectives for that firm. On this basis:

- When $h_t(M_3) > 0$, then the signal communicates positive perspectives, and individual investors expect that future prices may increase;

- When $h_t(M_3) < 0$, then the signal communicates negative perspectives, and individual investors expect that future prices may decrease;

- When $h_t(M_3) = 0$, then the signal conveys an uncertain message, and individual investors do not derive any expectation from it.

In sum, the collective signal based on qualitative method M_3 implies that:

$$F_3\left(h_t(M_3)\right) = m_3 h_t(M_3) \text{ with } m_3 > 0. \tag{20}$$

In conclusion, according to the above analysis of collective signals M_1, M_2 and M_3, driven by firm-specific information $Y_{t,j}$, the general function F_t (equation 4) becomes:

$$F_t = -\lambda m_1 \Delta_t(M_1) - \mu m_2 \Delta_t(M_2) + (1 - \lambda - \mu) m_3 h_t(M_3). \tag{21}$$

4 The formation of individual investor's expectations

This section analyzes how individual investors form their market price expectations. On the basis of these expectations, they will decide whether enter the Share Exchange by buying or selling, or simply wait (individual decision-making). For sake of simplicity, the basic model assumes that share market comprises only two types of investors: fundamentalists F, and speculators S. Each type comprises shareholding and potential investors: respectively investors inside the market $(F_I; S_I)$, and investors outside it $(F_O; S_O)$. The relative proportion of each type among shareholding investors (inside the market) is *endogenously* determined by the share market dynamics. This means that the impact of different types of individual strategy depends on this dynamics (that in turn relates to the overall institutional configuration), not only on subjective attitudes or beliefs.

Every type of investors forms specific and timely expectations on the basis of the available information set comprising market-driven and firm-specific information. The general model[7] for individual expectations $E_{i,t}\left(p_{t+1}\right)$ comprises the current price p_t, the revision of past price expectation $E_{i,t-1}\left(p_t\right)$, the price trend $\Delta_{t,t-1}(p_{t-1})$, and the composite signal from fundamental analysis F_t:

$$E_{i,t}\left(p_{t+1}\right) = E_{i,t}(p_t; E_{i,t-1}\left(p_t\right); \Delta_{t,t-1}(p); F_t) \ \forall t \tag{22}$$
$$\text{where } i = F_I, F_O, S_I, S_O.$$

In particular, each type of investors has definite preferences about the relative weights given to each component of the model:

- the speculators do not utilize signals from fundamental analysis, whilst fundamentalists do;

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[7] This model of price expectation $E_{S_{t,I}}\left(p_{t+1}\right)$ results from a combination between a "first order adaptive model": $E_t(P_{t+1}) = E_{t-1}(P_t) + \beta'\left(P_t - E_{t-1}(P_t)\right)$ where β' weights the revision of the most recent expectation error, and an "extrapolative expectation model": $E_t(P_{t+1}) - (P_t) = \gamma(P_t - P_{t-1})$ where γ weights the most recent price change (trend). Whenever $\gamma > 0$, any market price increase results in increasing the price expectation.

- investors outside the market (which currently do not hold shares) do not revise their past price expectation, whilst investors inside the market (which currently hold shares) do.[8]

4.1 The fundamentalists' mindset

Fundamentalists are expected to form their market price expectations by analyzing the whole set of information comprising the history of the share price p_t, and the firm-specifc informational subset $Y_{t,j}$. The latter includes history of dividends, accounting data, and qualitative information concerned with the overall performance and position generated by the business firm through time. For instance, fundamentalists may be considered as such investors that pay the cost of, or are capable to discover and process the firm-specific information provided by the accounting system of the business firm.

Fundamentalists may split in two groups: the shareholding fundamentalists $F_{I,t}$, and the potential fundamentalists $F_{O,t}$. The latter may decide if buying shares (and thus become shareholders) on the basis of their individual expected return involved by the future market price. Each group forms its own distinctive price expectation, respectively: $E_{F_I,t}(p_{t+1})$ and $E_{F_O,t}(p_{t+1})$.

Concerning shareholding fundamentalists, their market price expectation is:

$$E_{F_I,t}(p_{t+1}) = p_t + \varphi_i F_t(\cdot) + \alpha_F (p_t - p_{t-1}) + \beta_F (E_{F_I,t-1}(p_t) - p_t) \quad (23)$$

where the future price depends on the price at time t; the price trend $(p_t - p_{t-1})$; the revision of their previous price expectation $(E_{F_I,t-1}(p_t) - p_t)$; and the composite signal $F_t(\cdot)$ derived by fundamental analysis.

Concerning potential fundamentalists (who still rest outside the Share Exchange), their market price expectation is:

$$E_{F_O,t}(p_{t+1}) = p_t + \varphi_i F_t(\cdot) + \alpha_F (p_t - p_{t-1}). \quad (24)$$

Again, this latter expectation is featured by the absence of the revision on previous expectations, since potential fundamentalists did not hold shares, and thus did not have previous expectations to be revised.

Furthermore, according to the assumed design of signals by fundamental analysis based upon firm-specific information, the fundamentalists'market price expectations (equations 23 and 24) become:

$$E_{F_I,t}(p_{t+1}) = p_t + \varphi_i \left((1 - \lambda - \mu) m_3 h_t(M_3) - \lambda m_1 \Delta_t(M_1) - \mu m_2 \Delta_t(M_2) \right) +$$
$$+ \alpha_F (p_t - p_{t-1}) + \beta_F (E_{F_I,t-1}(p_t) - p_t) \quad (25)$$

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[8]This is equivalent to set the corresponding parameter $\beta_i = 0$, which implies an "extrapolative expectation model", since potential speculators do not hold shares and consequently do not have past expectations on past market prices.

and

$$E_{F_O,t}\left(p_{t+1}\right) = p_t + \varphi_i\left(\left(1 - \lambda - \mu\right)m_3 h_t(M_3) - \lambda m_1 \Delta_t(M_1) - \mu m_2 \Delta_t(M_2)\right) + \\ + \alpha_F\left(p_t - p_{t-1}\right). \tag{26}$$

4.2 The speculators' mindset

Speculators are expected to form their expectations by analyzing only the market-driven subset of information delivered by share market pricing. This peculiar mindset may be justified both by the cost (or lacked capability) of discovering and processing the firm-specific information, and by other personal preferences on investment choices, horizons and strategies.

Once again, speculators may split in two groups: shareholding speculators $S_{I,t}$, and potential speculators $S_{O,t}$. The latter may decide if buying shares (and becoming shareholders indeed) on the basis of their individual expected return driven by the future market price. Each group forms its own distinctive price expectation, respectively: $E_{S_I,t}\left(p_{t+1}\right)$ and $E_{S_O,t}\left(p_{t+1}\right)$.

Concerning shareholding speculators, their price expectation is:

$$E_{S_I,t}\left(p_{t+1}\right) = p_t + \alpha_s\left(p_t - p_{t-1}\right) + \beta_s\left(E_{S_I,t-1}\left(p_t\right) - p_t\right) \text{ with } \alpha_s, \beta_s > 0. \tag{27}$$

This means that their price expectation depends on the current price at time t, on the price trend $(p_t - p_{t-1})$, and on the revision of their previous price expectation $(E_{S_I,t-1}\left(p_t\right) - p_t)$.

Concerning potential speculators (who still rest outside the Share Exchange), their price expectation is:

$$E_{S_O,t}\left(p_{t+1}\right) = p_t + \alpha_s\left(p_t - p_{t-1}\right) \text{ with } \alpha_s > 0. \tag{28}$$

They cannot revise previous price expectation that they have not yet formed, since they did not enter the Share Exchange and did not hold shares in the past period $t - 1$.

4.3 Summing-up

In sum, let define the following price expectation function which includes all the possible mindsets of the generic investor i:

$$E_{i,t}\left(p_{t+1}\right) = p_t + \varphi_i F_t(\cdot) + \alpha_i\left(p_t - p_{t-1}\right) + \beta_i\left(E_{i,t-1}\left(p_t\right) - p_t\right) \tag{29}$$
$$\text{with } i = F_I, F_O, S_I, S_O$$
$$\text{where } \beta_i = 0 \text{ if } i = F_O, S_O; \text{ and } \varphi_i = 0 \text{ if } i = S_I, S_O.$$

The basic model denotes the parameter φ_i - which captures the heterogeneity between individual investors in interpreting signals $F_t(\cdot)$ based upon collective information $Y_{t,j}$ - as follows:

$$\varphi_i = \left\{ \begin{array}{l} \varphi \text{ if } i = F_I, F_O \\ \quad 0 \text{ otherwise.} \end{array} \right. \tag{30}$$

The following extended model shall introduce higher degree of heterogeneity among investors with $\varphi_i \in (0,1) \; \forall i$.

5 Individual investor's decision-making

This section analyzes how each type of investors decides its buy-or-sell strategy according to its expectations on share market price. Under conditions of radical heterogeneity and interaction between investors, and the dual informational (and institutional) structure, we maintain the basic rational expectation hypothesis that links individual strategies to the expected return (pays-off) R from the share investment:

$$R = E_{i,t}(G_{i,t} - p_t) \; \forall t \text{ with } i = F_I, F_O, S_I, S_O. \tag{31}$$

where $G_{i,t}$ is the prospective potential gain. Accordingly investors i choose between returns today and future returns including capital gains, that is a form of Bellman's equation:

$$G_{i,t} = \max \arg \left\{ E_{i,t}(p_{t+1}) + E_{i,t}(D_t), G_{t+1} \right\}. \tag{32}$$

This individual return relates then to the expected dividend flow, the expected equity premium on share price period by period, and the prospective potential gain on future periods. These pay-offs are influenced by the dual institutional configuration of the share market. In particular, the dividend flow fundamentally depends on the accounting system, whilst the equity premium fundamentally depends on the price system. The prospective potential gain combines options from both.

Note that, at time t, the dividend flow D_t and the related actual earnings (that constitute the upper bound of the dividend flow) are not known. For sake of simplicity, let assume that *all* investors form their dividend expectation as follows:

$$E_{i,t}(D_t) = D_{t-1} + a_D (D_{t-1} - D_{t-2}) \; \forall i. \tag{33}$$

The dividend flow D_t may be sufficiently stable to justify this common expectation model based on dividend trend. Modeling individual expectations may be avoided by assuming that investors receive this information by common expert system or standardized procedure, instead of building themselves their own

expectations on D_t. As every contracted or regulated business flow (especially taxation), the dividend flow is ultimately related to collective firm-specific information Y_t. A model refinement may then allow fundamentalists to exploit signals of fundamental analysis F_t (based upon Y_t) in order to better estimate the prospective dividend flow generated by the business firm over time (looking at permanent earnings instead of dividends, for instance). Such refined expectation model would be related to equation (8), which is identical to equation (33) when $a_D = a_{m1}$.

Furthermore, at this stage, these individual strategies are only *wishes*, since they *may or not* meet a willing counterpart to eventually perform a share exchange. This matching step would realize according to the aggregation of individual demands and supplies on the marketplace. Given radical heterogeneity between interacting investors, speculative capital gains are then made possible by inconsistent plans between them (Tirole 1982). The aggregate matching will be analyzed in the next section.

5.1 The speculators' strategy

At each period t, speculators decide their strategy by looking only at the market side of the Share Exchange. They will focalize only on two periods by taking into account time t and time $t+1$. They compare the current price p_t to expectations on the dividend flow D_t (which will be paid out during time $t+1$) and the future share price p_{t+1}. This implies that:

$$
\begin{aligned}
G_t &= \max \arg \left\{ E_{S_I,t}\left(p_{t+1}\right) + E_t\left(D_t\right), 0 \right\} \\
&= E_{S_I,t}\left(p_{t+1}\right) + E_t\left(D_t\right) \ \forall t.
\end{aligned}
\tag{34}
$$

Inside the market, shareholding speculators decide if sell or hold their shares as follows:

$$
\begin{aligned}
&\text{if } p_t \geq E_{S_I,t}\left(p_{t+1}\right) + E_t\left(D_t\right) \ \text{then } S_{I,t} \ wish \ to \ \text{sell} \\
&\text{if } p_t < E_{S_I,t}\left(p_{t+1}\right) + E_t\left(D_t\right) \ \text{then } S_{I,t} \ \text{hold.}
\end{aligned}
\tag{35}
$$

By assuming such truncated comparison between the current and the future period, the equation (35) shows their best strategy:

- if they sell their shares, then they will receive the price p_t;

- if they hold the shares for another period and wait for selling, then they will receive the dividend per share D_t, because they still hold the shares during the time $t+1$ (when the dividend flow is paid out). In addition, they keep the possibility to sell at the future price at time $t+1$.

Outside the market, potential speculators decide if buy or not shares as follows:

$$\text{if } p_t - E_t\left(D_t\right) \leq E_{S_O,t}\left(p_{t+1}\right) \text{ then } S_{O,t} \text{ wish not to buy}$$
$$\text{if } p_t - E_t\left(D_t\right) > E_{S_O,t}\left(p_{t+1}\right) \text{ then } S_{O,t} \text{ wish to buy.} \tag{36}$$

Again, by assuming such truncated comparison between the current and the future period, this equation shows their best strategy:

- if they buy shares, then they will pay the price p_t and receive the D_t, because they hold shares during the time $t + 1$;

- if they expect a lower share price, then they will not buy, but wait for a period.

5.2   The fundamentalists' strategy

At each period t, fundamentalists decide their strategy by looking at the market side and the firm-specific (non-market) side of the Share Exchange. Fundamental analysis allows them to analyze prospective expectations beyond the next period $t + 1$. They purport to derive all relevant and reliable information from signals of fundamental analysis based upon firm-specific information. Therefore, they compare the current price p_t to such prospective expectations on business performances that (are expected to) eventually lead the future price path in a longer horizon.

Inside the market, shareholding fundamentalists decide if sell or hold their shares as follows:

$$\text{if } p_t \geq E_{F_I,t}\left(G_t\right) \text{ then } F_{I,t} \text{ wish to sell}$$
$$\text{if } p_t < E_{F_I,t}\left(G_t\right) \text{ then } F_{I,t} \text{ hold.} \tag{37}$$

By assuming prospective expectations formed by shareholding fundamentalists, the equation (39) shows their best strategy:

- if they sell their shares, then they will receive the current price p_t;

- if they hold the shares for another period, then they will receive the dividend per share D_t, and maintain the prospective potential gain G_t, related to the possibility to choose tomorrow if sell the shares at the price expected at time $t + 1$ or keep holding them, receiving D_{t+1} and so on.

Accordingly, the prospective potential gain G_t is:

$$G_t = \max \arg\left\{E_{F_I,t}\left(p_{t+1}\right) + E_t\left(D_t\right), G_{t+1}\right\}. \tag{38}$$

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Note that, if shareholding fundamentalists form their expectations by analyzing only two periods, the (37) rejoins the shareholding speculators' formula as follows:

$$\text{if } p_t \geq E_{F_I,t}(p_{t+1}) + E_t(D_t) \text{ then } F_{I,t} \text{ wish to sell}$$
$$\text{if } p_t < E_{F_{t,I}}(p_{t+1}) + E_t(D_t) \text{ then } F_{I,t} \text{ hold.} \tag{39}$$

Outside the market, potential fundamentalists decide if buy or not shares as follows:

$$\text{if } p_t \geq E_{F_O,t}(G_t) \text{ then } F_{O,t} \text{ wish not to buy}$$
$$\text{if } p_t < E_{F_O,t}(G_t) \text{ then } F_{O,t} \text{ wish to buy.} \tag{40}$$

By assuming prospective expectations formed by potential fundamentalists, the equation (39) shows their best strategy:

- if they buy shares, then they will pay the price p_t and receive the dividend D_t, because they will hold the shares during the time t, and acquire the prospective potential gain G_t;

- if they expect an higher price, then they do not buy and wait for one period.

Again, if potential fundamentalists form their expectations by analyzing only two periods, the (40) rejoins the potential speculators' formula as follows:

$$\text{if } p_t - E_t(D_t) > E_{F_O,t}(p_{t+1}) \text{ then } F_{O,t} \text{ wish not to buy}$$
$$\text{if } p_t - E_t(D_t) \leq E_{F_O,t}(p_{t+1}) \text{ then } F_{O,t} \text{ wish to buy.} \tag{41}$$

6 The formation of aggregate share market price over time

6.1 Aggregate Demand and Supply

Investors inside and outside the market observe the aggregate share market price p_t of the business firm j and, according to their own expectations on p_{t+1}, they decide whether change their position through buying or selling, or simply wait for the next period.

However, during each time period t, a quota b_t of current shareholders (investors inside the market) may be forced to liquidate its position because of budget, liquidity, and credit access constraints, and other individual preferences independent from their expectations (or guesses) on future market price p_{t+1}. By assumption:

- this quota may be constant over time: $b_t = b > 0 \ \forall t$. A further extension may analyze the dynamics of this quota and its impact on the price formation;

- this quota b_t is the same for F_I and S_I. This means that, higher is the proportion of fundamentalist F_I (speculators S_I) inside the market, higher is the number of shareholders from this category that must liquidate their position.

Accordingly, at each time t, the composition of the shareholding basis comprises (with $b_t + s_t = 1 \ \forall t$):

- $b_t = b$, the quota of shareholders which must liquidate and go out of the share market;

- $(1 - b)s_t$, the quota of shareholders which are investors of type S_I;

- $(1 - b)(1 - s_t)$, the quota of shareholders which are investors of type F_I.

In this way, the quota $s_t \ \forall t \neq 0$ is endogenously generated by the market dynamics through time. At time $t = 0$, let assume $s_0 = \frac{1}{2}$, that is, both types of shareholding investors, S_I and F_I, hold shares in the same proportion.

Apart from the quota b, the aggregation of individual demand and supply depends on the making of individual expectations and related financial decisions. Therefore, a *focal price* exists that induces each type of investors to change its position and enter the Share Exchange. In particular:

- $(p_t^*)_{F,I}$ for the shareholding fundamentalists, inside the market;

- $(p_t^*)_{F,O}$ for the potential fundamentalists, outside the market;

- $(p_t^*)_{S,I}$ for the shareholding speculators, inside the market;

- $(p_t^*)_{S,O}$ for the potential speculators, outside the market.

Remember that, by assumption, the focal price of one type of investors may differ from the focal price of another. For instance, the focal price of shareholding fundamentalists $(p_t^*)_{F,O}$ may differ from the focal price of potential fundamentalists $(p_t^*)_{F,I}$, whilst the focal price of potential speculator $(p_t^*)_{S,O}$ may differ from the focal price of shareholding speculator $(p_t^*)_{S,I}$. Since the time $t = 0$ when the firm J offers its shares on the primary market, the conditions to place shares follow the aggregation of individual demands and supplies based on these focal prices.

Let $(p_t^*)_{F,I}$ the minimum price that induces the shareholding fundamentalists $F_{t,I}$ to sell their shares. On this basis,

if $p_t \geq (p_t^*)_{F,I}$, then all the $(1-b)(1-s_t)$ investors wish to sell their shares;

if $p_t < (p_t^*)_{F,I}$, then $(1-b)(1-s_t)$ investors do hold their shares.

If the current price p_t is higher than the maximum return expected by fundamentalists $F_{I,t}$ by holding their shares (and waiting), then they will prefer selling them and obtain the maximum expected return, since:

$$(p_t^*)_{F,I} \geq E_{F_I,t}(G_t). \tag{42}$$

Note that fundamentalists $F_{I,t}$ are not forced to sell immediately according to the results of their fundamental analysis. Even though they expect, on a longer horizon, that the market price will eventually decrease according to the fall of prospective gain G_t from the business firm J, they may nevertheless decide to hold shares if they believe that the next period price will be so high to deliver a convenient expected gain. In this case, fundamentalists act like short-termist speculators in order to maximize their expected return from share investment.

Let $(p_t^*)_{F,O}$ the maximum price that induces the fundamentalists $F_{O,t}$ outside the market to buy shares. On this basis,

if $p_t \leq (p_t^*)_{F,O}$, then all the potential fundamentalists $F_{O,t}$ wish to buy shares

if $p_t > (p_t^*)_{F,O}$, then potential fundamentalists $F_{O,t}$ do not buy shares.

If p_t is higher than the maximum return that the $F_{O,t}$ investors expect to obtain by entering the market this period and look, then it is not convenient for them to buy shares, since:

$$(p_t^*)_{F,O} \leq E_{F_O,t}(G_t). \tag{43}$$

Let $(p_t^*)_{S,I}$ the minimum price that induces the shareholding speculators $S_{I,t}$ to sell their shares. On this basis,

if $p_t \geq (p_t^*)_{S,I}$, then all the $(1-b)s_t$ speculators wish to sell shares

if $p_t < (p_t^*)_{S,I}$, then $(1-b)s_t$ investors do hold shares.

If p_t is higher than the maximum return that the shareholding speculators $S_{I,t}$ expect to obtain by holding shares and waiting, then it is convenient for them to sell their shares, since

$$(p_t^*)_{S,I} \geq E_{S_I,t}(p_{t+1}) + E_t(D_t). \tag{44}$$

Let $(p_t^*)_{S,O}$ the maximum price that induces potential speculators $S_{O,t}$ to enter the market by buying shares and look:

if $p_t \leq (p_t^*)_{S,O}$, then all the $S_{O,t}$ investors wish to buy shares

if $p_t > (p_t^*)_{S,O}$, then $S_{O,t}$ investors do not buy shares.

If p_t is higher than the maximum return that the potential speculators $S_{t,O}$ expect to obtain by buying shares and looking, then it is not convenient for them to buy, since

$$(p_t^*)_{F,O} \leq E_{S_O,t}(p_{t+1}) + E_t(D_t). \tag{45}$$

All together, these four focal prices determine a "marketable area" (that could not exist) where share exchanges are *whished* by some shareholding and potential investors (Figure 3).
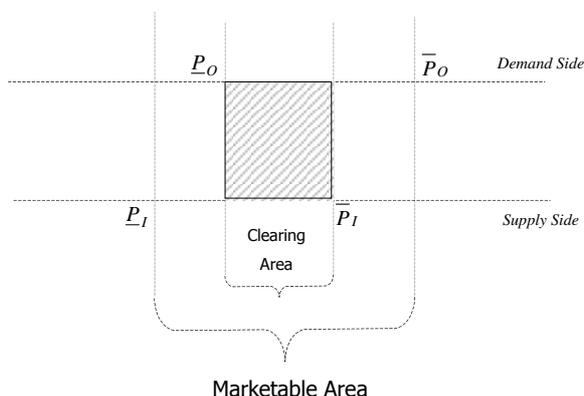

Figure 3: *Aggregate demand and supply when clearing is possible under the basic model*

The market price setting will decide which exchanges will be eventually performed within the "clearing area" (Figure 3).

6.2 Aggregate matching through Share Exchange

Three possible scenarios arise from this aggregation of individual demands and supplies and from the endogenous composition of the shareholding basis among speculators and fundamentalists:

- in the first scenario, aggregate demand meets aggregate supply (a clearing price exists);

- in the second scenario, the share market experiences an aggregate lack of demand;

- in the third scenario, the share market experience an aggregate lack of supply.

Let define

$$\overline{P}_i \equiv \max \arg \left((p_t^*)_{F,i} ; (p_t^*)_{S,i} \right) \tag{46}$$
$$\underline{P}_i \equiv \min \arg \left((p_t^*)_{F,i} ; (p_t^*)_{S,i} \right)$$
$$\text{where } i = I, O.$$

The **first scenario** may be described as follows:

$$\overline{P}_I \geq p_t \leq \underline{P}_O.$$

Here there exists a clearing price $\widehat{p}_t$ at which at least one type of shareholders decides to sell and at least one type of potential investors (outside the market) want to buy. In particular, from the supply side:

$F_{t,I}$ **sell** If $(p_t^*)_{F,I} > \widehat{p}_t \leq (p_t^*)_{S,I}$, then the price $\widehat{p}_t$ is, for the fundamentalists, higher than the expected future return to hold the shares. Consequently they sell their shares. Furthermore, $\widehat{p}_t$ is, for the speculators, lower than the expected future price. Consequently they do not sell their shares.

$S_{t,I}$ **sell** If $(p_t^*)_{S,I} > \widehat{p}_t \leq (p_t^*)_{F,I}$, then the price $\widehat{p}_t$ is, for the fundamentalists, lower than the expected future return to hold the shares. Consequently they hold their shares. Furthermore, $\widehat{p}_t$ is, for the speculators, higher than the expected future price. Consequently they sell their shares.

Both sell If $\widehat{p}_t \geq \underline{P}_I$, then the price $\widehat{p}_t$ is, for both categories, higher than the expected future. Consequently all sell their shares.

From the demand side:

$F_{t,O}$ **buy** if $(p_t^*)_{S,O} > \widehat{p}_t \leq (p_t^*)_{F,O}$, then the price $\widehat{p}_t$ is, for the fundamentalists, lower than the expected future return. Consequently they wish to buy shares. Furthermore, $\widehat{p}_t$ is, for the speculators, higher than the expected future price. Consequently they do not buy shares.

$S_{t,O}$ **buy** if $(p_t^*)_{F,O} > \widehat{p}_t \leq (p_t^*)_{S,O}$, then the price $\widehat{p}_t$ is, for the fundamentalists, higher than the expected future gain. Consequently they wish not to buy shares. On the contrary, $\widehat{p}_t$ is, for the speculators, lower than the expected future price. Consequently they wish to buy shares.

Both buy if $\widehat{p}_t \leq \overline{P}_O$ then the price $\widehat{p}_t$ is, for both categories, lower than the expected future gain. Consequently all wish to buy shares.

Let assume that, if both types of *potential* investors wish to buy, the shares sold in the Share Exchange are equally acquired by both types (fundamentalists and speculators), as it was assumed for $t = 0$. A further refinement may analyze the case in which this proportion differs and evolves over time, for example in line of the current *shareholding* basis: s_t for speculators, and $(1 - s_t)$ for fundamentalists.

The **second scenario** may be described as follows:

$$\overline{P}_I \geq p_t > \underline{P}_O.$$

In this scenario, the market price, $\widetilde{p}_t$, does not *clear* the Share Exchange. At this price, at least one category of shareholders decides to sell whilst no potential investors (outside the market) want to buy: the latter prefer to wait for a price decrease. The next section will analyze the price dynamics in this case. However, if $p_t = \widehat{\widetilde{p}}_t = \underline{P}_O$, then at least one type of potential investors want to buy, and this second scenario rejoins the first one.

The **third scenario** may be described as follows:

$$\overline{P}_I < p_t \leq \underline{P}_O.$$

In this scenario, the price, $\overline{p_t}$, does not clear the Share Exchange. At this price, at least one type of potential investors (outside the market) wish to buy shares, but no shareholders want to sell: the latter prefer to wait for a price increase. Anyway, by assumption, a quota of shareholders b exists that is forced to sell by individual constraints or shocks. As for the proportion s_0, let assume that this quota is equally bought by both types of potential investors (fundamentalists and speculators). However, when $p_t = \widehat{\overline{p}_t} = \overline{P}_I$, then at least one category of shareholders want to sell and the third scenario rejoins the first one.

6.3   The evolution of the shareholding basis

In sum, the dynamic matching of individual wishes to buy or sell impacts on the ongoing composition of the shareholding basis comprising fundamentalists and speculators. In fact, this composition partly depends on the following assumptions:

- the initial composition ($t = 0$) is assumed to be symmetric (half F and half S);

- under the first scenario, if both types of potential investors wish to buy shares, the matching supply is assumed to be symmetric (half F and half S).

Accordingly, the market dynamics endogenously affects the ongoing composition of the shareholding basis over time, depending on the evolution of share market prices and individual expectations on them. The composition of the

shareholding basis and its evolution over time capture the aggregate response to different signals from the market and the firm-specific (non-market) dimension, which shape individual expectations and decisions through time. The following tables resume their dual impact on the shareholding basis under the three distinctive scenarios. In particular Table 1 shows the different cases of the first scenario:

	only F sell	both F and S sell	only S sell
only F buy	no change	$F_{t-1,I} \nearrow S_{t+1,I} \searrow$	all are $F_{t+1,I}$
both buy	$F_{t-1,I} \searrow S_{t+1,I} \nearrow$	no change	$F_{t-1,I} \nearrow S_{t+1,I} \searrow$
only S buy	all are $S_{t+1,I}$	$F_{t-1,I} \searrow S_{t+1,I} \nearrow$	no change

Table 1: *Changes in the shareholding basis under the first scenario*

Whilst Table 2 describes the third scenario, where the only shareholders which sell their shares are those that are forced by constraints and shocks to liquidate their position:

	b sell
only F buy	$F_{t-1,I} \nearrow$
both buy	no change
only S buy	$S_{t+1,I} \nearrow$

Table 2: *Changes in the shareholding basis under the third scenario*

Obviously enough, these tables do not concern the second scenario because no trading arises (and thus no change in the shareholding basis occurs).

6.4 The evolution of aggregate share market price

The Share Exchange is the institutional device that, according to its institutional framework, collectively generates the market price p_t over time. Such price generation may depend either on an exogenous time series (ε_t), or an institutionally designed response to aggregate demand lack or excess (d_t) depending on the clearing of heterogeneous individual expectations described by the three scenarios:

$$p_t = p(p_{t-1}; d_t; \varepsilon_t) \ \forall t \tag{47}$$

Therefore, the evolution of the aggregate market price is determined by the mechanism of supply and demand, and by an exogenous shock ε_t. This means that the collective pricing does not simply result from spontaneous and always perfect matching of individual expectations, since the latter may differ each from another and also be disappointed in some cases. Conversely, this implies individual investors must form their own opinion on aggregate market pricing starting from the composite information set collectively available for financial decision-making.

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From one side, the exogenous shock ε_t may be designed as a stationary or a random walk ($\varepsilon_t = \theta\epsilon_{t-1} + N(0,1)$, respectively with $|\theta| < 1$ or $|\theta| = 1$), a white noise $\varepsilon_t \sim N(0, \sigma_\epsilon^2)$, or another noise having a skewed distribution capturing the asymmetric likelihood of greater shocks. Theoretically speaking, this shock denotes all the influences on the market price evolution that are not included in individual expectations and decisions. In a sense, it relates to the actual degree of stock market efficiency in Fama's terms.

For another side, the endogenous response to the mechanism of demand and supply may be understood as an *auctioneer-like* device calling a price series in search of bettering aggregate clearing and matching of individual demands and supplies over time.[9] This response is described by two joint dummies variables:

$$d_S = \begin{cases} 1 \text{ if the supply is equal to } b \text{ (minimum supply)} \\ 0 \text{ otherwise} \end{cases}$$

$$d_D = \begin{cases} 1 \text{ if the demand is equal to zero} \\ 0 \text{ otherwise.} \end{cases}$$

Accordingly, the resulting evolution of the aggregate market price p_t is

$$p_t = p_{t-1} + \varepsilon d_S - \varepsilon d_D \pm \varepsilon_t \qquad (48)$$
$$\text{with } \varepsilon > \varepsilon_t.$$

Therefore, following the three scenarios:

- in the first scenario, both dummies are equal to zero and the market price at time t is the price of the previous period with a (positive or negative) shock: $p_t = p_{t-1} \pm \varepsilon_t$;

- in the second scenario, no trading arises (because aggregate demand is equal to zero), and consequently the price decreases: $p_t = p_{t-1} - \varepsilon \pm \varepsilon_t$;

- in the third scenario, only the quota b of liquidating shareholders sell its shares; the aggregate supply then is not sufficient, and consequently the price increases: $p_t = p_{t-1} + \varepsilon \pm \varepsilon_t$.

In this way, the Share Exchange has generated a time series of aggregate market prices. This series is jointly influenced by two distinct sources of information: one driven by the market, another generated by other institutional devices such as established accounting processes of reporting and disclosure. This theoretical framework may be applied to analyze different paths of market prices depending on alternative institutional configurations that generate distinctive informational subsets. For this, the following extended model shall refine the market mechanism by introducing a higher degree of heterogeneity among investors.

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[9] The presence of ϵ_t may then result from the working of a drunk auctioneer !

Part II
The extended model for theoretical simulation

7 The extended model

This part extends the basic model to an higher heterogeneity between investors, and simulate numerically some results. The extended model maintains the main assumptions discussed above. The number of investors is now normalized to one and there exists a biunivocal correspondence - period by period - between each investor i and the parameter $\varphi_i \in (0, 1)$ both inside the market (among shareholders) and outside (potential investors). For sake of simplicity, most of the individual heterogeneity is concentrated in this parameter. Then, the generic expectation of an investor i becomes:

$$E_i\left(p_{t+1}\right) = p_t + \alpha\left(p_t - p_{t-1}\right) + \beta_b \varepsilon_i \left|t + \varphi_i\left(F\right)\right. \tag{49}$$

where

$$\varepsilon_i \left|t \equiv E_i\left(p_t\right) - p_t\right. \tag{50}$$

and

$$\beta_b = \left\{ \begin{array}{l} 0 \text{ if investors do not hold shares (potential demand)} \\ \quad \beta \text{ if investors hold shares (potential supply).} \end{array} \right.$$

As in the basic model, $F(\cdot)$ is the collective signal that investors receive by fundamental analysis based upon the firm-specific subset Y_t. This signal concerns the change in the performance generated by the business firm to shareholders, which investors may expect to be positively connected with changes in the share price over longer time horizon.

For sake of simplicity, contrary to the previous formulation (29), we assume that $\alpha_i = \alpha$. Whilst the basic model captured the individual heterogeneity through four classes of investors, the extended model denotes it through the individual parameter φ_i and the parameter β_b. The first parameter represents the degree of confidence on the signal $F(\cdot)$, whilst the second parameter identifies if the investor currently holds shares (and then revises his expectations on price over time), or not (and then he does not have previous expectations to be revised). The four classes of investors described above are then included as a special case with $\varphi_i = 0, 1$. In fact, it is still possible to define four representative investors: two with $\varphi_i = 0$, who are "pure speculators" (inside or outside the market) and do not care of the signal $F(\cdot)$; and two with $\varphi_i = 1$, who are "pure fundamentalists" (inside or outside the market) and attribute the highest confidence to the signal $F(\cdot)$ derived by fundamental analysis.

7.1 Individual mindsets

The mindset of the (previous class of) speculators inside the market (equation 27) corresponds here to that of the "pure speculator" who holds shares, with $\varphi_i = 0$, as follows:

$$E_{\varphi_i=0,I}\left(p_{t+1}\right) = p_t + \alpha\left(p_t - p_{t-1}\right) + \beta\varepsilon_0\left|t\right. \tag{51}$$

where

$$\varepsilon_0\left|t\right. = E_{\varphi_i=0,I}\left(p_t\right) - p_t. \tag{52}$$

Furthermore, the mindset of the (previous class of) speculators outside the market (equation 28) corresponds to that of the "pure speculator" who may potentially demand the shares, with $\varphi_i = 0$ and $\beta = 0$, as follows:

$$E_{\varphi_i=0,O}\left(p_{t+1}\right) = p_t + \alpha\left(p_t - p_{t-1}\right). \tag{53}$$

Moreover, the mindset of the (previous class of) fundamentalists inside the market (equation 23) corresponds to that of the "pure fundamentalist" who holds shares, that is, the shareholder with $\varphi_i = 1$, as follows:

$$E_{\varphi_i=1,I}\left(p_{t+1}\right) = p_t + \alpha\left(p_t - p_{t-1}\right) + \beta\varepsilon_1\left|t\right. + F \tag{54}$$

where

$$\varepsilon_1\left|t\right. = E_{\varphi_i=1,I}\left(p_t\right) - p_t. \tag{55}$$

Finally, the mindset of the (previous class of) speculators outside the market (equation 24) corresponds to the "pure fundamentalist" who may potentially demand shares, with $\varphi_i = 1$ and $\beta = 0$, as follows:

$$E_{\varphi_i=1,O}\left(p_{t+1}\right) = p_t + \alpha\left(p_t - p_{t-1}\right) + F \tag{56}$$

In this way, investors are ranked according to the relative degree of confidence $\varphi_i \in (0,1)$ on the signal $F(\cdot)$. Therefore, individual mindsets remain essentially the same. The main change introduced by the extended model is the continuos whole of investors ranked according to their distinctive φ_i.

7.2 Summing-up

In sum, the expectations of the four representative investors (speculators 0 and investor 1, inside the market I and outside the market O) are:

$$E_{0,O}\left(p_{t+1}\right) = p_t + \alpha\left(p_t - p_{t-1}\right) \equiv P_0 \tag{57}$$
$$E_{0,I}\left(p_{t+1}\right) = P_0 + \beta\varepsilon_0\left|t\right. \tag{58}$$
$$E_{1,O}\left(p_{t+1}\right) = P_0 + (F) \tag{59}$$
$$E_{1,I}\left(p_{t+1}\right) = P_0 + (F) + \beta\varepsilon_1\left|t\right.. \tag{60}$$

The expectation of the generic investor (inside or outside the market) i (φ_i) is

$$E_{\varphi_i,O}\left(p_{t+1}\right) = P_0 + \varphi_i\left(F\right) \tag{61}$$

$$E_{\varphi_i,I}\left(p_{t+1}\right) = P_0 + \varphi_i\left(F\right) + \beta\varepsilon_{\varphi_i}\left|t\right. . \tag{62}$$

For sake of simplicity, let assume that $\varepsilon_{\varphi_i}\left|t\right.$ - defined by equation (50) - is the linear combination of $\varepsilon_0\left|t\right.$ (the error of the pure speculator) and $\varepsilon_1\left|t\right.$ (the error of the pure fundamentalist), as follows:

$$\varepsilon_{\varphi_i}\left|t \equiv \left(1 - \varphi_i\right)\varepsilon_0\right|t + \varphi_i\varepsilon_1\left|t\right. . \tag{63}$$

7.3 Aggregate Demand and Supply

This section replicates the analysis of the demand and the supply side under the extended model. As in the equation (46), let define

$$\overline{P}_i \equiv \max\arg\left(E_{0,i}\left(p_{t+1}\right); E_{1,i}\left(p_{t+1}\right)\right) \tag{64}$$

$$\underline{P}_i \equiv \min\arg\left(E_{0,i}\left(p_{t+1}\right); E_{1,i}\left(p_{t+1}\right)\right)$$

$$\text{where } i = O, I$$

By assuming a linear distribution of investors, the supply S is defined by the following system:[10]

$$S = \left(\begin{array}{c} 0 \ \ \text{if} \ \ p_t \leq \underline{P}_I \\ \frac{p_{t+1} - \overline{P}_I}{\overline{P}_I - \underline{P}_I} \ \ \text{if} \ \ \underline{P}_I < p_t < \overline{P}_I \\ 1 \ \ \text{if} \ \ p_t \geq \overline{P}_I. \end{array} \right. \tag{65}$$

Furthermore, the demand D is defined by the following system:

$$D = \left(\begin{array}{c} 1 \ \ \text{if} \ \ p_t \leq \underline{P}_O \\ \frac{\overline{P}_O - p_{t+1}}{\overline{P}_O - \underline{P}_O} \ \ \text{if} \ \ \underline{P}_O < p_t < \overline{P}_O \\ 0 \ \ \text{if} \ \ p_t \geq \overline{P}_O. \end{array} \right. \tag{66}$$

The clearing price arises when the demand is equal to the supply ($S = D$):

$$S = D \implies \left\{ \begin{array}{c} \frac{p_{t+1} - \overline{P}_I}{\overline{P}_I - \underline{P}_I} = \frac{\overline{P}_O - p_{t+1}}{\overline{P}_O - \underline{P}_O} \ \text{if} \ \max\arg\ \left(\underline{P}_I; \underline{P}_O\right) < p_t < \min\arg\ \left(\overline{P}_I; \overline{P}_O\right) \\ \text{never otherwise}. \end{array} \right.$$

$$\tag{67}$$

The figures 3 and 4 show the matching between aggregate demand and supply. In particular, the figure 4 shows the case when the clearing price exists,

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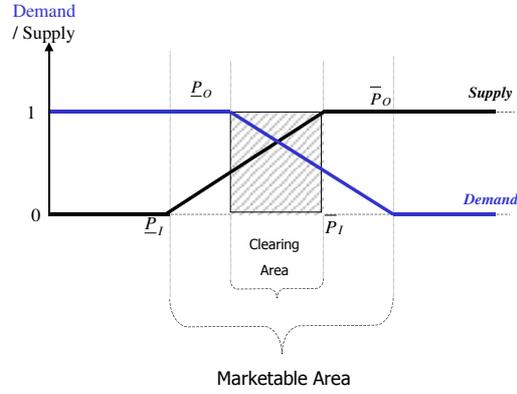[10] The hypothesis of forced selling does not apply to the extended model, that is, $b_t = 0 \ \forall t$.

Figure 4: *Aggregate demand and supply when clearing is possible under the extended model*

whilst figure 5 shows the case when it does not. In the latter case, no share exchanges occur.
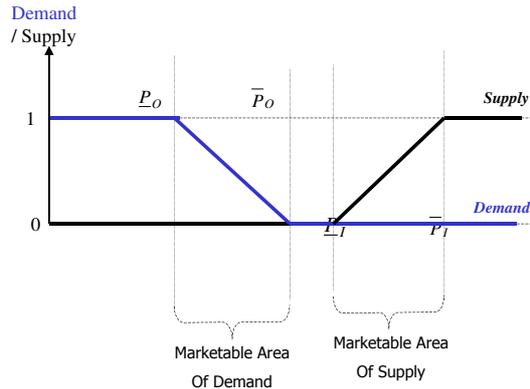

Figure 5: *Aggregate demand and supply when clearing is impossible under the extended model*

By computing, the clearing price is:

$$
p_{t+1}^* = \begin{cases} \dfrac{(\overline{P}_I - \underline{P}_I)\underline{P}_O + (\overline{P}_O - \underline{P}_O)\overline{P}_I}{(\overline{P}_O - \underline{P}_O) + (\overline{P}_I - \underline{P}_I)} & \text{if } \max \arg \ (\underline{P}_I; \underline{P}_O) < p_t < \min \arg \ (\overline{P}_I; \overline{P}_O) \\ 0 & \text{otherwise.} \end{cases}
$$

(68)

The determination of the clearing price passes through the identification of $\overline{P}_I, \underline{P}_I, \overline{P}_O$ and $\underline{P}_O$. In particular, their ranking depends on the sign of the signal $F_t(\cdot)$. In particular, if $F_t > 0$, then:

$$\overline{P}_O = E_{1,O}\left(p_{t+1}\right) = P_0 + (F)$$
$$\overline{P}_I = E_{1,I}\left(p_{t+1}\right) = P_0 + (F) + \beta\varepsilon_1 \left|t\right.$$
$$\underline{P}_O = E_{0,O}\left(p_{t+1}\right) = P_0$$
$$\underline{P}_I = E_{0,I}\left(p_{t+1}\right) = P_0 + \beta\varepsilon_0 \left|t\right. .$$

Otherwise, if $F_t < 0$, then:

$$\overline{P}_O = E_{0,O}\left(p_{t+1}\right) = P_0$$
$$\overline{P}_I = E_{0,I}\left(p_{t+1}\right) = P_0 + \beta\varepsilon_0 \left|t\right.$$
$$\underline{P}_O = E_{1,O}\left(p_{t+1}\right) = P_0 + (F)$$
$$\underline{P}_I = E_{1,I}\left(p_{t+1}\right) = P_0 + (F) + \beta\varepsilon_1 \left|t\right. .$$

In both cases, the clearing price is:

$$p_{t+1}^* = P_0 + \left.\frac{F\left(F + \beta\varepsilon_1\right)}{2F + \beta\varepsilon_1 - \beta\varepsilon_0}\right|_t \tag{69}$$

or, equivalently,

$$p_{t+1}^* = p_t + \alpha\left(p_t - p_{t-1}\right) + \left.\frac{F\left(F + \beta\varepsilon_1\right)}{2F + \beta\varepsilon_1 - \beta\varepsilon_0}\right|_t . \tag{70}$$

8   Theoretical simulation

This economic analysis may improve the theoretical understanding of the share price formation over time. In a further exercise, numerical simulations with montecarlo method and econometric analysis may better estimate and calibrate the structure of the model. This section will briefly investigate two theoretical issues:

- the occurrence of speculative bubbles;

- the impact of different accounting systems providing distinctive subsets of firm-specific information.

8.1   The occurrence of speculative bubbles

The model allows to investigate the theoretical conditions under which a "speculative bubble"[11] may occur and consequently explode. A overvaluation bubble is likely to occur when the market price walk persistently disconnects from the

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[11] Bubbles cannot be defined here on a collective concept of fundamental value that no longer exists. They may be denoted according to the stability and resilience of share price formation over time.

fundamental signal through time on a growing path. In this case, *fundamentalist* shareholders would increasingly sell their shares (after an initial time when they mimic speculators and hold the shares even if they believe that their shares are overvalued), whilst potential *fundamentalists* outside the market would not buy. This dynamics will modify the composition of the shareholding basis and of the participants to the Share Exchange. Only speculators will go on buying and selling the shares, following their mindset based on the price trend and a shorter time horizon. Accordingly, since speculators dominate the market, a small reversal in the price trend will eventually lead to a massive selling of shares by (speculative) shareholders, involving a price fall. At that moment, the market may attract again fundamentalist buyers, who start considering the price as undervalued, following the Rockefeller's adage: "the time to buy is when there is blood in the streets".

8.2 Assessing alternative accounting systems

Furthermore, this framework of analysis may be applied to assessing the relative impact on market price formation over time by alternative accounting systems. In particular, two distinctive accounting systems exist, one based on fair value accounting, another on historical cost accounting (Anthony 2004). They may respectively perform:

- the collective provision of such firm-specific information that follows the information provided by the market (so-called fair value accounting model);

- the collective provision of firm-specific accounting information that constitutes an autonomous source of firm-specific information (so-called historical cost accounting model).

In the first case, the model may be simply calibrated on a signal $F(\cdot)$ that reproduces the relative change in the market price level with one period lag (that is, $\frac{p_{t-1}-p_{t-2}}{p_{t-2}}$) plus a small white error. In the second case, the model may be calibrated on a signal $F(\cdot)$ that constitutes an exogenous time series N(0; 1), that is, a white error with a variance of magnitude ten times inferior to the initial price $p_0 = 10$. To be clear, *no accounting system may avoid the occurrence of "speculative bubbles"*. Nevertheless, the likelihood and magnitude of a price walk far distant from the initial price of reference are greater under the first system (fair value accounting) than under the second system (historical cost accounting), since the price walk becomes auto-referential and does not have any not-market stabilizing device in the first case. Figures 6 and 7 capture this broad message.[12]

In the first case, the market price walk remains quite near to the theoretical level of 10, and shows a lack of clearing price on period 17 (Figure 6).

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[12]For sake of simulation, the "drunk auctionner" is expected to intervene - by announcing the last available market price plus a small white noise - only if the current clearing price does not exist.
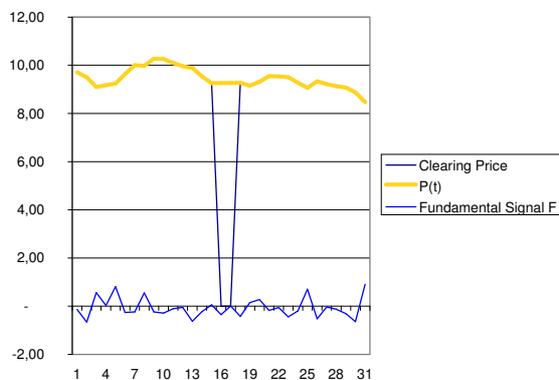
In the second case, the market price walk shows a persistent decreasing pattern until a level of 4, with two periods without clearing price (Figure 7).
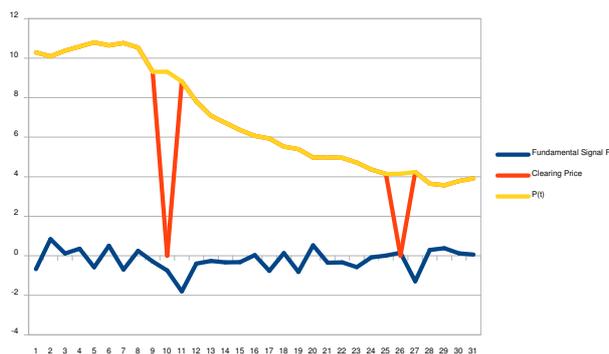

Figure 7: *Numerical simulation under fair value accounting model*

Theoretically speaking, fair value accounting is then expected to involve greater pro-cyclical effects on share market dynamics than historical cost accounting, which provides a stabilizing lighthouse in the market price walk confronted with hazard, learning, and interaction.

9   Conclusion

Responding to the claim by Khotari (2001) for improved theorizing on accounting information and the share market, this paper has developed a novel theoretical framework of analysis for the market price formation over time under a dual informational structure. This approach maintains the focus on the flow of aggregate market prices through time, but it assumes that this flow is jointly influenced by two distinct sources of information: one driven by the market,

another generated by other institutional devices such as established accounting processes of reporting and disclosure. This approach is then concerned with different paths of aggregate market prices depending on alternative institutional configurations, which feature this dual informational structure.

This theoretical perspective is then expected to enhance our understanding of the behavior of market price through time. Accordingly, the paper has provided theoretical simulations to capture this special accounting role in the share price formation over time. The accounting system is then supposed to constitute a lighthouse in the market price walk confronted with through hazard, learning and interaction. Theoretically informed implications and recommendations were provided on the occurrence of speculative bubbles, and the relative impact of alternative accounting systems, distinctively based on fair value or historical cost accounting.

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